Direct-integration time-history analysis

Direct-integration time-history analysis is a nonlinear, dynamic analysis method in which the equilibrium equations of motion are fully integrated as a structure is subjected to dynamic loading. Analysis involves the integration of structural properties and behaviors at a series of time steps which are small relative to loading duration. The equation of motion under evaluation is given as follows:

\[ M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t) \]

**NOTE:** Integration is performed at every time step of the input record, regardless of the output increment.

**Stability conditions**

For well-behaved problems, we recommend using the Newmark method with \( \beta = \frac{1}{4} \), which yields the constant average acceleration method, and for poorly converging nonlinear time-history cases, the Hilber-Hughes-Taylor (HHT) method with \( 0 < \beta < \frac{1}{3} \). During HHT application, when \( \beta = 0 \), formulation is identical to the average acceleration method, so HHT will actually suffice for all problems. Note that all nodes should have mass, and rotational inertia may be added to improve dynamic response.

The Newmark method is unconditionally stable only when \( 2^{1/2} \), which is why we recommend using \( \beta = \frac{1}{4} \). When \( \beta = \frac{1}{6} \), which yields the linear acceleration method, formulation is only conditionally stable, and can become unstable when \( \frac{\delta t}{T} > 2 \frac{3}{\alpha} \), where \( \delta t \) is the time step and \( T \) is the shortest structural period which is excited by loading. The shortest period may be obtained during modal analysis such that time step may be coordinated to ensure convergence.

**See Also**

- Mass and stiffness-proportional damping – Damping of stiff elements with inelastic softening
- Output accuracy – Time-history output-acceleration accuracy
- Ground motion at specific supports – Manual multi-support excitation