okAgenda

Subject: BUCKLING AND P-DELTA EFFECTS

Subtask:

Prepared by: ok   Date: 1/8/2009

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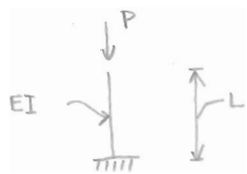
GOAL: Illustrate how P-Delta forces are related to or used in buckling analysis.

MODEL:



Buckling analysis involves the solution of the generalized eigenvalue problem:

$$[\ K - \lambda\ G(r)\ ]\ \Psi = 0$$

Ref.: CSI Analysis Reference Manual, chapter „Analysis Cases", section „Linear Buckling Analysis"

K... stiffness matrix

G(r) . geometric stiffness matrix due to load vector r

λ . unknown eigenvavalues diagonal matrix

Ψ .. matrix of corresponding eigenvectors

For our cantilever problem, the equation can be rewritten as follows:

$$\big[\underbrace{K_e + \lambda K_G}\big]\ \Psi = 0$$

non-trivial solution

to obtain non-trivial solution, this matrix must be singular

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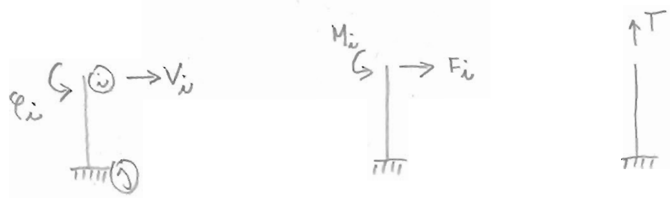
where

$$K_e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L \\ 6L & 4L^2 \end{bmatrix} \qquad \text{note: } \begin{Bmatrix} F_i \\ M_i \end{Bmatrix} = K_e \begin{Bmatrix} v_i \\ \varphi_i \end{Bmatrix}$$

$$K_G = \frac{T}{30L}\begin{bmatrix} 36 & 3L \\ 3L & 4L^2 \end{bmatrix} \qquad \text{note: } \begin{Bmatrix} F_i \\ M_i \end{Bmatrix} = K_G \begin{Bmatrix} v_i \\ \varphi_i \end{Bmatrix}$$

Edward L. Wilson: Static & Dynamic Analysis of Structures, 2004, p. 120, 121



SIGN CONVENTION FOR v_i, φ_i, F_i, M_i and T

Then $$K_t = K_e + \lambda K_G = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L \\ 6L & 4L^2 \end{bmatrix} + \lambda \frac{T}{30L}\begin{bmatrix} 36 & 3L \\ 3L & 4L^2 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{12EI}{L^3} + \frac{6}{5}\frac{1}{L}T\lambda & \frac{6EI}{L^2} + \frac{1}{10}T\lambda \\ \frac{6EI}{L^2} + \frac{1}{10}T\lambda & \frac{4EI}{L} + \frac{2}{15}TL\lambda \end{bmatrix}$$

To obtain nontrivial solution for the eigen value problem, the determinant of the K_t matrix must be equal to zero:

$$\det(k_t) = 0$$

$$\left(\frac{12EI}{L^3} + \frac{6}{5}\frac{T}{L}\lambda\right)\left(\frac{4EI}{L} + \frac{2}{15}LT\lambda\right) - \left(\frac{6EI}{L^2} + \frac{1}{10}T\lambda\right)^2 = 0$$

| solve for unknown λ

$$\frac{48(EI)^2}{L^4} + \frac{24}{15}\frac{EI}{L^2}T\lambda + \frac{24}{5}\frac{EI}{L^2}T\lambda + \frac{12}{75}T^2\lambda^2 -$$

$$- \frac{36(EI)^2}{L^4} - 2\,\frac{6}{10}\frac{EI}{L^2}T\lambda - \frac{1}{100}T^2\lambda^2 = 0$$

$$12\left(\frac{EI}{L^2}\right)^2 + \lambda\frac{EI}{L^2}T\underbrace{\left[\frac{24}{15}+\frac{24}{5}-\frac{12}{10}\right]}_{\frac{48+144-36}{30}=\frac{156}{30}=\frac{26}{5}} + \lambda^2 T^2 \underbrace{\left[\frac{12}{75}-\frac{1}{100}\right]}_{\frac{48-3}{300}=\frac{45}{300}=\frac{3}{20}} = 0$$

$$\underbrace{\frac{3}{20}T^2}_{a}\lambda^2 + \underbrace{\frac{26}{5}\frac{EI}{L^2}T}_{b}\lambda + \underbrace{12\left(\frac{EI}{L^2}\right)^2}_{c} = 0$$

$$ax^2+bx+c=0$$

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

solve this quadratic equation:

$$D = b^2-4ac = \frac{26^2}{5^2}\left(\frac{EI}{L^2}\right)^2T^2 - 4\left(\frac{3}{20}T^2\right)\left(12\left(\frac{EI}{L^2}\right)^2\right) = \left(\frac{EI}{L^2}\right)^2T^2\left(\frac{26^2}{5^2}-\frac{36}{5}\right)$$

$$= \left(\frac{EI}{L^2}\right)^2T^2\,\frac{676-180}{25} = \left(\frac{EI}{L^2}\right)^2T^2\,\frac{496}{25}$$

$$\sqrt{D} = \frac{EI}{L^2}T\,\frac{\sqrt{496}}{5} = \frac{4\sqrt{31}}{5}\frac{EI}{L^2}T$$

$$\lambda_{1,2} = \frac{-\frac{26}{5}\frac{EI}{L^2}T \pm \frac{4\sqrt{31}}{5}\frac{EI}{L^2}T}{2\,\frac{3}{20}T^2} = \frac{\frac{EI}{L^2}T\left(-\frac{26}{5}\pm\frac{4\sqrt{31}}{5}\right)}{\frac{3}{10}T^2} = \frac{EI}{L^2T}\cdot\frac{10}{3}\cdot\frac{-26\pm4\sqrt{31}}{5} =$$

$$= \frac{2}{3}\left(-26\pm4\sqrt{31}\right)\frac{EI}{L^2T} = \frac{4}{3}\left(-13\pm2\sqrt{31}\right)\frac{EI}{L^2T} = \begin{cases} -2.4860\,\dfrac{EI}{L^2T} \\ -32.1807\,\dfrac{EI}{L^2T}\end{cases}$$

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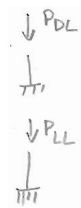
when compressive force P is applied, then

$T = -P$ and $\lambda_1 = 2.4860 \frac{EI}{L^2 P}$ (lowest root)

Accounting for pre-existing P-Delta forces in SAP2000:



Load case 1: DL ... nonlinear with P-Delta effects

Load case 2: LL ... linear buckling using stiffness at the end of case 1

↓

loads from case 1 are <u>NOT</u> included in case 2, but P-Delta effects <u>ARE</u> included

Therefore $P_{cr} = P_{DL} + f\, P_{LL}$

↑ buckling factor determined and reported by SAP2000 for P_{LL}, taking into account the effect of P_{DL} already acting on the structure