Multi-pendulum model (Newton's cradle)

Test Problem

Name:	Pendulum
Description:	Model a pendulum system in SAP2000 using large-displacement time-history analysis.
Program:	SAP2000
Version:	15.1.0
Model ID:	516

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The multi-pendulum model described on this page is primarily an academic exercise, not a typical structural problem, to illustrate that SAP2000 is capable of handling large displacements in time history analysis.

Newton's cradle model

As shown in Figures 1 and 2 below, a Newton's cradle pendulum system is modeled using frame objects and gap (compression-only) links, located at the points of contact. Attached is an animation of the system oscillating.

The time of impact is extremely short compared to the period of oscillation of the pendulum. The propagation of the momentum between the essentially rigid balls (gaps) is a high-speed wave propagation problem, not a typical structural problem. Typically wave-propagation problems are solved using explicit direct-integration methods that are unstable except at very small time steps. This requires some special consideration when choosing the time integration method:

- We normally use implicit direct-integration methods that are unconditionally stable for linear problems, but may not be so for nonlinear problems such as this one, because the problem here is the change from a relatively long time scale to a very short time scale, not a typical structural problem.
- Using Hilber-Hughes-Taylor with alpha<0 with shorter time sub-steps is probably the best solution. It's going to run for a while. A little damping wouldn't hurt.
- If you want to try explicit integration, use Newmark with Beta = 0, and no stiffness proportional damping. This requires extremely small time sub-steps (less than 1/10th the shortest eigen period when the gaps are closed). It runs faster than an implicit method, but will still take quite a while.

Detailed description of the model including the parameters for the time integration method are provided in the attached "analysis notes.pdf" file.

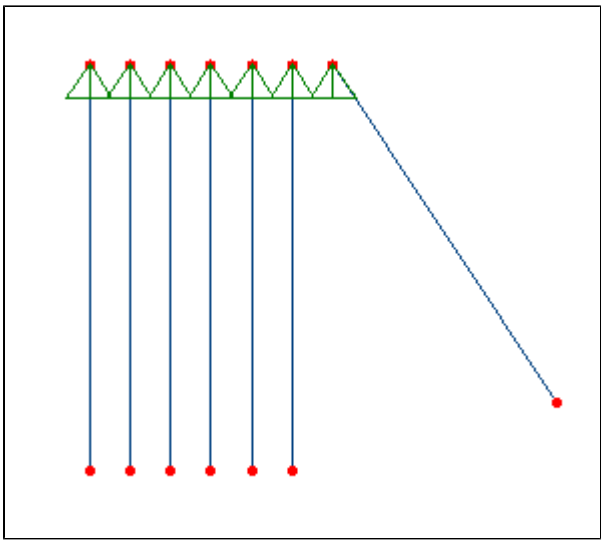

Figure 1 - Newton's cradle pendulum system

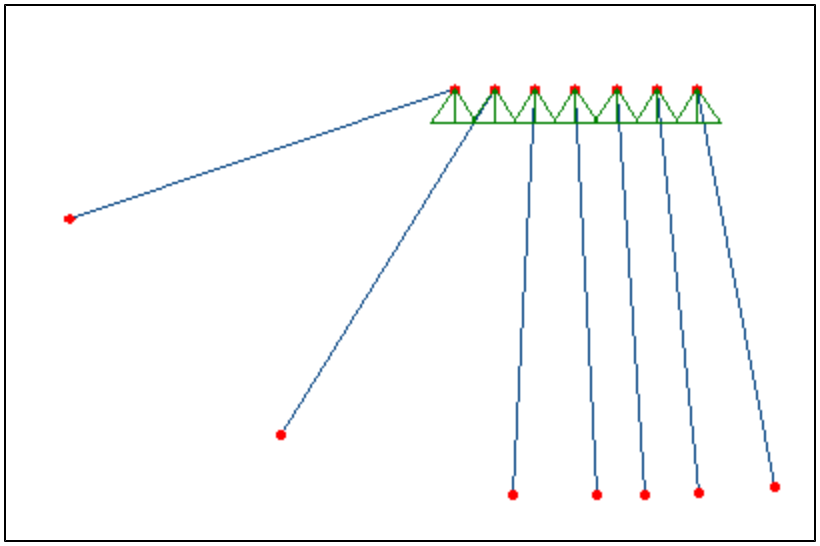

Figure 2 - Newton's cradle pendulum system

General notes on modeling pendulum systems

Pendulum systems may be modeled in SAP2000 using nonlinear time-history analysis with large deflections. Typically, the restoring force comes from a linear spring modeled either as a torsional spring at the support or as a horizontal spring attached to the free end. Pendulum systems may be useful for simulation and solution of isolation and tuned-mass damper problems.

See Also

- Contact surface article

Attachments

- Analysis notes (PDF file)
- SAP2000 V15.1.0 model (zipped SDB file)
- Pendulum video (zipped AVI file)