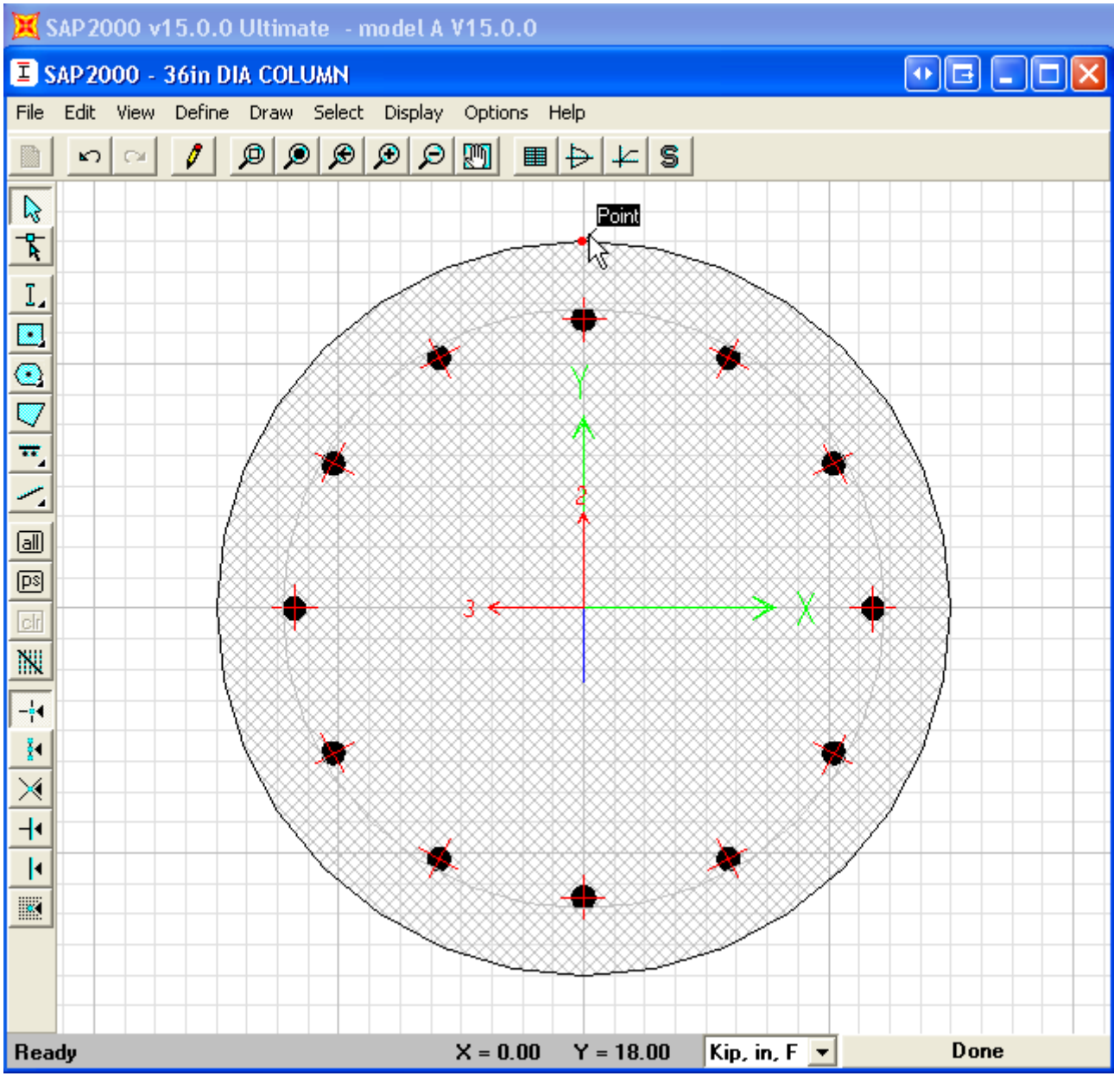
Moment curvature and cracked moment of inertia

- The purpose of this test problem is to explain the meaning of various parameters reported for the moment curvature curve and illustrate how is the cracked moment of inertia calculated.
- The moment curvature analysis is performed for circular column section described in detail in Example 2 of Reference [1].

Cross-Section



Explanation of Moment-Curvature Output Parameters

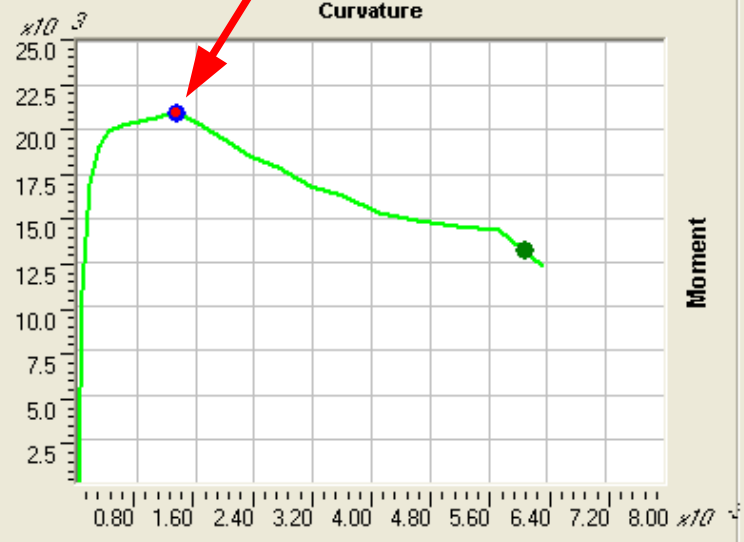
Exact Integration Curve Phi-Conc, M-Conc

- Phi-Conc is the curvature and M-Conc is the moment for which ultimate concrete strain is reached in extreme compressive fiber

SAP2000 v15.0.0 Ultimate - model A V15.0.0

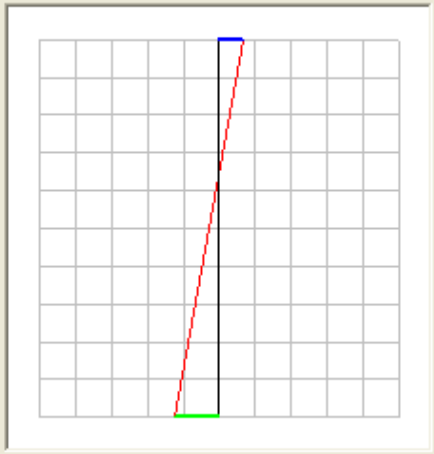
SAP2000 - 36in DIA COLUMN

Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)



Select Type of Graph: Moment-Curvature

Specify Scales/Headings... (1.351E-03 , 20931.10)



Concrete Strain	-0.0151
Steel Strain	0.0283
Neutral Axis	6.7872

Curves

New Curve

Selected Curve Color

Click to: Add Curve, Delete Curve

Plot Exact-Integration Curve

Plot 3x3 Fiber Model Curve

Show Numerical Results for Exact-Integration Curve

Show Numerical Results for Fiber Model Curve

Caltrans Idealized Model

No. of Points: 20

P [Tension +ve]: -1000

Angle (Deg): 0.

Max Curvature: 0

Phi-Conc = .00134155 M-Conc = 20949.858

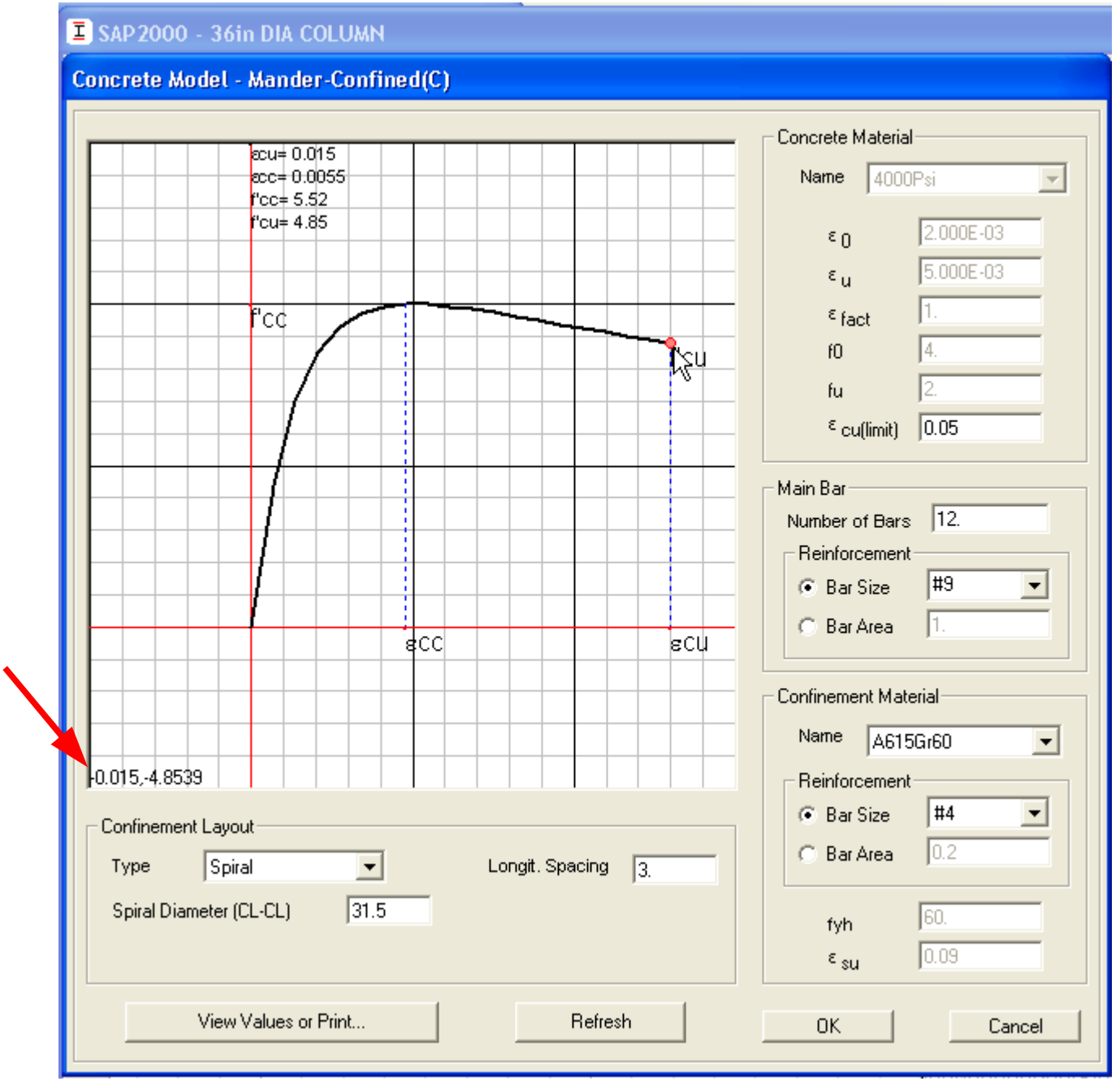
Phi-Steel = .00610152 M-Steel = 13114.275

Analysis Control

- Concrete Failure (Lowest Ultimate Strain)
- Concrete Failure (Highest Ultimate Strain)
- First Rebar/Tendon Failure
- User Defined Curvature

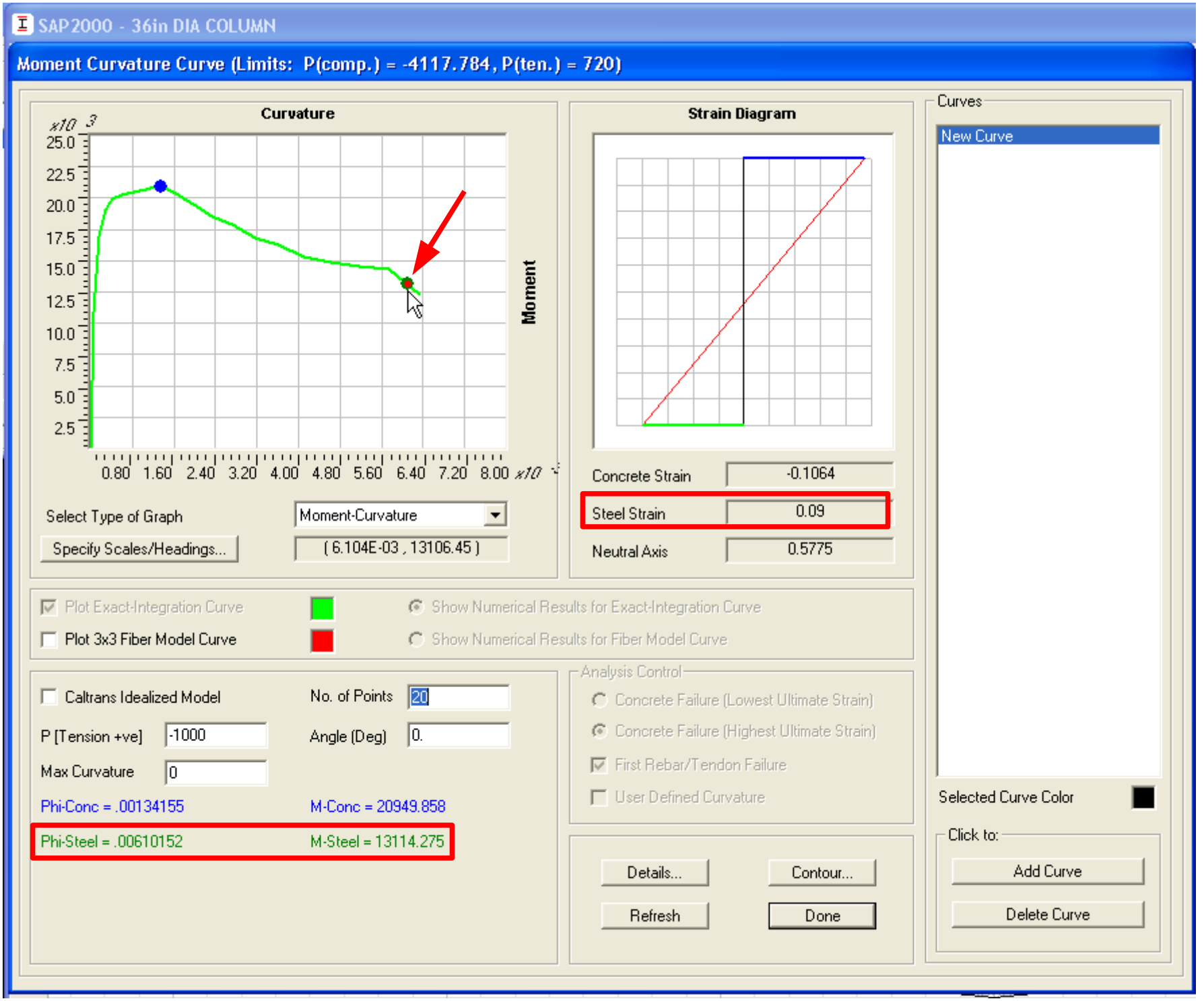
Details... Contour... Refresh Done

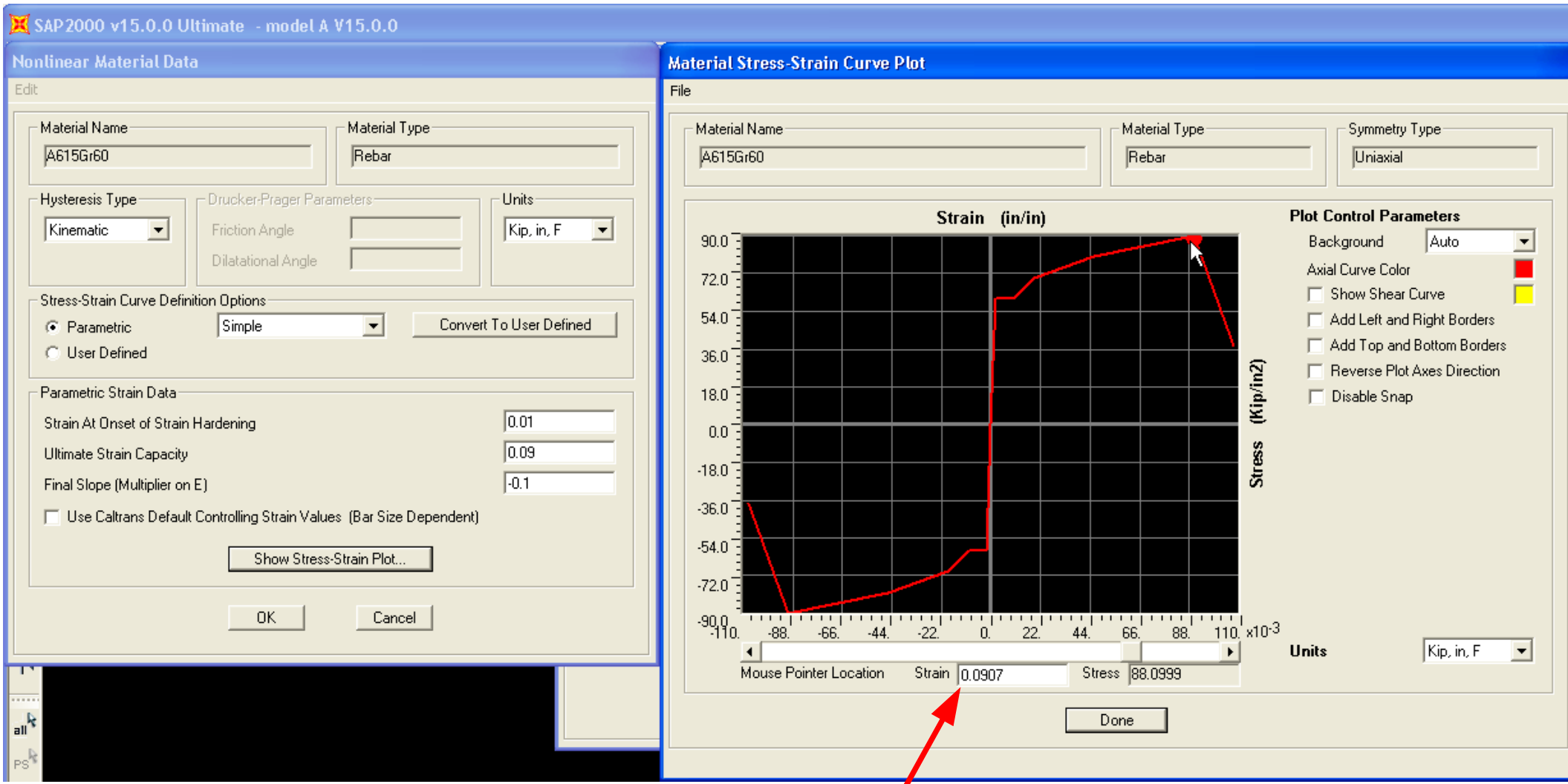
Ready X = -11.38 Y = 20.93 Kip, in, F Done


SAP2000 - 36in DIA COLUMN
Concrete Model - Mander-Confined(C)
εcu= 0.015
εcc= 0.0055
f'cc= 5.52
f'cu= 4.85
f'CC
εCU
εCC
εCU
-0.015,-4.8539
Concrete Material
Name
4000Psi
ε 0
2.000E-03
ε u
5.000E-03
ε fact
1.
f0
4.
fu
2.
ε cu(limit)
0.05
Main Bar
Number of Bars
12.
Reinforcement
Bar Size
#9
Bar Area
1.
Confinement Material
Name
A615Gr60
Reinforcement
Bar Size
#4
Bar Area
0.2
fyh
60.
ε su
0.09
Confinement Layout
Type
Spiral
Longit. Spacing
3.
Spiral Diameter (CL-CL)
31.5
View Values or Print...
Refresh
OK
Cancel

Exact Integration Curve Phi-Steel, M-Steel

- Phi-Steel is the curvature and M-Steel is the moment for which failure reinforcement stress is reached in the any reinforcement bar


SAP2000 - 36in DIA COLUMN
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)
Curvature
x10 3
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.80 1.60 2.40 3.20 4.00 4.80 5.60 6.40 7.20 8.00 x10
Moment
Select Type of Graph
Moment-Curvature
Specify Scales/Headings...
(6.104E-03 , 13106.45)
Strain Diagram
Concrete Strain
-0.1064
Steel Strain
0.09
Neutral Axis
0.5775
Curves
New Curve
Plot Exact-Integration Curve
Plot 3x3 Fiber Model Curve
Show Numerical Results for Exact-Integration Curve
Show Numerical Results for Fiber Model Curve
Caltrans Idealized Model
No. of Points
20
P [Tension +ve]
-1000
Angle (Deg)
0.
Max Curvature
0
Phi-Conc = .00134155
M-Conc = 20949.858
Phi-Steel = .00610152
M-Steel = 13114.275
Analysis Control
Concrete Failure (Lowest Ultimate Strain)
Concrete Failure (Highest Ultimate Strain)
First Rebar/Tendon Failure
User Defined Curvature
Details...
Contour...
Refresh
Done
Selected Curve Color
Click to:
Add Curve
Delete Curve

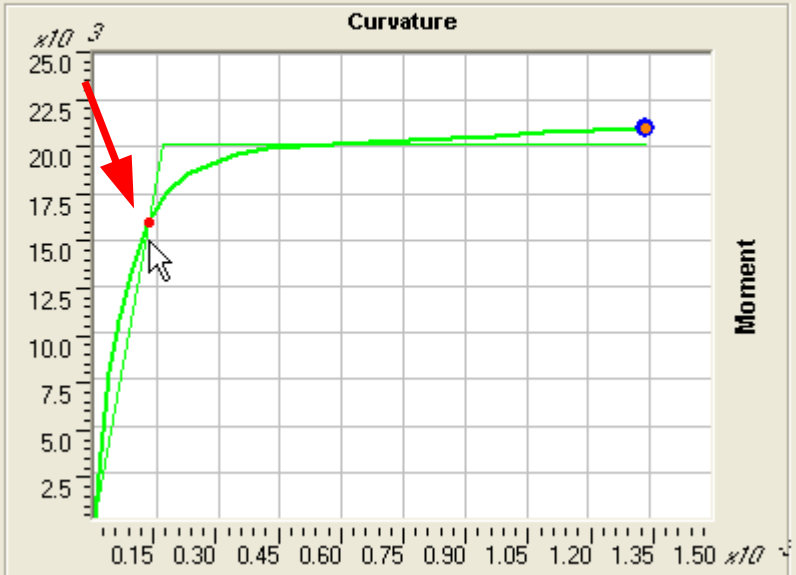

SAP2000 v15.0.0 Ultimate - model A V15.0.0
Nonlinear Material Data
Edit
Material Name
A615Gr60
Material Type
Rebar
Hysteresis Type
Kinematic
Drucker-Prager Parameters
Friction Angle
Dilatational Angle
Units
Kip, in, F
Stress-Strain Curve Definition Options
Parametric
User Defined
Simple
Convert To User Defined
Parametric Strain Data
Strain At Onset of Strain Hardening
0.01
Ultimate Strain Capacity
0.09
Final Slope (Multiplier on E)
-0.1
Use Caltrans Default Controlling Strain Values (Bar Size Dependent)
Show Stress-Strain Plot...
OK
Cancel
Material Stress-Strain Curve Plot
File
Material Name
A615Gr60
Material Type
Rebar
Symmetry Type
Uniaxial
Strain (in/in)
Stress (Kip/in2)
90.0
72.0
54.0
36.0
18.0
0.0
-18.0
-36.0
-54.0
-72.0
-90.0
-110.
-88.
-66.
-44.
-22.
0.
22.
44.
66.
88.
110.
x10-3
Plot Control Parameters
Background
Auto
Axial Curve Color
Show Shear Curve
Add Left and Right Borders
Add Top and Bottom Borders
Reverse Plot Axes Direction
Disable Snap
Units
Kip, in, F
Mouse Pointer Location
Strain
0.0907
Stress
88.0999
Done

Caltrans Model
Phi-yield(initial), M-yield

- Phi-yield(initial) is the curvature and M-yield is the moment for the the first rebar in the cross-section starts to yield

SAP2000 - 36in DIA COLUMN

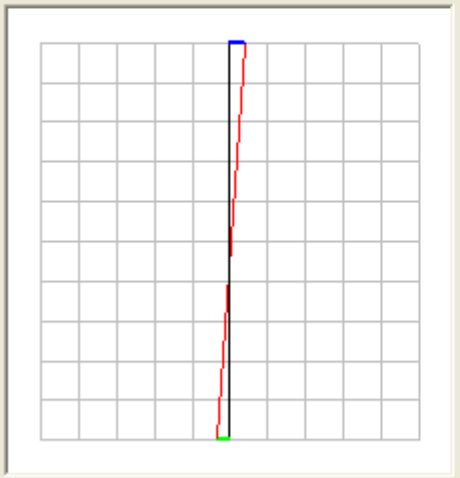
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)



Select Type of Graph: Moment-Curvature

Specify Scales/Headings...

(1.315E-04 , 15919.87)



Concrete Strain	-2.175E-03
Steel Strain	2.058E-03
Neutral Axis	1.4338

Curves

New Curve

Selected Curve Color

Click to:

Add Curve

Delete Curve

Plot Exact-Integration Curve

Show Numerical Results for Exact-Integration Curve

Plot 3x3 Fiber Model Curve

Show Numerical Results for Fiber Model Curve

Caltrans Idealized Model

No. of Points: 20

P [Tension +ve]: -1000

Angle (Deg): 0

Max Curvature: 1.342E-03

Mmax = 20966.92

Phi-Conc = .00134155

M-Conc = 20966.92

Phi-Steel = N/A

M-Steel = N/A

Phi-yield(Initial) = .00013205

M-yield = 15941.855

Phi-yield(Idealized) = .00016702

Mp = 20163

ICrack = 33487.397

Analysis Control

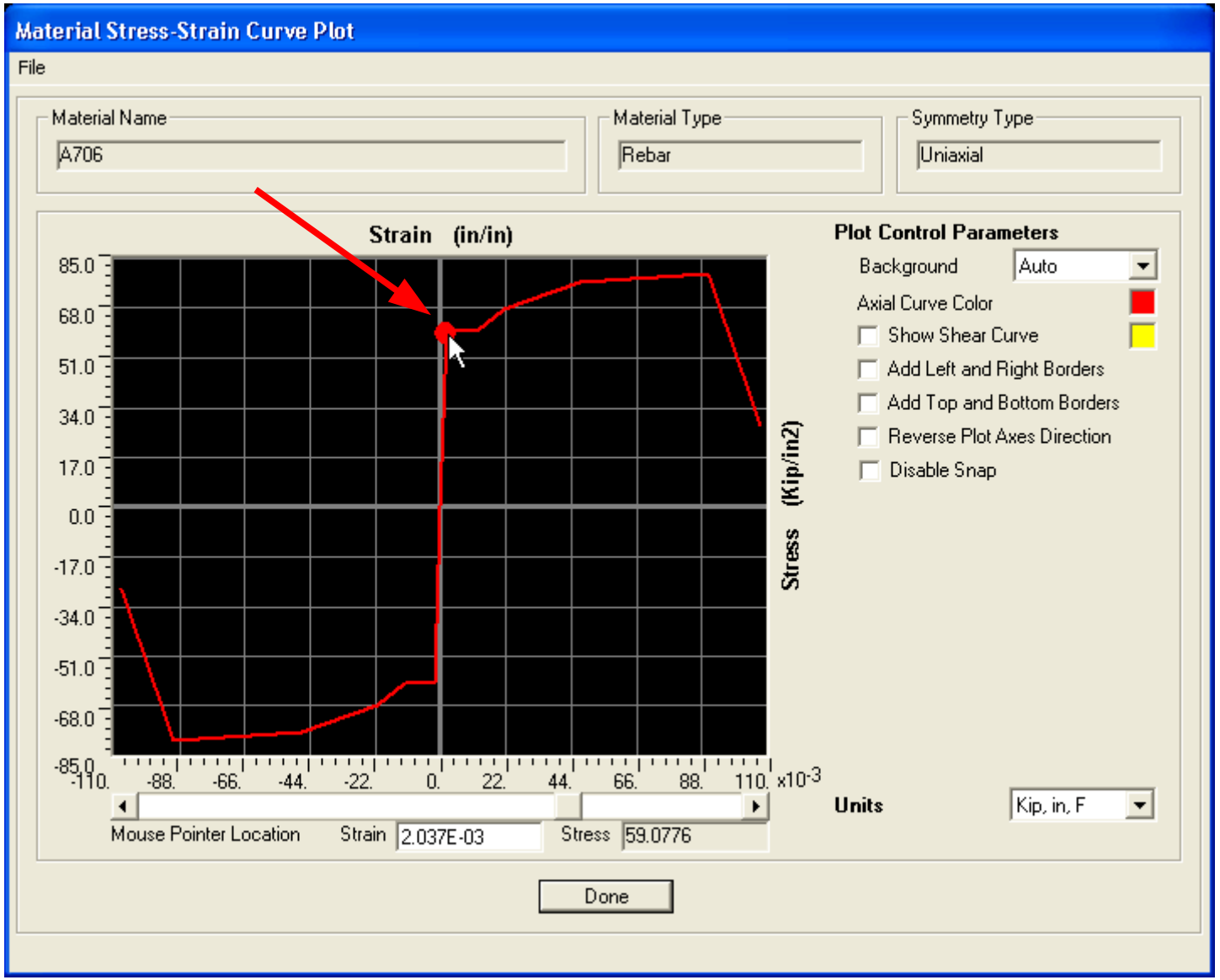
- Concrete Failure (Lowest Ultimate Strain)
- Concrete Failure (Highest Ultimate Strain)
- First Rebar/Tendon Failure
- User Defined Curvature

Details...

Contour...

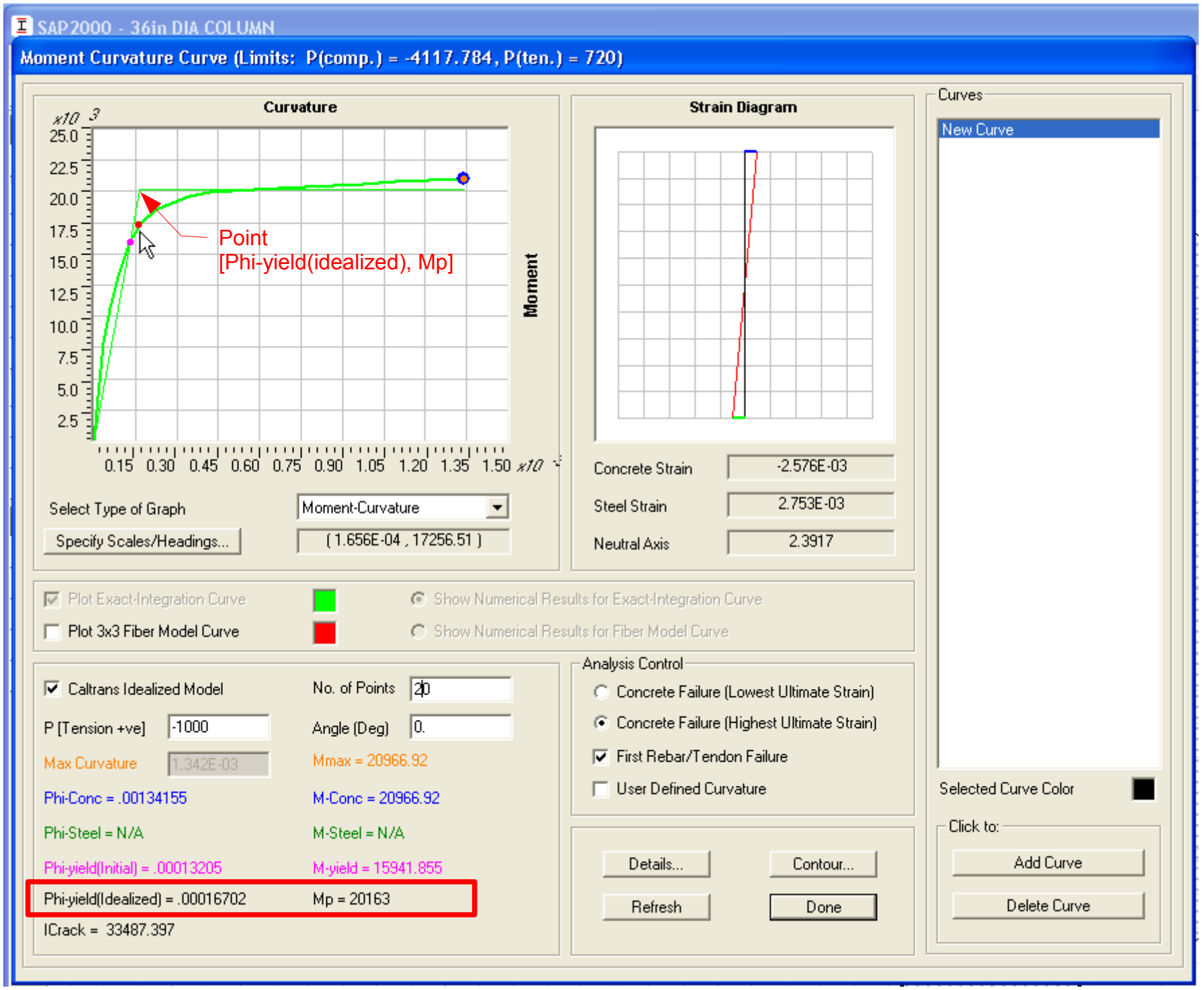
Refresh

Done


Material Stress-Strain Curve Plot
File
Material Name
A706
Material Type
Rebar
Symmetry Type
Uniaxial
Strain (in/in)
85.0
68.0
51.0
34.0
17.0
0.0
-17.0
-34.0
-51.0
-68.0
-85.0
-110.
-88.
-66.
-44.
-22.
0.
22.
44.
66.
88.
110.
x10-3
Stress (Kip/in2)
Plot Control Parameters
Background
Auto
Axial Curve Color
Show Shear Curve
Add Left and Right Borders
Add Top and Bottom Borders
Reverse Plot Axes Direction
Disable Snap
Units
Kip, in, F
Mouse Pointer Location
Strain
2.037E-03
Stress
59.0776
Done

Caltrans Model
Phi-yield(idealized), Mp

- Phi-yield(idealized) is the curvature and Mp is the moment for the initial yield on the Caltrans idealized moment curvature curve.
- The idealized curve is obtained as per Caltrans Seismic Design Criteria [2].


SAP2000 - 36in DIA COLUMN
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)
Curvature
x10 -3
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.15 0.30 0.45 0.60 0.75 0.90 1.05 1.20 1.35 1.50 x10 -3
Moment
Point
[Phi-yield(idealized), Mp]
Select Type of Graph
Moment-Curvature
Specify Scales/Headings...
(1.656E-04 , 17256.51)
Strain Diagram
Concrete Strain
-2.576E-03
Steel Strain
2.753E-03
Neutral Axis
2.3917
Curves
New Curve
Plot Exact-Integration Curve
Show Numerical Results for Exact-Integration Curve
Plot 3x3 Fiber Model Curve
Show Numerical Results for Fiber Model Curve
Caltrans Idealized Model
No. of Points
20
P [Tension +ve]
-1000
Angle (Deg)
0.
Max Curvature
1.342E-03
Mmax = 20966.92
Phi-Conc = .00134155
M-Conc = 20966.92
Phi-Steel = N/A
M-Steel = N/A
Phi-yield(Initial) = .00013205
M-yield = 15941.855
Phi-yield(Idealized) = .00016702
Mp = 20163
ICrack = 33487.397
Analysis Control
Concrete Failure (Lowest Ultimate Strain)
Concrete Failure (Highest Ultimate Strain)
First Rebar/Tendon Failure
User Defined Curvature
Details...
Contour...
Refresh
Done
Selected Curve Color
Click to:
Add Curve
Delete Curve

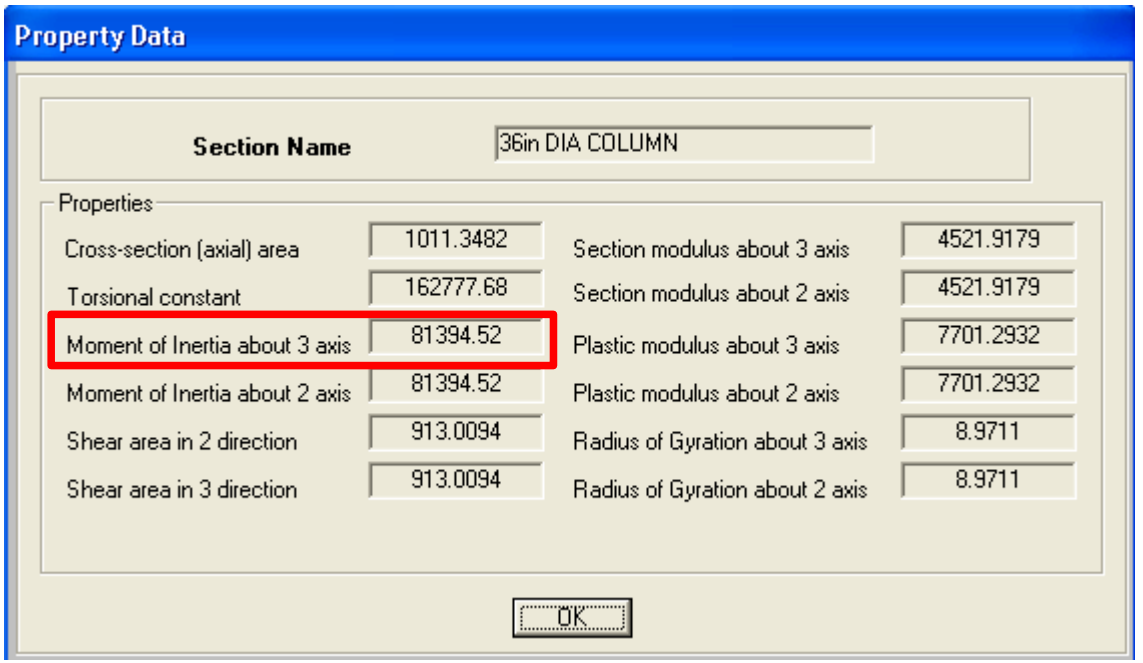
Caltrans Model
Icrack

- Icrack is calculated as follows:

$$I_{crack} = \frac{M_p}{C_p E} = \frac{20163\,\text{kip-in}}{(0.00016702\,\text{rad/in})(3604\,\text{ksi})} = 33483\,\text{in}^4$$

$$I_{crack} = \frac{M_y}{C_y E}$$

- Compare Icrack = 33483 in^4 with gross moment of inertia of 81394 in^4



References

[1] Robert Mathews: Moment Curvature, 2001.
http://www.structsource.com/pdf/Momcurv_web.pdf

[2] Caltrans Seismic Design Criteria, Version 1.4
http://www.dot.ca.gov/hq/esc/techpubs/manual/othermanual/other-engin-manual/seismic-design-criteria/sdc.html