d is the average effective depth of the slab at the critical section
y_c is the horizontal distance from the x-axis to the centroid of the segment

Horizontal segments, ℓ_3 and ℓ_7 in Fig. 13-49, have

$$I_x = I_{x,\text{own}} + Ay_c^2 \qquad \text{(13-36)}$$

where

$$I_{x,\text{own}} = A_3 v_3^2/12$$

$$v_3 = \ell_3$$

and the second term drops out if $y_3 = 0$. (See Fig. 13-55.)

***Inclined segments*:** The angles between the inclined segments, 2, 4, 6, and 8, and the x-axis depend on the relative lengths of the shear reinforcement in the two directions. For segment 2,

$$I_x = I_{x,\text{own}} + Ay_c^2 \qquad \text{(13-36)}$$

where

$$I_{x,\text{own}} = (\ell_2 d)v_2^2/12 \qquad \text{(13-38)}$$

$I_{x,\text{own}}$ is the moment of inertia of segment 2 about its own centroidal axis parallel to the x-axis. The second term on the right-hand side of (13-36) transfers the I_x about the axis of the segment to the centroidal x-axis of the entire section.

***Product of inertia*:**

$$I_{xy} = \Sigma[(\ell_i d)x_i y_i] \qquad \text{(13-39)}$$

summed over all the segments. The sign of I_{xy} depends on the signs of x_i and y_i in the quadrants around the column. For any one segment,

x positive x is to the right
y positive y is down
I_{xy} is the summation, over the cross-sectional area, of the product of the area of the segment $A = (\ell_i d)$ times the x_{ci} and y_{ci} distances from the centroid of the section to the centroid of the elemental area. For a given segment, I_{xy} is negative if the sign of either x or y is negative; that is, I_{xy} is positive if more of the area lies in the quadrants for which the product of the x and y distances is positive. $I_{xy} = 0$ if one or both axes are axes of symmetry.

General stress equation: Calculations of shear stresses using I_x, I_y, I_{xy} and principal axes must use the general stress equation (13-40).

$$v_u = \frac{V_u}{b_o d} + \left(\frac{\gamma_{vx}M_{ux}I_y - \gamma_{vy}M_{uy}I_{xy}}{I_x I_y - I_{xy}^2}\right)y + \left(\frac{\gamma_{vy}M_{uy}I_x - \gamma_{vx}M_{ux}I_{xy}}{I_x I_y - I_{xy}^2}\right)x \qquad \text{(13-40)}$$

where

γ_{vx} and γ_{vy} are the values of γ_v relative to the x- and y-axes, respectively I_x, I_y, and I_{xy} are as defined previously

Substituting this into (13-40) gives (13-23a). If calculations are carried out via the general combined stress equation, (13-40), for edge or corner columns, revised values of γ_v and γ_f should be used. See Ghali [13-21].

ACI Committee 318 did not accept the proposal from Ghali because a comparison of the two calculation procedures to test data for corner columns without shear reinforcement showed that the orthogonal axes method was adequately safe, while the principal axes procedure was quite conservative.

If either of the orthogonal x- and y-axes is an axis of symmetry of the critical section and the moments, M_{ux} and M_{uy}, under consideration are about the x- or y-axis, the product of inertia, I_{xy}, will vanish, and (13-23a) can be written as

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_{vx} M_{ux} y}{I_x} + \frac{\gamma_{vy} M_{uy} x}{I_y} \tag{13-23b}$$

In the *general combined stress equation*, M_{ux} is the factored moment about the x-axis, where $M_{ux} = M_{u1} - M_{u2}$, shown in Fig. 13-50, and M_{uy} has a similar relationship about the y-axis. This is true only if one or both of the x- and y-axes is an axis of symmetry. By definition an axis of symmetry is a principal axis of the critical section. The original derivation of (13-23a) assumed that the x- and y-axes were principal axes, but erred in not specifically requiring them to be such.

The sign convention for directions, forces, and moments must be strictly followed to get the correct shear stresses:

Fig. 13-5
Interactic
moment

x positive x is to the right, when the critical section is viewed in plan

y positive y is down

V positive shear, V, is assumed to act upwards along the axis of the column. (Positive V causes downward stresses on the slab outside the critical section and downwards on the slab between the critical section and the face of the column.)

M_x positive M_x about the x-axis of the critical section causes downward shear stresses on the portion of slab located below the x-axis of the critical section between the column face and the critical section, as shown in Figs. 13-50 and 13-51, and so causes tension at the bottom of the slab at positive y, by the right-hand rule.

M_y positive M_y causes tension at positive x, by the right-hand rule.

I_{xy} is positive if more of the area is in the $+x$, $+y$ quadrant.

Design of Headed Shear Reinforcement in Slabs Subjected to Combined Moment and Shear Transfer from Slab to Column

Design follows the rules presented in the last subsection of Section 13-7. Headed shear studs are placed in rows that extend out from the column until the shear stresses, ϕv_n, due to V and M are less than $2\sqrt{f'_c}$ at the critical section located $d/2$ outside the outermost line of shear studs that surrounds the column. Once again it should be noted that shear studs are not specifically sanctioned by the ACI Code.

Interaction Between Shear and Moment at Edge-Column to Slab Connections

Figure 13-58 is a plot of the moments and shears transferred at failure in a number of test specimens of edge columns in slabs without edge beams [13-26]. The vertical axis represents the ratio of the ultimate moment in a test, M_s, taken about the centroid of the critical shear perimeter, to the nominal moment capacity, M_n, of the flexural reinforcement within a