Static pushover methods - explanation, comparison and implementation

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This paper reviews and explains a number of static push-over methods, compares them for accuracy, and identifies the most promising method. It must be emphasized in advance that the procedure used to compare the methods is neither detailed nor scientific. Rather, it represents the type of study that a practicing engineer might use to gain confidence in push-over methods. The paper also shows that push-over analysis can give useful sensitivity information for redesign.

Introduction

Static push-over analysis is an attractive tool for performance assessment because it involves less calculation than nonlinear dynamic analysis, and uses a response spectrum rather than a suite of ground accelerograms. Its main weakness is that it uses static analysis to capture dynamic effects, and hence may be inaccurate. Fig. 1 shows the main steps.

There are several static push-over methods, all with the same overall steps but different details. This paper considers the following methods: ATC 40 Capacity Spectrum Method (ATC 1996); FEMA 356 Coefficient Method (FEMA 2000); FEMA 440 Coefficient (or Displacement Modification) Method (FEMA 2005); and FEMA 440 Linearization Method (FEMA 2005).

When nonlinear dynamic analysis is used for performance assessment, it is common to calculate response histories for several (often 7) ground motions, calculate the maximum values of the demand-capacity (D/C) ratios for each motion, average these maximum values, and use these averages for performance assessment. Push-over analysis is less direct. Push-over analysis uses a single response spectrum to represent the several ground motions, and uses this spectrum and the push-over curve to estimate the average of the maximum displacements caused by the earthquakes. The D/C ratios for performance assessment are calculated at this displacement.

Bilinear Approximation of Push-Over Curve

Most push-over methods use a bilinear approximation of the actual push-over curve, for example as shown in Fig. 2. This figure shows the FEMA 356 procedure. The figure also shows a simple single DOF structure. This type of structure is used for the analyses in this paper.
It may be noted that the push-over curve for an actual structure can be plotted using a number of different measures for H and D. Two examples are Base Shear vs. Roof Displacement and Spectral Acceleration vs. Spectral Displacement. Some publications imply that the curve must be in Spectral Acceleration vs. Spectral Displacement, or “ADRS”, form. This is not the case. In the author’s opinion the ADRS is an unnecessary complication.

**Capacity Spectrum, Coefficient and Linearization Methods**

Fig. 3 shows the computational steps for the Capacity Spectrum, Coefficient, and FEMA 440 Linearization Methods. Following are some key points.
Figure 3. Steps for Capacity Spectrum, Coefficient, and FEMA 440 Linearization Methods. For each method follow from Step 1 to Step 5 to see the computation sequence.
1. The Capacity Spectrum and Linearization Methods require a response spectrum family, with spectra for a number of damping ratios. The Coefficient Method uses only one spectrum, usually for 5% damping.

2. The Coefficient and Linearization Methods use empirical formulas that are obtained by calibration against a large number of dynamic analyses. For example, empirical formulas are used in the Coefficient Method for coefficient C1, and in the Linearization Method for the stiffness $K_{eff}$ and damping ratio $\alpha_{eff}$.

3. The Capacity Spectrum Method can be interpreted as an extension of response spectrum analysis for a linear structure, but using the secant period and damping ratio rather than the elastic period and 5% damping. With this interpretation, this method is rational rather than empirical (although the ATC 40 implementation has empirical features). This does not mean, however, that the method is accurate.

4. The most complex step in the computation is the conversion from $S_\alpha$ to $H$. For the simple structure in Fig. 2 the conversion is trivial ($H = M S_\alpha$, where $M = mass$). For a real multi DOF structure it is not so simple, and it is important to apply consistent structural dynamics principles. FEMA 356 is a little weak on this point.

### Cyclic Degradation

Degradation in strength and stiffness can occur under cyclic loading. It is usual to account for strength degradation in the “backbone” curves used for inelastic components. This then affects the shape of the push-over curve. Stiffness degradation is accounted for more directly, by considering the shape of the hysteresis loop for inelastic behavior. Fig. 4 shows three hysteresis loops, two of which have stiffness degradation.

![Figure 4. Hysteresis loops. The loop in Fig. (b) is the basis for the FEMA 440 Linearization method, and essentially also for the FEMA 356 and FEMA 440 Coefficient Methods. There are many other possible loops, for example as in Fig. (c). The shaded bands are the loops for a small amplitude cycle.](image)

One measure of the amount of stiffness degradation is the area of the degraded loop divided by the area of the non-degraded loop. This can be termed the “energy ratio”, since the loop area is the dissipated inelastic energy. The smaller this ratio, the larger the amount of degradation. For the loop in Fig. 4(b) the energy ratio is 0.5 for all ductility ratios if $K_{eff} = 0$. If $K_{eff} > 0$ the ratio increases somewhat (less energy degradation) as the ductility ratio increases. The loop in Fig. 4(c) also has an energy ratio of 0.5. Other ratios can be obtained by changing the details of the loops. It may be noted that the energy ratios are for full amplitude loops. For smaller amplitude loops, shown shaded in Fig. 4, the ratio of the degraded loop area to the area of a comparable loop with no degradation is approximately 0.6 in Fig. 4(b) and 0.85 in Fig. 4(c).

Energy degradation is considered in all of the push-over methods, but they use different procedures, as follows:

1. In the ATC 40 Capacity Spectrum Method, the amount of energy degradation depends on the non-degraded energy and the structure type (Type A, B or C, representing progressively smaller energy ratios). For the analyses in this paper the ATC 40 energy ratios are not used. Instead, the ratio is specified directly as a function of the non-degraded energy. For the loop in Fig. 4(a) the ratio is 1, and for the loops in Figs. 4(b) and 4(c) it is 0.5, for all non-degraded energies.

2. In the FEMA 356 Coefficient Method, the coefficient $C_2$ depends on whether the structure degrades. For a degrading structure, $C_2$ also depends on the performance level (Immediate Occupancy, Life Safety or Collapse Prevention), increasing from the IO to the CP level. In effect, this means that the energy ratio decreases as the ductility ratio increases, but this ratio is not explicitly specified. For the analyses in this paper the ductility ratios are in the LS range, and $C_2$ is calculated for this performance level.

3. The FEMA 440 Coefficient Method is calibrated for the loops in Figs 4(a) and 4(b), with different equations for $C_2$ in the non-degraded and degraded cases. The energy ratio is automatically 0.5 in the degraded case.

4. The FEMA 440 Linearization Method is also calibrated for the loops in Figs 4(a) and 4(b), with separate sets of equations for the non-degraded and degraded cases. The energy ratio is automatically 0.5 in the degraded case.

### Example Structure

Fig. 5 shows a simple structure and four different push-over curves. This structure has been analyzed to assess the accuracy of the various push-over methods.
There are 4 structures (Cases B1, B2, C1 and C2), 4 push-over methods (FEMA 356 Coefficient Method, FEMA 440 Coefficient Method, FEMA 440 Linearization Method, and Capacity Spectrum Method) and 2 levels of cyclic degradation (the hysteresis loops in Figs. 4(a) and 4(b)), for a total of 32 push-over analyses. The displacement demands from these analyses are compared with the displacements from dynamic analyses. A method is accurate if it gives displacement demands that are close to those calculated by dynamic analysis. The comparison procedure is described in the next section. This is not a rigorous procedure, but it is one that an engineer might use to gain confidence in the method for practical performance evaluation.

**Accuracy**

For the dynamic analyses a set of 6 spectrum-matched ground motions is used. These motions are matched to a 5% spectrum of the shape used in FEMA 356 and FEMA 440 (for the period range of interest, $S_a$ is constant to a period $T_o$, then varies inversely with period). The spectra are shown in Fig. 6.

Fig. 6(a) shows that the spectrum matching is close for 5% damping, but not for 20%. For linear dynamic analysis of a single DOF structure, using 5% damping, the 6 motions give similar maximum displacements. For nonlinear dynamic analysis, where the effective damping ratio is larger than 5%, there is substantial scatter. The mean spectra in Fig. 6(b) are used for the pushover analyses. The displacement demands from the push-over analysis are compared with the mean values of the maximum displacements from dynamic analysis.

The period $T_o$ for the 5% spectrum is 0.4 seconds. This is between the 0.3 and 0.45 second periods of the example structures B1 and B2. For the FEMA 440 Coefficient Method, the coefficient C1 depends on the site class (class B, C or D, with progressively increasing C1 values). The spectrum is consistent with site class B, and this class is assumed for the analyses.

**Results and Comparison**

Figure 7 shows the results.
Figure 7. Results for simple structure. The bars show displacements, normalized to the mean of the maximum dynamic displacement for each case (these mean displacements are not the same for all cases).

For each analysis the results are normalized to the mean of the maximum dynamic displacements for the six ground motions. This is the continuous dashed horizontal line in Fig. 7. The standard deviation of the maximum dynamic displacements is shown for each analysis, as a shorter dashed line. The coefficients of variation for the dynamic displacements range from 12.3% to 16.5%, indicating substantial scatter even though the ground motions are spectrum matched. Table 1 summarizes the results.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean of $\Delta_{PO}/\Delta_{DM}$</th>
<th>CV of $\Delta_{PO}/\Delta_{DM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient, FEMA 356</td>
<td>1.14</td>
<td>9.0%</td>
</tr>
<tr>
<td>Coefficient, FEMA 440</td>
<td>1.26</td>
<td>9.8%</td>
</tr>
<tr>
<td>Linearization, FEMA 440</td>
<td>1.07</td>
<td>7.3%</td>
</tr>
<tr>
<td>Capacity Spectrum</td>
<td>1.03</td>
<td>22.9%</td>
</tr>
</tbody>
</table>
Table 1. Summary of Results. $\Delta_{PO}$ = displacement from static push-over analysis. $\Delta_{DM}$ = mean of maximum displacements from dynamic analyses, considering the first 8 analyses in Fig. 7 (i.e., not considering the alternative degradation). The summary results using all 12 analyses are similar. CV = coefficient of variation.

The following are some key points.

1. For a push-over method to be accurate, the mean value of $\Delta_{PO}/\Delta_{DM}$ must be close to 1.0, and the coefficient of variation must be small. On this basis the FEMA 440 Linearization Method is the most accurate.
2. Even though the FEMA 440 Coefficient and Linearization Methods are both calibrated for bilinear push-over curves, these methods give substantially different results for the bilinear cases.
3. The FEMA 440 Coefficient Method is intended to be an improvement over the FEMA 356 Coefficient Method. However, in this study the FEMA 440 method is significantly less accurate than the FEMA 356 method.
4. The Capacity Spectrum Method is inaccurate, with a large coefficient of variation.

Another point is that a small change in the ratio of the area under the actual push-over curve to the area under the bilinear approximation can have a significant effect on the result. For example, for structure C2 and the non-degraded case, 2% change in the area ratio, from 0.99 to 1.01, causes an 8% change in $H_y$. For the Coefficient Method this causes a 4.5% change in the calculated displacement. The Linearization and Capacity Spectrum Methods are less sensitive.

Can the Capacity Spectrum Method be Resurrected?

The author admits to having a soft spot for the Capacity Spectrum Method, largely because it has at least the semblance of a rational basis. However, it is well known that the method is inaccurate. A major reason for the inaccuracy may be that the basic Capacity Spectrum Method uses the maximum displacement to calculate the effective stiffness and damping ratio. Intuitively, it seems that the effective displacement should be smaller than this maximum. Another possible reason is that the effective energy ratios are not necessarily those used for the analysis (1.0 and 0.5, respectively, for the non-degraded and degraded cases). Analyses by the author suggest that the effective energy ratio for the non-degraded case may be smaller than 1.0, and for the degraded case this ratio may be larger than 0.5.

The results of a rough study of a Modified Capacity Spectrum Method are summarized in Table 2. For this study the effective displacement is 0.75 times the distance from the yield displacement to the target displacement, the energy ratio for the non-degraded case is 0.95 rather than 1.0, and the energy ratio for the degraded case is 0.8 rather than 0.5.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean of $\Delta_{PO}/\Delta_{DM}$</th>
<th>CV of $\Delta_{PO}/\Delta_{DM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Capacity Spectrum</td>
<td>1.09</td>
<td>12.2%</td>
</tr>
</tbody>
</table>

Table 2. Summary of Results for Modified Capacity Spectrum Method.

A comparison with Table 1 suggests that this method is comparable in accuracy to the FEMA 356 and FEMA 440 Coefficient Methods. This is by no means a definitive study, but it indicates that the Capacity Spectrum Method can be improved, and perhaps that it should not be consigned to the trash heap.

Sensitivity Information

The steps for the various push-over methods are illustrated in Fig. 3. These diagrams assume that the displacement demand (or target displacement) is determined by iteration. This iterative procedure can be used, but there is an alternative procedure that uses the same calculations and gives additional and useful sensitivity information. This is illustrated in Fig. 8.
Fig 8 shows the push-over (capacity) curve, and also a demand curve. The target displacement is the point where the demand curve intersects the capacity curve. To get the demand curve a number of trial points are selected. For each trial point a bilinear approximation is constructed, as in Fig. 2. In Fig. 8(a) the strength, $H$, of the structure is increased or decreased for each trial point, keeping constant, until the target displacement is equal to the trial displacement. The required change in $H$ can be calculated by iteration. The resulting demand curve shows how the target displacement changes when the strength and stiffness change in the same proportion. In Fig. 8(b), both $H$ and $C$ are increased or decreased in the same proportion (which keeps the stiffness constant), until the target displacement is equal to the trial displacement. The demand curve shows how the target displacement changes when the strength changes keeping the stiffness constant. This sensitivity information is useful for redesign when the target displacement is larger or smaller than the displacement required to satisfy the performance requirements.

**Computer Implementation**

The static and dynamic analyses for this paper were carried out using the Perform-3D computer program (CSI 2006). Version 4 of Perform-3D implements all of the push-over methods described in this paper. The implementation includes the plotting of demand curves using the procedures in Fig. 8, and the calculation of D/C ratios at the target displacement.

**Conclusion**

Based on the (very limited) comparisons in this paper, the FEMA 440 Linearization Method is the most accurate and consistent. This method is based on the work of Iwan and Guyader (FEMA 2005). The FEMA 356 and FEMA 440 Coefficient Methods are significantly less accurate. The basic Capacity Spectrum Method is inaccurate, but a rough study in this paper suggests that a Modified Method could be comparable in accuracy to the Coefficient methods.

An important question is whether push-over analysis is effective as a design tool. For a real building, push-over analysis is reliable only if the building behaves essentially as a single DOF structure. This limits its application to low rise buildings. (A number methods have been proposed for multi-mode push-over analysis, but they are complex and of questionable accuracy.) For most low-rise buildings it requires only modest computational effort to run dynamic analyses, and push-over analysis may not offer much reduction in the overall effort. An exception is a class of low-rise buildings with many shear walls, which are often used for hospitals. Nonlinear analysis models for such structures can be complex, and the cost of dynamic analysis can be large. For most other structures, push-over analysis is definitely useful for preliminary performance evaluation, but final evaluation may best be done using dynamic analysis.

**References**

ATC 1996. ATC 40 : Recommended Methodology for Seismic Evaluation and Retrofit of Existing Concrete Buildings, Applied Technology Council, Redwood City, CA.