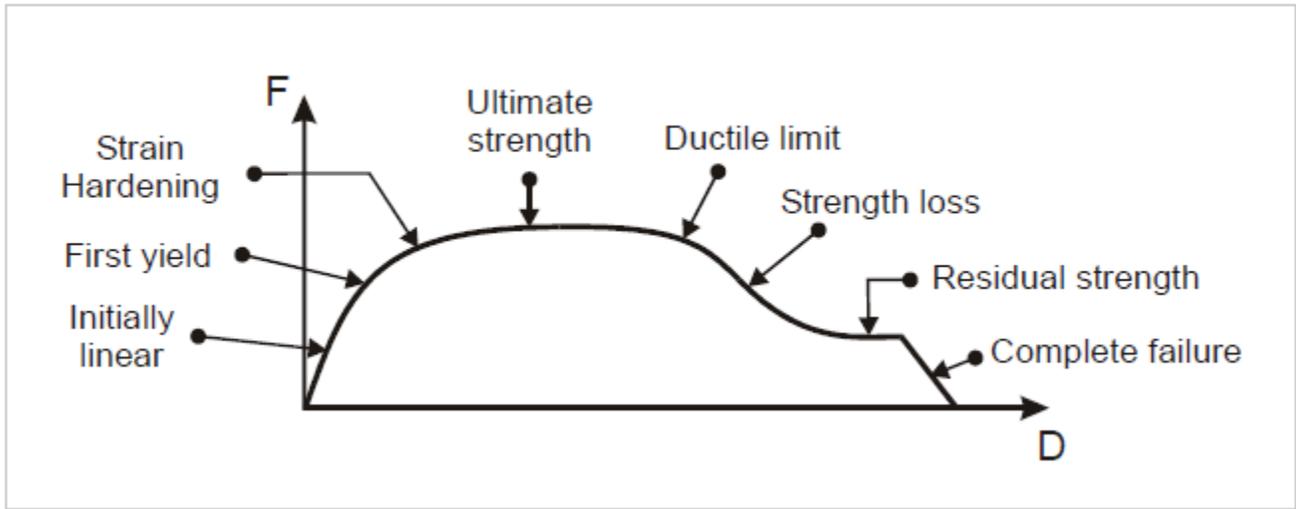# Material nonlinearity

Material nonlinearity is associated with the inelastic behavior of a component or system. Inelastic behavior may be characterized by a force-deformation (F-D) relationship, also known as a backbone curve, which measures strength against translational or rotational deformation. The general F-D relationship shown to the right indicates that once a structure achieves its yield strength, additional loading will cause response to deviate from the initial tangent stiffness (elastic behavior). Nonlinear response may then increase (hardening) to an ultimate point before degrading (softening) to a residual strength value.

A variety of F-D relationships may characterize material nonlinearity, including the following:

- Monotonic curve
- Hysteretic cycle
- Interaction surface

Monotonic curve

A **monotonic curve** is produced when a load pattern is progressively applied to a component or system such that the deformation parameter (independent variable) continuously increases from zero to an ultimate condition. The corresponding force-based parameter (dependent variable) is then plotted across this range, indicating the pattern of material nonlinearity.

Static-pushover analysis is a nonlinear method which generates a monotonic response curve. The P-M2-M3 hinge is best suited for modeling a condition of static pushover. Some examples of monotonic F-D relationships (and their associated physical mechanism) include stress-strain (axial), moment-curvature (flexure), and plastic-hinging (rotation).

To simplify the expression of a monotonic F-D relationship, and to provide for numerically-efficient formulation, the nonlinear curve may be idealized as a series of linear segments. Figure 2 presents one such model. When the general curve is compared with the idealized, it is evident that an exact formulation may be simplified with minimal compromise to accuracy.
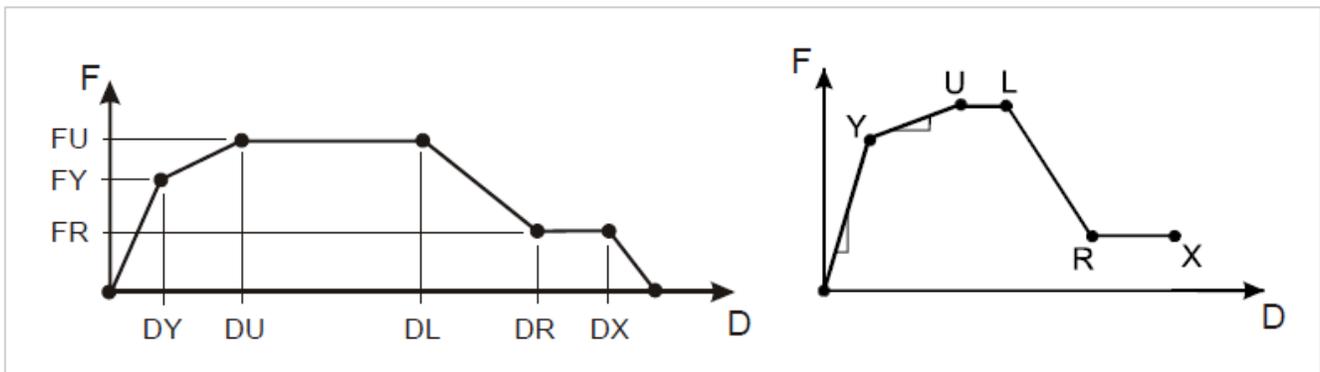

Figure 2 - Idealized monotonic backbone curve

Serviceability parameters may then be superimposed onto the nonlinear F-D relationship to provide insight into structural performance. Property owners and the general public are very much interested in performance measures which relate to daily use. Therefore it may be useful to introduce such **limit states** as immediate-occupancy (IO), life-safety (LS), and collapse-prevention (CP), which indicate the correlation between material nonlinearity and deterministic projections for structural damage sustained. Figure 3 depicts the serviceability limit states of a F-D relationship.
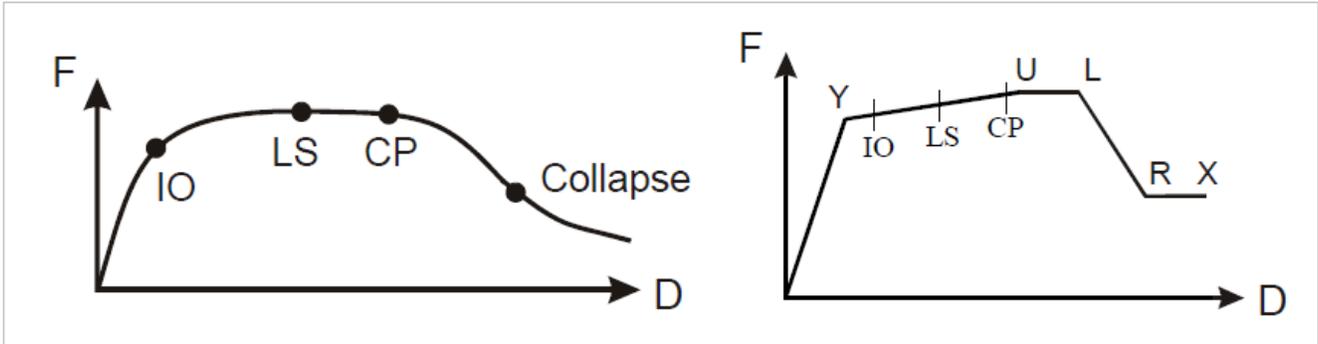

Figure 3 - Serviceability limit states

Limit states may also be specific to inelastic behavioral thresholds. For example, under static pushover, a confined reinforced-concrete column may experience 1). yielding of longitudinal steel; 2). spalling of cover concrete; 3). crushing of core concrete; 4). fracture of transverse reinforcement; and 5). fracture of longitudinal steel.

Hysteretic cycle

Another relationship type which indicates material nonlinearity is the **hysteretic cycle**. When the F-D relationship is developed for a component or system subjected to cyclic loading, hysteretic loops are produced. When modeling hysteretic dynamics, the fiber hinge is best applied.

Figure 4 illustrates hysteretic behavior. Again, translational or rotational deformation is the independent variable. As the orientation of loading continually reverses, a strength-based parameter is plotted against the physical oscillation of the system. Hysteresis is useful for characterizing dynamic response under application of a time-history record.
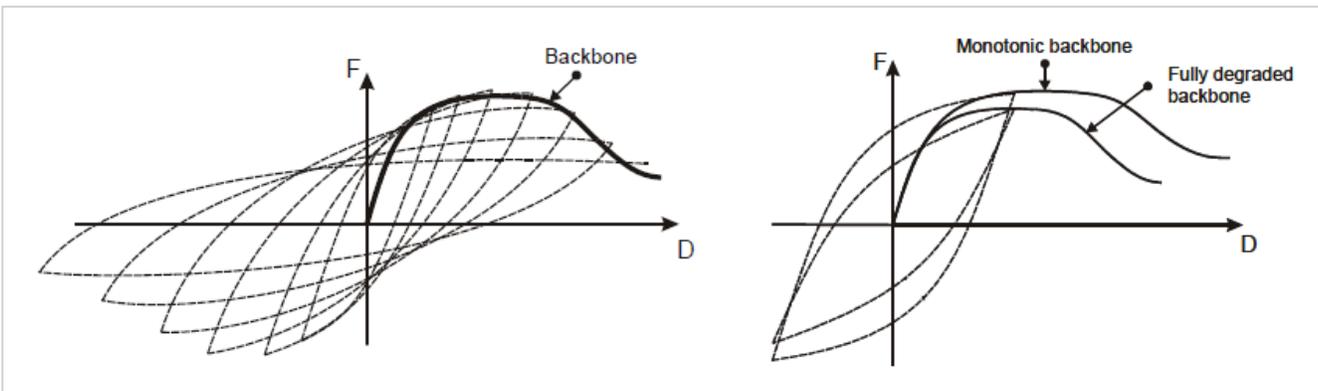

Figure 4 - Hysteresis loop

As seen in Figure 4, both stiffness and strength deviate from their initial relationship once yielding occurs. This behavior advances with additional hysteretic cycles, and becomes more pronounced with greater inelastic deformations. Initially, strength may increase through hardening behavior, though ultimately, stiffness and strength will both degrade through softening behavior. Whereas strength gain or loss is indicated by the strength level achieved, the decrease in slope upon load reversal indicates degradation of stiffness. **Ductility** describes the ability of a system to maintain post-peak strength levels during hysteretic behavior and increasing levels of deformation.

As hysteresis loops develop, the profile of peak values forms the cyclic envelope. The backbone curve produced by the cyclic envelope will be less than the monotonic curve which would result from the same structure being subjected to monotonic loading. This may be attributed to strength and stiffness degradation. An important provision of nonlinear modeling is the accurate characterization of strength and stiffness relationships as a structure progresses through hysteretic behavior. PERFORM-3D is a computational tool which offers this capability.

Depending on structural geometry and materials, a hysteretic cycle may follow one of many different possible patterns. Four possible hysteretic-behavior types are illustrated in Figure 5:
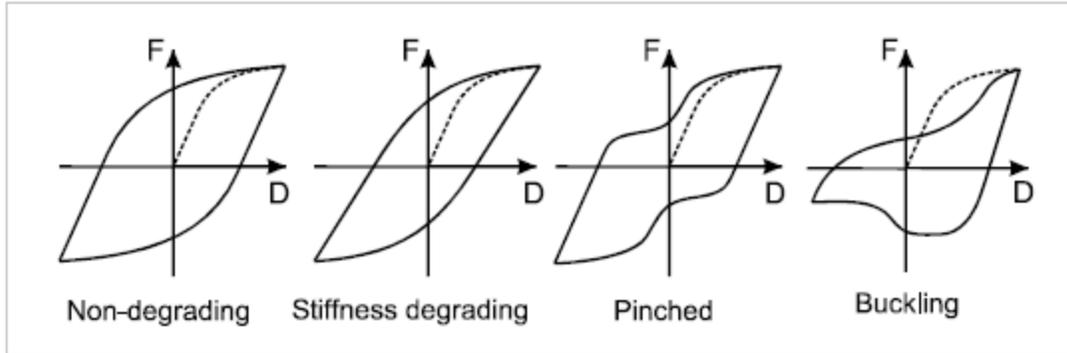

Figure 5 - Hysteresis loop types

Information on plotting hysteresis loops is available in the Plotting link hysteresis article.

Interaction surface

An **interaction surface** is developed for a structural element when the combined relationship between various strength parameters is plotted. Von Mises, Mroz, or another such plasticity theory may be used to develop a 2D or 3D surface which represents a performance envelope for a given limit state. Behavior exceeds the limit state when the performance measure is outside the envelope. An example may be a 3D P-M2-M3 interaction surface describing the yielding of a column under combined axial, strong-axis, and weak-axis bending. These three performance measures interact in a way which may be plotted to create a 3D ellipse. A response measure outside of the P-M-M envelope would indicate that the column has yielded.