Ritz vs. Eigen vectors

An overview of **Ritz and Eigen vectors**, taken from the CSI *Analysis Reference Manual* (Modal Analysis > Overview, page 323), is given as follows:

- Eigenvector analysis determines the undamped free-vibration mode shapes and frequencies of the system. These natural modes provide an excellent insight into the behavior of the structure.

- Ritz-vector analysis seeks to find modes that are excited by a particular loading. Ritz vectors can provide a better basis than do Eigenvectors when used for response-spectrum or time-history analyses that are based on modal superposition.

  The user should determine the type of modes which are the most appropriate.

Eigenvectors

Eigen modes are most suitable for determining response from horizontal ground acceleration, though a missing-mass (residual-mass) mode may need to be included to account for missing high-frequency effects. Mass participation is a common measure for determining whether or not there are enough modes, though it does not provide information about localized response.

Eigen analysis is useful for checking behavior and locating problems within the model. Another benefit is that natural frequencies indicate when resonance should be expected under different loading conditions. Users may control the convergence tolerance. Orthogonality is strictly maintained to within the accuracy of the machine (15 decimal digits). Sturm sequence checks are performed and reported to avoid missing Eigen vectors when using shifts. Internal accuracy checks are performed and used to automatically control the solution. Ill-conditioned systems are detected and reported, then still produce Eigen vectors which may be used to trace the source of the modeling problem.

Ritz vectors

Load-dependent Ritz vectors are most suitable for analyses involving vertical ground acceleration, localized machine vibration, and the nonlinear FNA method. Ritz vectors are also efficient and widely used for dynamic analyses involving horizontal ground motion. Their benefit here is that, for the same number of modes, Ritz vectors provide a better participation factor, which enables the analysis to run faster, with the same level of accuracy.

Further, missing-mass modes are automatically included, there is no need to determine whether or not there are enough modes, and when determining convergence of localized response with respect to the number of modes, Ritz vectors converge much faster and more uniformly than do Eigen vectors. Ritz vectors are not subject to convergence questions, though strict orthogonality of vectors is maintained, similar to Eigen vectors.

Sources of documentation on Load-dependent Ritz vectors include:

- Structural-analysis textbooks (Wilson, 2004)

- Finite-element textbooks (Cook et al., 2001)

- Structural-dynamics textbooks (Chopra, 2000)

Tips

- Both Eigen and Ritz modes may be calculated simultaneously, in the same model, and in the same analysis run, such that their behavior may be compared.

- Sources of flexibility available during calculation of Eigen and Ritz modes include:

  - Consideration of P-Delta and other nonlinear effects
  - Modes at different stages of construction
  - Frequency shifts for specialized loading