# Steel Structures Design Manual To AS 4100

First Edition

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## **PREFACE**

\_\_\_\_\_

This book introduces the design of steel structures in accordance with AS 4100, the Australian Standard, in a format suitable for beginners. It also contains guidance and worked examples on some more advanced design problems for which we have been unable to find simple and adequate coverage in existing works to AS 4100.

The book is based on materials developed over many years of teaching undergraduate engineering students, plus some postgraduate work. It follows a logical design sequence from problem formulation through conceptual design, load estimation, structural analysis to member sizing (tension, compression and flexural members and members subjected to combined actions) and the design of bolted and welded connections. Each topic is introduced at a beginner's level suitable for undergraduates and progresses to more advanced topics. We hope that it will prove useful as a textbook in universities, as a self-instruction manual for beginners and as a reference for practitioners.

No attempt has been made to cover every topic of steel design in depth, as a range of excellent reference materials is already available, notably through ASI, the Australian Steel Institute (formerly AISC). The reader is referred to these materials where appropriate in the text. However, we treat some important aspects of steel design, which are either:

- (i) not treated in any books we know of using Australian standards, or
- (ii) treated in a way which we have found difficult to follow, or
- (iii) lacking in straightforward, realistic worked examples to guide the student or inexperienced practitioner.

For convenient reference the main chapters follow the same sequence as AS 4100 except that the design of tension members is introduced before compression members, followed by flexural members, i.e. they are treated in order of increasing complexity. Chapter 3 covers load estimation according to current codes including dead loads, live loads, wind actions, snow and earthquake loads, with worked examples on dynamic loading due to vortex shedding, crane loads and earthquake loading on a lattice tank stand. Chapter 4 gives some examples and diagrams to illustrate and clarify Chapter 4 of AS 4100. Chapter 5 treats the design of tension members including wire ropes, round bars and compound tension members. Chapter 6 deals with compression members including the use of frame buckling analysis to determine the compression member effective length in cases where AS 4100 fails to give a safe design. Chapter 7 treats flexural members, including a simple explanation of criteria for classifying cross sections as fully, partially or laterally restrained, and an example of an I beam with unequal flanges which shows that the approach of AS 4100 does not always give a safe design. Chapter 8 deals with combined actions including examples of (i) in-plane member capacity using plastic analysis, and (ii) a beam-column with a tapered web. In Chapter 9, we discuss various existing models for the design of connections and present examples of some connections not covered in the AISC connection manual. We give step-bystep procedures for connection design, including options for different design cases. Equations are derived where we consider that these will clarify the design rationale.

A basic knowledge of engineering statics and solid mechanics, normally covered in the first two years of an Australian 4-year B.Eng program, is assumed. Structural analysis is treated only briefly at a conceptual level without a lot of mathematical analysis, rather than using the traditional analytical techniques such as double integration, moment area and moment distribution. In our experience, many students get lost in the mathematics with these methods and they are unlikely to use them in practice, where the use of frame analysis software

packages has replaced manual methods. A conceptual grasp of the behaviour of structures under load is necessary to be able to use such packages intelligently, but knowledge of manual analysis methods is not.

To minimise design time, Excel spreadsheets are provided for the selection of member sizes for compression members, flexural members and members subject to combined actions.

The authors would like to acknowledge the contributions of the School of Engineering at Griffith University, which provided financial support, Mr Jim Durack of the University of Southern Queensland, whose distance education study guide for Structural Design strongly influenced the early development of this book, Rimco Building Systems P/L of Arundel, Queensland, who have always made us and our students welcome, Mr Rahul Pandiya a former postgraduate student who prepared many of the figures in AutoCAD, and the Australian Steel Institute.

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# **NOTATION**

The following notation is used in this book. In the cases where there is more than one meaning to a symbol, the correct one will be evident from the context in which it is used.

| $A_{g}$                                   | =           | gross area of a cross-section  |
|---|-------------|--|
| $\mathbf{A}_{\mathrm{n}}$                 | =           | net area of a cross-section  |
| $A_{o}$                                   | =           | plain shank area of a bolt   |
| $A_s$                                     | = =         | tensile stress area of a bolt; or area of a stiffener or stiffeners in contact with a flange   |
| $A_{\rm w}$                               | =           | gross sectional area of a web  |
| $a_e$                                     | =           | minimum distance from the edge of a hole to the edge of a ply measured in the direction of the component of a force plus half the bolt diameter. |
| d   | =           | depth of a section   |
| $\begin{array}{c} d_e \\ d_f \end{array}$ | =<br>=<br>= | effective outside diameter of a circular hollow section<br>diameter of a fastener (bolt or pin); or<br>distance between flange centroids         |
| $d_p$                                     | = =         | clear transverse dimension of a web panel; or<br>depth of deepest web panel in a length  |
| $d_1$                                     | =           | clear depth between flanges ignoring fillets or welds  |
| $d_2$                                     | =           | twice the clear distance from the neutral axes to the compression flange.  |
| E   | =           | Young's modulus of elasticity, 200x10 <sup>3</sup> MPa   |
| e   | =           | eccentricity   |
| F   | =           | action in general, force or load   |
| $f_{ m u}$                                | =           | tensile strength used in design  |
| $f_{ m uf}$                               | =           | minimum tensile strength of a bolt   |
| $f_{ m up}$                               | =           | tensile strength of a ply  |
| $f_{ m uw}$                               | =           | nominal tensile strength of weld metal   |
| $f_{ m y}$                                | =           | yield stress used in design  |
| $f_{ m ys}$                               | =           | yield stress of a stiffener used in design   |
| G   | =           | shear modulus of elasticity, 80x10 <sup>3</sup> MPa; or nominal dead load  |
| I   | =           | second moment of area of a cross-section   |
| $I_{cy}$                                  | =           | second moment of area of compression flange about the section minor principal y- axis  |
|   |             |  |

 $I_m = I$  of the member under consideration

 $I_w$  = warping constant for a cross-section

 $I_x = I$  about the cross-section major principal x-axis

 $I_v = I$  about the cross-section minor principal y-axis

J = torsion constant for a cross-section

k<sub>e</sub> = member effective length factor

 $k_f$  = form factor for members subject to axial compression

 $k_l$  = load height effective length factor

 $k_r$  = effective length factor for restraint against lateral rotation

l = span; or,

= member length; or,

= segment or sub-segment length

 $l_e/r$  = geometrical slenderness ratio

 $l_i$  = length of a bolted lap splice connection

 $M_b$  = nominal member moment capacity

 $M_{bx} = M_b$  about major principal x-axis

 $M_{cx}$  = lesser of  $M_{ix}$  and  $M_{ox}$ 

 $M_o$  = reference elastic buckling moment for a member subject to bending

 $M_{oo}$  = reference elastic buckling moment obtained using  $l_e = l$ 

 $M_{os} = M_{ob}$  for a segment, fully restrained at both ends, unrestrained against lateral rotation and loaded at shear centre

 $M_{ox}$  = nominal out-of-plane member moment capacity about major principal x-axis

 $M_{pr}$  = nominal plastic moment capacity reduced for axial force

 $M_{prx} = M_{pr}$  about major principal x-axis

 $M_{pry} = M_{pr}$  about minor principal y-axis

 $M_{rx} = M_s$  about major principal x-axis reduced by axial force

 $M_{ry} = M_s$  about minor principal y-axis reduced by axial force

M<sub>s</sub> = nominal section moment capacity

 $M_{sx} = M_s$  about major principal x-axis

 $M_{sv} = M_s$  about the minor principal y-axis

 $M_{tx}$  = lesser of  $M_{rx}$  and  $M_{ox}$ 

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M\* = design bending moment

 $N_c$  = nominal member capacity in compression

 $N_{cy} = N_c$  for member buckling about minor principal y-axis

N<sub>om</sub> = elastic flexural buckling load of a member

 $N_{omb} = N_{om}$  for a braced member

 $N_{oms} = N_{om}$  for a sway member

 $N_s$  = nominal section capacity of a compression member; or

= nominal section capacity for axial load

 $N_t$  = nominal section capacity in tension

 $N_{tf}$  = nominal tension capacity of a bolt

N\* = design axial force, tensile or compressive

 $n_{ei}$  = number of effective interfaces

Q = nominal live load

 $R_b$  = nominal bearing capacity of a web

R<sub>bb</sub> = nominal bearing buckling capacity

 $R_{bv}$  = nominal bearing yield capacity

R<sub>sb</sub> = nominal buckling capacity of a stiffened web

 $R_{sy}$  = nominal yield capacity of a stiffened web

r = radius of gyration

 $r_v$  = radius of gyration about minor principle axis.

S = plastic section modulus

s = spacing of stiffeners

 $S_g$  = gauge of bolts

 $S_p$  = staggered pitch of bolts

t = thickness; or

= thickness of thinner part joined; or

= wall thickness of a circular hollow section; or

= thickness of an angle section

 $t_f$  = thickness of a flange

 $t_p$  = thickness of a plate

 $t_s$  = thickness of a stiffener

 $t_{\rm w}$  = thickness of a web

 $t_w$ ,  $t_{w1}$ ,  $t_{w2}$  = size of a fillet weld

 $V_b$  = nominal bearing capacity of a ply or a pin; or

= nominal shear buckling capacity of a web

 $V_f$  = nominal shear capacity of a bolt or pin – strength limit state

 $V_{sf}$  = nominal shear capacity of a bolt – serviceability limit state

V<sub>u</sub> = nominal shear capacity of a web with a uniform shear stress distribution

 $V_v$  = nominal shear capacity of a web

 $V_{vm}$  = nominal web shear capacity in the presence of bending moment

V<sub>w</sub> = nominal shear yield capacity of a web; or

= nominal shear capacity of a pug or slot weld

V\* = design shear force

 $V_b^*$  = design bearing force on a ply at a bolt or pin location

 $V_{f}^{*}$  = design shear force on a bolt or a pin – strength limit state

 $V_{w}^{*}$  = design shear force acting on a web panel

 $y_0$  = coordinate of shear centre

Z = elastic section modulus

 $Z_c = Z_e$  for a compact section

 $Z_e$  = effective section modulus

 $\alpha_b$  = compression member section constant

 $\alpha_c$  = compression member slenderness reduction factor

 $\alpha_{\rm m}$  = moment modification factor for bending

 $\alpha_s$  = slenderness reduction factor.

 $\alpha_{\rm v}$  = shear buckling coefficient for a web

 $\beta_e$  = modifying factor to account for conditions at the far ends of beam

members

 $\xi$  = compression member factor defined in Clause 6.3.3 of AS 4100

 $\eta$  = compression member imperfection factor defined in Clause 6.3.3 of AS 4100

 $\lambda$  = slenderness ratio

 $\lambda_e$  = plate element slenderness

 $\lambda_{\rm ed}$  = plate element deformation slenderness limit

 $\lambda_{ep}$  = plate element plasticity slenderness limit

 $\lambda_{ey}$  = plate element yield slenderness limit

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 $\lambda_n$  = modified compression member slenderness

 $\lambda_s$  = section slenderness

 $\lambda_{sp}$  = section plasticity slenderness limit

 $\lambda_{sy} \quad \ = \quad \quad section \ yield \ slenderness \ limit$ 

v = Poisson's ratio, 0.25

 $\vartheta \qquad = \qquad I_{cy}/I_y$ 

 $\phi$  = capacity factor

# 1 INTRODUCTION: THE STRUCTURAL DESIGN PROCESS

#### 1.1 PROBLEM FORMULATION

Before starting to design a structure it is important to clarify what purpose it is to serve. This may seem so obvious that it need not be stated, but consider for example a building, e.g. a factory, a house, hotel, office block etc. These are among the most common structures that a structural engineer will be required to design. Basically a building is a box-like structure, which encloses space.

Why enclose the space? To protect people or goods? From what? Burglary? Heat? Cold? Rain? Sun? Wind? In some situations it may be an advantage to let the sun shine in the windows in winter and the wind blow through in summer (Figure 1.1). These considerations will affect the design.

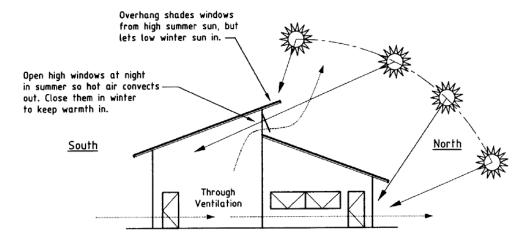


Figure 1.1 Design to use sun, wind and convection

How much space needs to be enclosed, and in what layout? Should it be all on ground level for easy access? Or is space at a premium, in which case multi-storey may be justified (Figure 1.2). How should the various parts of a building be laid out for maximum convenience? Does the owner want to make a bold statement or blend in with the surroundings?

The site must be assessed: what sort of material will the structure be built on? What local government regulations may affect the design? Are cyclones, earthquakes or snow loads likely? Is the environment corrosive?

#### 1.2 CONCEPTUAL DESIGN

Architects rather than engineers are usually responsible for the problem formulation and conceptual design stages of buildings other than purely functional industrial buildings. However structural engineers are responsible for these stages in the case of other industrial

#### 2 Introduction

structures, and should be aware of the issues involved in these early stages of designing buildings. Engineers sometimes accuse architects of designing weird structures that are not sensible from a structural point of view, while architects in return accuse structural engineers of being concerned only with structural issues and ignoring aesthetics and comfort of occupiers. If the two professions understand each other's points of view it makes for more efficient, harmonious work.





Figure 1.2 Low industrial building and high rise hotel

The following decisions need to be made:

- 1. Who is responsible for which decisions?
- 2. What is the basis for payment for work done?
- 3. What materials should be used for economy, strength, appearance, thermal and sound insulation, fire protection, durability? The architect may have definite ideas about what materials will harmonise with the environment, but it is the engineer who must assess their functional suitability.
- 4. What loads will the structure be subjected to? Heavy floor loads? Cyclones? Snow? Earthquakes? Dynamic loads from vibrating machinery? These questions are firmly in the engineer's territory.

Besides buildings, other types of structure are required for various purposes, for example to hold something vertically above the ground, such as power lines, microwave dishes, wind turbines or header tanks. Bridges must span horizontally between supports. Marine structures such as jetties and oil platforms have to resist current and wave forces. Then there are moving steel structures including ships, trucks and railway rolling stock, all of which are subjected to dynamic loads.

Once the designer has a clear idea of the purpose of the structure, he or she can start to propose conceptual designs. These will usually be based on some existing structure, modified to suit the particular application. So the more you notice structures around you in everyday life the better equipped you will be to generate a range of possible conceptual designs from which the most appropriate can be selected.

For example a tower might be in the form of a free standing cantilever pole, or a guyed pole, or a free-standing lattice (Figure 1.3). Which is best? It depends on the particular application. Likewise there are many types of bridges, many types of building, and so on.





Figure 1.3 Towers Left: "Tower of Terror" tube cantilever at Dream World theme park, Gold Coast. Right: bolted angle lattice transmission tower.

#### 1.3 CHOICE OF MATERIALS

Steel is roughly three times more dense than concrete, but for a given load-carrying capacity, it is roughly 1/3 as heavy, 1/10 the volume and 4 times as expensive. Therefore concrete is usually preferred for structures in which the dead load (the load due to the weight of the structure itself) does not dominate, for example walls, floor slabs on the ground and suspended slabs with a short span. Concrete is also preferred where heat and sound insulation are required. Steel is generally preferable to concrete for long span roofs and bridges, tall towers and moving structures where weight is a penalty. In extreme cases where weight is to be minimised, the designer may consider aluminium, magnesium alloy or FRP (fibre reinforced plastics, e.g. fibreglass and carbon fibre). However these materials are much more expensive again. The designer must make a rational choice between the available materials, usually but not always on the basis of cost.

Although this book is about steel structures, steel is often used with concrete, not only in the form of reinforcing rods, but also in composite construction where steel beams support concrete slabs and are connected by shear studs so steel and concrete behave as a single structural unit (Figs.1.4, 1.5). Thus the study of steel structures cannot be entirely separated from concrete structures.

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Figure 1.4 Steel bridge structure supporting concrete deck, Adelaide Hills



Figure 1.5 Composite construction: steel beams supporting concrete slab in Sydney Airport car park

#### 1.4 ESTIMATION OF LOADS (STRUCTURAL DESIGN ACTIONS)

Having decided on the overall form of the structure (e.g. single level industrial building, high rise apartment block, truss bridge, etc.) and its location (e.g. exposed coast, central business district, shielded from wind to some extent by other buildings, etc.), we can then start to estimate what loads will act on the structure. The former SAA Loading code AS 1170 has now been replaced by AS/NZS 1170, which refers to loads as "structural design actions." The main categories of loading are dead, live, wind, earthquake and snow loads. These will be discussed in more detail in Chapter 2. A brief overview is given below.

**1.4.1 Dead loads or permanent actions** (the permanent weight of the structure itself). These can be estimated fairly accurately once member sizes are known, but these can only be determined after the analysis stage, so some educated guesswork is needed here, and numbers may have to be adjusted later and re-checked. This gets easier with experience.

- **1.4.2 Live loads (imposed actions)** are loads due to people, traffic etc. that come and go. Although these do not depend on member cross sections, they are less easy to estimate and we usually use guidelines set out in the Loading Code AS 1170.1
- **1.4.3 Wind loads (wind actions)** will come next. These depend on the geographical region whether it is subject to cyclones or not, the local terrain open or sheltered, and the structure height.
- **1.4.4 Earthquake and snow loads** can be ignored for some structures in most parts of Australia, but it is important to be able to judge when they must be taken into account.
- 1.4.5 Load combinations (combinations of actions). Having estimated the maximum loads we expect could act on the structure, we then have to decide what load combinations could act at the same time. For example dead and live load can act together, but we are unlikely to have live load due to people on a roof at the same time as the building is hit by a cyclone. Likewise, wind can blow from any direction, but not from more than one direction at the same time. Learners sometimes make the mistake of taking the most critical wind load case for each face of a building and applying them all at the same time. If we are using the limit state approach to design, we will also apply load factors in case the loads are a bit worse than we estimated. We can then arrive at our design loads (actions).

#### 1.5 STRUCTURAL ANALYSIS

Once we know the shape and size of the structure and the loads that may act on it, we can then analyse the effects of these loads to find the maximum load effects (action effects), i.e. axial force, shear force, bending moment and sometimes torque on each member. Basic analysis of statically determinate structures can be done using the methods of engineering statics, but statically indeterminate structures require more advanced methods. Before desktop computers and structural analysis software became generally available, methods such as moment distribution were necessary. These are laborious and no longer necessary, since computer software can now do the job much more quickly and efficiently. An introduction to one package, Spacegass, is provided in this book. However it is crucial that the designer understands the concepts and can distinguish a reasonable output from a ridiculous output, which indicates a mistake in data input.

#### 1.6 MEMBER SIZING, CONNECTIONS AND DOCUMENTATION

After the analysis has been done, we can do the **detailed design** – deciding what cross section each member should have in order to be able to withstand the design axial forces, shear forces and bending moments. The principles of solid mechanics or stress analysis are used in this stage. As mentioned above, dead loads will depend on the trial sections initially assumed, and if the actual member sections differ significantly from those originally assumed it will be necessary to adjust the dead load and repeat the analysis and member sizing steps.

We also have to design **connections:** a structure is only as strong as its weakest link and there is no point having a lot of strong beams and columns etc that are not joined together properly.

Finally, we must **document** our design, i.e. provide enough information so someone can build it. In the past, engineers generally provided dimensioned sketches from which draftsmen prepared the final drawings. But increasingly engineers are expected to be able to prepare their own CAD drawings.

## 2 STEEL PROPERTIES

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#### 2.1 INTRODUCTION

To design effectively it is necessary to know something about the properties of the material. The main properties of steel, which are of importance to the structural designer, are summarised in this chapter.

#### 2.2 STRENGTH, STIFFNESS AND DENSITY

Steel is the strongest, stiffest and densest of the common building materials. Spring steels can have ultimate tensile strengths of 2000 MPa or more, but normal structural steels have tensile and compressive yield strengths in the range 250-500 MPa, about 8 times higher than the compressive strength and over 100 times the tensile strength of normal concrete. Tempered structural aluminium alloys have yield strengths around 250 MPa, similar to the lowest grades of structural steel.

Although yield strength is an important characteristic in determining the load carrying capacity of a structural element, the elastic modulus or Young's modulus E, a measure of the stiffness or stress per unit strain of a material, is also important when buckling is a factor, since buckling load is a function of E, not of strength. E is about 200 GPa for carbon steels, including all structural steels except stainless steels, which are about 5% lower. This is about 3 times that of Aluminium and 5-8 times that of concrete. Thus increasing the yield strength or grade of a structural steel will not increase its buckling capacity.

The specific gravity of steel is 7.8, i.e. its mass is about 7.8 tonnes/m³, about three times that of concrete and aluminium. This gives it a strength to weight ratio higher than concrete but lower than structural aluminium.

#### 2.3 DUCTILITY

Structural steels are ductile at normal temperatures under normal conditions. This property has two important implications for design. First, high local stresses due to concentrated loads or stress raisers (e.g. holes, cracks, sudden changes of cross section) are not usually a major problem as they are with high strength steels, because ductile steels can yield locally and relive these high stresses. Some design procedures rely on this ductile behaviour. Secondly, ductile materials have high "toughness," meaning that they can absorb energy by plastic deformation so as not to fail in a sudden catastrophic manner, for example during an earthquake. So it is important to ensure that ductile behaviour is maintained.

The factors affecting brittle fracture strength are as follows:

- (1) Steel composition, including grain size of microscopic steel structures, and the steel temperature history.
- (2) Temperature of the steel in service.
- (3) Plate thickness of the steel.
- (4) Steel strain history (cold working, fatigue etc.)
- (5) Rate of strain in service (speed of loading).
- (6) Internal stress due to welding contraction.

In general slow cooling of the steel causes grain growth and a reduction in the steel toughness, increasing the possibility of brittle fracture. Residual stresses, resulting from the manufacturing process, reduce the fracture strength, whilst service temperatures influence whether the steel will fail in brittle or ductile manner.

#### 2.3.1 Metallurgy and transition temperature

Every steel undergoes a transition from ductile behaviour (high energy absorption, i.e. toughness) to brittle behaviour (low energy absorption) as its temperatures falls, but this transition occurs at different temperatures for different steels, as shown in Fig.2.1 below. For low temperature applications L0 (guaranteed "notch ductile" down to 0°C) or L15 (ductile down to -15°C) should be specified.

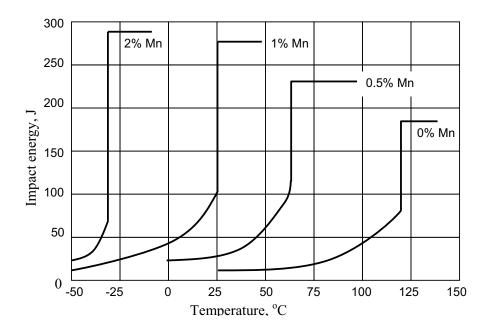


Figure 2.1 Impact energy absorption capacity and ductile to brittle transition temperatures of steels as a function of manganese content (adapted from Metals Handbook [1])

#### 2.3.2 Stress effects

Ductile steel normally fails by shearing or slipping along planes in the metal lattice. Tensile stress in one direction implies shear stress on planes inclined to the direction of the applied stress, as shown in Fig.2.2, and this can be seen in the necking that occurs in the familiar tensile test specimen just prior to failure. However if equal tensile stress is applied in all three principal directions the Mohr's circle becomes a dot on the tension axis and there is no shear stress to produce slipping. But there is a lot of strain energy bound up in the material, so it will reach a point where it is ready to fail suddenly. Thus sudden brittle fracture of steel is most likely to occur where there is triaxial tensile stress. This in turn is most likely to occur in heavily welded, wide, thick sections where the last part of a weld to cool will be unable to contract as it cools because it is restrained in all directions by the solid metal around it. It is therefore in a state of residual triaxial tensile stress and will tend to pull apart, starting at any defect or crack.

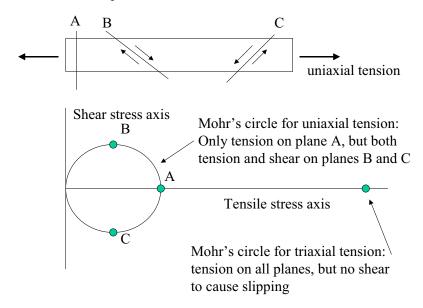


Figure 2.2 *Uniaxial or biaxial tension produces shear and slip, but uniform triaxial tension does not* 

#### 2.3.3 Case study: King's St Bridge, Melbourne

The failure of King's St Bridge in Melbourne in 1962 provided a good example of brittle fracture. One cold morning a truck was driving across the bridge when one of the main girders suddenly cracked (Fig.2.3). Nobody was injured but the subsequent enquiry revealed that some of the above factors had combined to cause the failure.

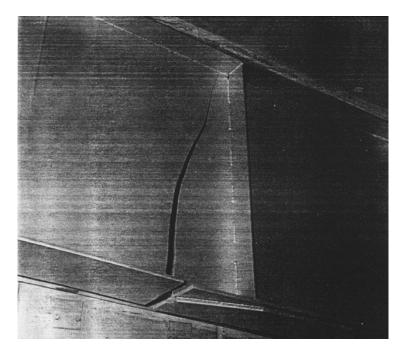


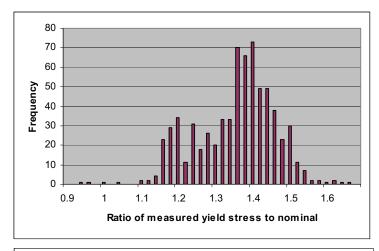
Figure 2.3 Brittle Crack in King's St. Bridge Girder, Melbourne

- 1. A higher yield strength steel than normal was used, and this steel was less ductile and had a higher brittle to ductile transition temperature than the lower strength steels the designers were accustomed to.
- 2. Thick (50 mm) cover plates were welded to the bottom flanges of the bridge girders to increase their capacity in areas of high bending moment.
- 3. These cover plates were correctly tapered to minimise the sudden change of cross section at their ends (Fig.2.2), but the welding sequence was wrong in some cases: the ends were welded last, and this caused residual triaxial tensile stresses at these critical points where stresses were high and the abrupt change of section existed.

Steelwork can be designed to avoid brittle fracture by ensuring that welded joints impart low restraint to plate elements, since high restraint could initiate failure. Also stress concentrations, typically caused by notches, sharp re-entrant angles, abrupt changes in shape or holes should be avoided.

#### 2.4 CONSISTENCY

The properties of steel are more predictable than those of concrete, allowing a greater degree of sophistication in design. However there is still some random variation in properties, as shown in Fig.2.4.



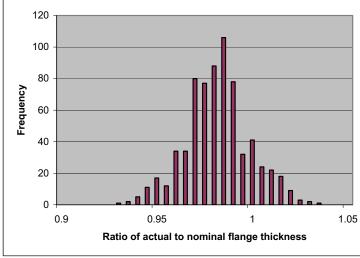
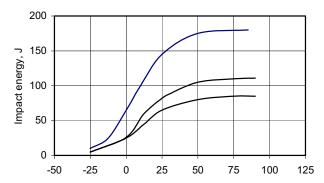


Figure 2.4 Random variation in measured properties of nominally identical steel specimens (adapted from Byfield and Nethercote [2])

Although steel is usually assumed to be a homogeneous, isotropic material this is not strictly true, as all steel includes microscopic impurities, which tend to be preferentially oriented in the direction of mill rolling. This results in lower toughness perpendicular to the plane of rolling (Fig.2.5).



Variation of Charpy V-notch impact energy with notch orientation and temperature for steel plate containing 0.012% C.

Figure 2.5 Lower toughness perpendicular to the plane of rolling (Metals Handbook [1])

Some impurities also tend to stay near the centre of the rolled item due to their preferential solubility in the liquid metal during solidification, i.e. near the centre of rolled plate, and at the junction of flange and web in rolled sections. The steel microstructure is also affected by the rate of cooling: faster cooling will result in smaller crystal grain sizes, generally resulting in some increase in strength and toughness. (Economical Structural Steel Work [3])

As a result, AS 4100 [4] Table 2.1 allows slightly higher yield stresses than those implied by the steel grade for thin plates and sections, and slightly lower yield stresses for thick plates and sections. For example the yield stress for Grade 300 flats and sections less than 11 mm thick is 320 MPa, for thicknesses from 11 to 17 mm it is 300 MPa and for thicknesses over 17 mm it is 280 MPa.

#### 2.5 CORROSION

Normal structural steels corrode quickly unless protected. Corrosion protection for structural steelwork in buildings forms a special study area. If the structural steelwork of a building includes exposed surfaces (to a corrosive environment) or ledges and crevices between abutting plates or sections that may retain moisture, then corrosion becomes an issue and a protection system is then essential. This usually involves consultation with specialists in this area. The choice of a protection system depends on the degree of corrosiveness of the environment. The cost of protection varies and is dependent on the significance of the structure, its ease of access for maintenance as well as the permissible frequency of maintenance without inconvenience to the user. Depending on the degree of corrosiveness of the environment, steel may need:

- Epoxy paint
- ROZC (red oxide zinc chromate) paint
- Cold galvanising (i.e. a paint containing zinc, which acts as a sacrificial coating, i.e. it corrodes more readily than steel)
- Hot dip galvanising (each component must be dipped in a bath of molten zinc after fabrication and before assembly)
- Cathodic protection, where a negative electrical potential is maintained in the steel, i.e. an oversupply of electrons that stops the steel losing electrons and forming Fe +++ or Fe and hence an oxide.
- Sacrificial anodes, usually of zinc, attached to the structure, which lose electrons more readily than the steel and so keep the steel supplied with electrons and inhibit oxide formation.

#### 2.6 FATIGUE STRENGTH

The application of cyclic load to a structural member or connection can result in failure at a stress much lower than the yield stress. Unlike aluminium, steel has an "endurance limit" for applied stress range, below which it can withstand an indefinite number of stress cycles, as shown in Fig 2.6.

However Fig.2.6 oversimplifies the issue and the assessment of fatigue life of a member or connection involves a number of factors, which may be listed as follows:

- (1) Stress concentrations
- (2) Residual stresses in the steel.
- (3) Welding causing shrinkage strains.
- (4) The number of cycles for each stress range.
- (5) The temperature of steel in service.
- (6) The surrounding environment in the case of corrosion fatigue.

For most static structures fatigue is not a problem, but fatigue calculations are usually carried out for the design of structures subjected to many repetitions of large amplitude stress cycles such as railway bridges, supports for large rotating equipment and supports for large open structures subject to wind oscillation.

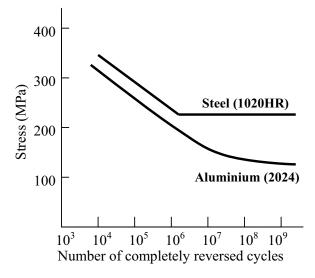


Figure 2.6 Stress cycles to failure as a function of stress level (adapted from Mechanics of Materials [5])

To be able to design against fatigue, information on the loading spectrum should be obtained, based on research or documented data. If this information is not available, then assumptions must be made with regard to the nature of the cyclic loading, based on the design life of the structure. A detailed procedure on how to design against fatigue failure is outlined in Section 11 of AS4100 [4].

#### 2.7 FIRE RESISTANCE

Although steel is non-combustible and makes no contribution to a fire it loses strength and stiffness at temperatures exceeding about 200°C. Fig.2.7 shows the twisted remains of a steel framed building gutted by fire.



Figure 2.7 Remains of a steel framed building gutted by fire, Ashmore, Gold Coast

Regulations require a building structure to be protected from the effects of fire to allow a sufficient amount of time before collapse for anyone in the building to leave and for fire fighters to enter if necessary. Additionally, it ought to delay the spread of fire to adjoining property. Australian Building Regulations stipulate fire resistance levels (FRL) for structural steel members in many types of applications.

The fire resistance level is a measure of the time, in minutes; it will take before the steel heats up to a point where the building collapses. The FRL required for a particular application is related to,

- the likely fire load inside the building (this relates to the amount of combustible material in the building)
- the height and area of the building
- the fire zoning of the building locality and the onsite positioning.

In order to achieve the fire resistance periods, (specified in the Building Regulations) systems of fire protection are designed and tested by their manufactures. A fire protection system consists of the fire protection material plus the manner in which it is attached to the steel member. Apart from insulating structural elements, building codes call for fireproof

walls (in large open structures) at intervals to reduce the hazard of a fire in one area spreading to neighbouring areas.

There is a range of fire protection systems to choose from, such as non-combustible paints or encasing steel columns in concrete. The manufactures of these materials can provide the necessary accreditation and technical data for them. These should be references to tests conducted at recognised fire testing stations. Their efficiency for achieving the required FRL as well as the cost of these materials should be taken into consideration. Concerning the protection of steel, the most feasible way is to cover or encase the bare steelwork in a non-combustible, durable, and thermally protective material. In addition, the chosen material must not produce smoke or toxic gases at an elevated temperature. These may be either sprayed onto the steel surface, or take the form of prefabricated casings clipped round the steel section.

#### 2.8 REFERENCES

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- 3. Australian Institute of Steel Construction (1996). *Economical Structural Steelwork*, 4<sup>th</sup> edn.
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## 3 LOAD ESTIMATION

#### 3.1 INTRODUCTION

Before any detailed sizing of structural elements can start, it is necessary to start to estimate the loads that will act on a structure. Once the designer and the client have agreed on the purpose, size and shape of a proposed structure and what materials it is to be made of, the process of load estimation can begin. Loads will always include the self-weight of the structure, called the "dead load." In addition there may be "live" loads due to people, traffic, furniture, etc., that may or may not be present at any given time, and also loads due to wind, snow, earthquakes etc. The required sizes of the members will depend on the weight of the structure but will also contribute to the weight. So load estimation and member sizing are to some extent an iterative process in which each affects the other. As the designer gains experience with a particular type of structure it becomes easier to predict approximate loads and member sizes, thereby reducing the time taken in trial and error. However the inexperienced designer can save time by intelligent use of some short cuts. For example the design of structures carrying heavy dead loads such as concrete slabs or machinery may be dominated by dead load. In this case it may be best to size the slabs or machinery first so the dead loads acting on the supporting structure can be estimated. On the other hand many steelframed industrial buildings in warm climates where snow does not fall can be designed mainly on the basis of wind loads, since dead and live loads may be small enough in relation to the wind load to ignore for preliminary design purposes. The wind load can be estimated from the dimensions of the structure and its location. Members can then be sized to withstand wind loads and then checked to make sure they can withstand combinations of dead, live and wind load. Where snowfall is significant, snow loads may be dominant. Earthquake loads are only likely to be significant for structures supporting a lot of mass, so again the mass should be estimated before the structural elements are sized.

#### 3.2 ESTIMATING DEAD LOAD (G)

Dead load is the weight of material forming a permanent part of the structure, and in Australian codes it is given the symbol G. Dead load estimation is generally straightforward but may be tedious. The best way to learn how to estimate G is by examples.

#### 3.2.1 Example: Concrete slab on columns

Probably the simplest form of structure – at least for load estimation - is a concrete slab supported directly on a grid of columns, as shown in Fig.3.1.

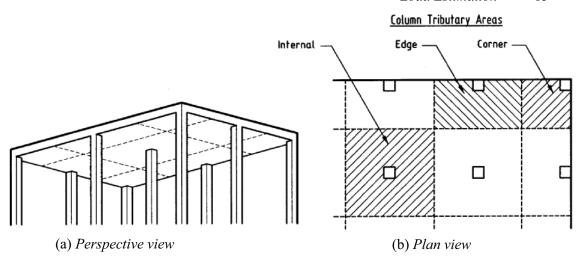


Figure 3.1 Concrete Slab on Columns

Suppose the concrete (including reinforcing steel) weighs 25 kN/m³, the slab is 200 mm (0.2 m) thick, and the columns are spaced 4 m apart in both directions. We want to know how much dead load each column must support.

First, we work out the area load, i.e. the dead weight G of one square metre of concrete slab. Each square m contains  $0.2 \text{ m}^3$  of concrete, so it will weigh  $0.2 \text{x} 25 = 5 \text{ kN/m}^2$  or G = 5 kPa.

Next, we multiply the area load by the tributary area, i.e. the area of slab supported by one column. We assume that each piece of slab is supported by the column closest to it. So we can draw imaginary lines half way between each row of columns in each direction. Each internal column (i.e. those that are not at the edge of the slab) supports a tributary area of  $16 \text{ m}^2$ , so the total dead load of the slab on each column is 16x5 = 80 kN.

Assuming there is no overhang at the edges, edge columns will support a little over half as much tributary area because the slab will presumably come to the outer edge of the columns, so the actual tributary area will be 2.1x4 = 8.4 m<sup>2</sup> and the load will be 42 kN. Corner columns will support 2.1x2.1 = 4.42 m<sup>2</sup> and a load of 22.1 kN.

To find the load acting on a cross-section at the bottom of each column where G is maximum, we must also consider the self-weight of the column. Suppose columns are 150UC30 sections (i.e steel universal columns with a mass of 30 kg/m, 4m high between the floor and the suspended slab. The weight of one column will therefore be 30x9.8/1000x4 = 1.2kN approximately. Thus the total load on a cross section of an internal column at the bottom will be 80 + 1.2 = 81.2 kN.

If there are two or more levels, as in a multi-level car park or an office building, the load on each ground floor column would have to be multiplied by the number of floors. Thus if our car park has 3 levels, a bottom level internal column would carry a total dead load G = 3x81.2 = 243.6 kN.

#### 3.2.2 Concrete slab on steel beams and column

A more common form of construction is to support the slab on beams, which are in turn supported on columns as shown in Figs.3.2 and 3.3 below. Because the beams are deeper and stronger than the slab, they can span further so the columns can be further apart, giving more clear floor space.

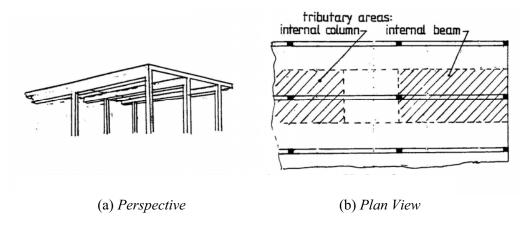


Figure 3.2 Slab, Beams and Columns





Figure 3.3 Car parks. Left: Sydney Airport: concrete slabs on steel beams and concrete columns. Right: Petrie Railway Station, Brisbane: concrete slab on steel beams and steel columns

To calculate the dead load on the beams and columns, we now add another step in the calculation. Assuming we still have a 200mm thick slab, the area load due to the slab is still the same, i.e. 5 kPa.

Assume columns are still of 200x200mm section, at 4 m spacing in one direction. But we now make the slab span 4m between beams, and the beams span 8m between columns. So we have only half as many columns. But we now want to know the load on a beam. We could work out the total load on one 8m span of beam. But it is normal to work out a line load, i.e. the load per m along the beam. The tributary area for each internal beam in this case is a strip 4m wide, as shown in the diagram above. So the line load on the beam due to the slab only is  $5 \text{ kN/m}^2 \times 4 \text{ m} = 20 \text{ kN/m}$ . Note the units.

We must also take into account the self-weight of the beam. Suppose the beams are 610UB101 steel universal beams weighing approximately 1 kN/m. The total line load G on the internal beams is now 20 + 1 = 21 kN/m. This will be the same on each floor because each beam supports only one floor. The lower columns take the load from upper floors but the beams do not. A line load diagram for an internal beam is shown in Fig.3.4 below. Note that we specify the span (8 m), spacing (4 m), load type (G) and load magnitude (21 kN/m).

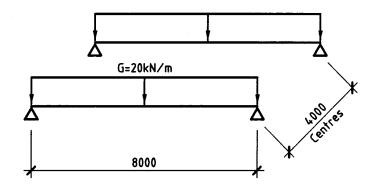


Figure 3.4 Line load diagram for Dead Load G on Beam

#### 3.2.3 Walls

Unlike car parks, most buildings have walls, and we can estimate their dead weight in the same way as we did with slabs, columns and beams. Sometimes walls are structural, i.e. they are designed to support load. Other walls may be just partitions, which contribute dead weight but not strength. These non-structural partition walls are common because it is very useful to be able to knock out walls and change the floor plan of a building without having to worry about it falling down.

Suppose a wall is 100 mm thick and is made of reinforced concrete weighing 25 kN/m³. The weight will be 25 x 0.1 = 2.5 kN/m² of wall area. If it is 4 m high, it will weigh  $4\text{m} \times 2.5$  kN/m² = 10 kN/m of wall sitting on the floor, i.e. the line load it will impose on a floor will be 10 kN/m. The SAA Loading Code AS 1170 Part 1, Appendix A, contains data on typical weights of building materials and construction. For example a concrete hollow block masonry wall 150 mm thick, made with standard aggregate, weighs about 1.73 kN/m² of wall area. A 2.4 m high wall of this type of blocks will impose a line load of  $1.73 \times 2.4 = 4.15$  kN/m.

#### 3.2.4 Light steel construction

Although the dead weight of steel and timber roofs and floors is much less than that of concrete slabs, it must still be allowed for. The principles are still the same: sheeting is supported on horizontal "beam" elements, i.e. members designed to withstand bending.

However it is common in steel and timber roof and floor construction to have two sets of "beams," i.e. flexural members, running at right angles to each other. These have special names, which are shown in the diagrams below.

#### 3.2.5 Roof construction

Corrugated metal (steel or aluminium) roof sheeting is normally supported on relatively light steel or timber members called purlins which run horizontally, i.e. at right angles to the corrugations which run down the slope. In domestic construction, tiled roofing is common. Tiles require support at each edge of each tile, so they are supported on light timber or steel members called battens, which serve the same purpose as purlins but are at much closer spacing, usually 0.3m.

The purlins or battens are in turn supported on rafters or trusses. Rafters are heavier, more widely spaced steel or timber beams running at right angles to the purlins or battens, as shown in Fig.3.5, and spanning between walls or columns.

Purlins usually span about 5 to 8 m and are usually spaced about 0.9 to 1.5 m apart. This spacing is dictated partly by the distance the sheeting can span between purlins, and partly by the fact that it is easier to erect a building if the purlins are close enough to be able to step from one to another before the sheeting is in place.



Figure 3.5 Roof Sheeting is Supported by Purlins, Rafters and Columns

Trusses are commonly used to support battens in domestic construction. These are usually timber but may be made of light, cold-formed steel.

#### 3.2.6 Floor construction

Light floors are usually made of timber floor boards or sheets of particle board. Light floors are supported on floor joists, just as roof sheeting is supported on purlins or battens. Floor joists are typically spaced at 300, 450 or 600 mm centres and are in turn supported by bearers, as shown in Fig.3.7 below. Finally the whole floor is held up either by walls or by vertical columns called stumps. Note the similarity in principle between the roof structure shown in Fig.3.5 and the floor structure in Fig.3.6. This similarity is shown schematically in Fig.3.7.



Figure 3.6 Typical Steel-Framed Floor Construction Showing Timber Sheeting and Steel Floor Joists, Bearers and Stumps

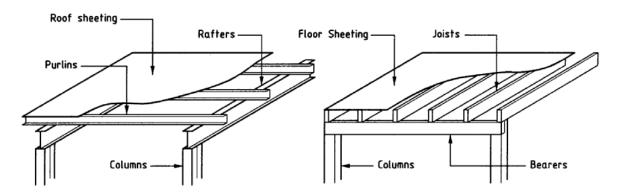


Figure 3.7 Similarity in Principle Between Floor and Roof Construction

#### 3.2.7 Sample calculation of dead load G for a steel roof

We start by finding the weight of each component of the roof, i.e.

- sheeting
- purlins
- rafters

Let us assume the roof sheeting is "Custom Orb" (the normal corrugated steel sheeting) 0.48 mm thick. In theory we could work out the weight of this material from the density of steel, but we would need to allow for the corrugations and the overlap where sheets join. So it is simpler to look it up in a published table which includes these allowances, such as the one in Appendix A of this study guide. From this table, the weight of 0.48mm Orb is 5.68 kg/m<sup>2</sup>.

Let us now assume this roof sheeting is supported on cold-formed steel Z section purlins of Z15019 section (i.e. 150 mm deep, made of 1.9 mm thick sheet metal formed into a Z profile. See Appendix B). This section weighs 4.46 kg/m.

Assume the purlins are at 1.2 m centres (i.e. their centre lines are spaced 1.2 m apart). These purlins span 6 m between rafters of hot rolled 310UB40.4 section (the "310" means 310 mm deep, and the "40.4" means 40.4 kg/m). Assume the rafters span 10 m and are, of course spaced at 6 m centres, the same as the purlin span. This arrangement is shown in Fig.3.10.

"Cold formed steel Z section" means a flat steel strip is bent so it cross section resembles a letter Z. This is done while the steel is cold, and the cold working increases its yield strength but decreases its ductility. Cold formed sections are usually made from thin (1 to 3 mm) galvanized steel. This contrasts with the heavier hot rolled I, angle and channel sections which are formed while hot enough to make the steel soft. Hot rolled sections are usually supplied "black," i.e. as-rolled, with no special surface finish or corrosion protection, so they usually have some rust on the surface.

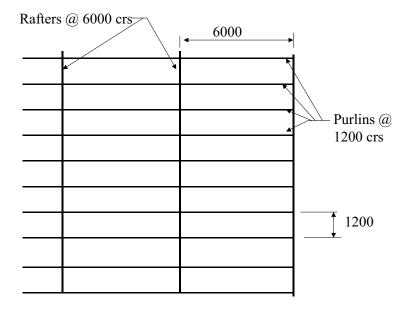


Figure 3.8 Layout of Purlins and Rafters

#### 3.2.7.1 Dead load on purlins

To calculate dead loads acting on purlins, the principles are the same as for concrete construction, i.e.

- 1. Work out area loads of roof sheeting
- 2. Multiply by the spacing of purlins to get line loads on purlins.

In this case, the area load due to sheeting is  $5.68 \text{ kg/m}^2 = 5.68 \times 9.8 \text{ N/m}^2 = 5.68 \times 9.8 \text{ N/m}^2 = 0.0557 \text{ kPa}.$ 

The line load on the purlins due to sheeting will be  $0.0557 \text{ kN/m}^2 \text{ x } 1.2\text{m} = 0.0668 \text{ kN/m}$ .

But the total line load for G on purlins consists of the sheeting weight plus the purlin self-weight, which is 4.46 kg/m = 0.0437 kN/m.

Thus the total line load due to dead weight G on the purlin is G = 0.0668 + 0.0437 = 0.11 kN/m.

This is shown as a line load diagram in Fig.3.9 below, in the same form as the line load diagram for the beam in Fig.3.4.

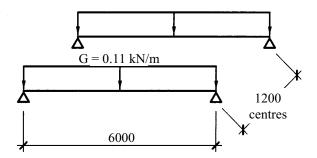


Figure 3.9 Line Load Diagram for Dead load G on Purlin

#### 3.2.7.2 Dead load on rafters

This is a bit more tricky than the load on the purlins because the weight of the sheeting and the purlins is applied to the rafter at a series of points, i.e. it is not strictly a uniformly distributed load (UDL). However the point loads will all be equal and they are close enough together to treat them as a UDL.

Each metre of a typical, internal rafter (i.e. not an end rafter) supports 6 m<sup>2</sup> of roof, as shown in Fig.3.10. Thus the weight of sheeting supported by each metre of rafter is simply weight/m<sup>2</sup> x rafter spacing =  $0.0557 \text{ kPa} \times 6 \text{ m} = 0.334 \text{ kN/m}$ .

The weight of the purlins per m of rafter can be calculated either of 2 ways:

- 1. Purlin weight = 0.0437 kN/m of purlin =  $0.0437/1.2 \text{ kN/m}^2$  of roof, since they are 1.2 m apart.  $\therefore$  purlin weight/m of rafter =  $0.0437/1.2 \text{ kN/m}^2 \times 6 \text{ m} = 0.2185 \text{ kN/m}$ .
- 2. Every 1.2 m of rafter supports 6 m of purlin (3 m each side). ∴on average, every 1 m of rafter supports 6/1.2 = 5 m of purlin.∴purlin weight/m of rafter = 0.0437 kN/m x 6m/1.2m = 0.2185 kN/m.

So the total dead load on 1 m of rafter =  $0.334 + 0.2185 + 40.4 \times 9.8/1000 = 0.948 \text{ kN/m}$ . Again, this can be represented as a line load diagram for G on the rafter.

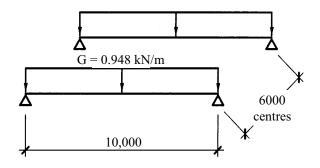


Figure 3.10 Line Load Diagram for G on the Rafter

#### 3.2.8 Dead load due to a timber floor

The same procedure can be used to estimate the dead weight in a timber floor. Density of timber can vary from 1150 kg/m³ (11.27 kN/m³) for unseasoned hardwood down to about half that for seasoned softwood. Detailed information for actual species is contained in AS 1720, the SAA Timber Structures Code, but for most purposes it can be assumed that hardwood weighs 11 kN/m³ and softwood weighs 7.8 kN/m³.

So the designers calculates the volume per  $m^2$  of floor area, or per m run of supporting member, and hence the weight. e.g. for a 30 mm thick softwood floor the area load is 7.8  $kN/m^3 \times 0.03 \text{ m} = 0.234 \text{ kN/m}^2 = kPa$ .

For a 100 x 50 mm hardwood joist the weight per m run =  $11 \text{ kN/m}^3 \text{ x } 0.1 \text{ m x } 0.05 \text{ m} = 0.055 \text{ kN/m}$ .

#### 3.2.9 Worked Examples on Dead Load Estimation

#### **Example 3.2.9.1**

A 0.42 mm Custom orb steel roof sheeting is supported on Z15015 purlins at 1200 centres. Rafters of 200UB29.8 section are at 5 m centres and span 10 m. Find line load G on (a) purlins, (b) rafters.

#### Solution

```
0.42 mm Custom orb steel roof sheeting has a mass of 4.3 kg/m<sup>2</sup> Weight = 4.3 x 9.8 / 1000 = 0.042 kPa (kN/m<sup>2</sup>). This the area load due to sheeting. Line load on purlin due to sheeting = area load x spacing = 0.042 kN/m<sup>2</sup> x 1.2m = 0.0506 kN/m
```

```
Mass of Z15015 purlins = 3.54 \text{ kg/m}
Weight = 3.54 \times 9.8 / 1000 \text{ kN/m} = 0.0347 \text{ kN/m}
```

- (a) Line load G on purlins = weight per m of sheeting plus purling self-weight = 0.0506 + 0.0347 kN/m = 0.085 kN/m
- (b) Line load on rafters (i.e. load supported by 1 m run of rafter) is made up of 3 components:
- (i)  $5 \text{ m}^2 \text{ of roof sheeting} = 0.042 \text{ x } 5 = 0.21 \text{ kN/m}$
- (ii) Average weight of purlins supporting 5 m<sup>2</sup> of roofing =  $0.0347 \text{ kN/m} \times 5 \text{m long} / 1.2 \text{m spacing} = 0.145 \text{ kN/m}$
- (iii) Self weight of rafter =  $29.8 \text{ kg/m} \times 9.8/1000 = 0.292 \text{ kN/m}$ Hence total line load G on rafter = 0.21 + 0.145 + 0.292 = 0.647 kN/m

#### **Example 3.2.9.2**

Concrete tiles (0.53 kN/m²) are supported on steel top hat section battens (0.62 kg/m) at 300 mm centres. Find the line load on trusses at 900 centres due to tiles plus battens.

#### Solution

```
Line load on trusses due to tiles plus battens
= weight of 0.9m<sup>2</sup> tiles + weight/m of battens x span / spacing
= 0.53 x 0.9 kN/m + (0.62 x 9.8/1000)x0.9/0.3 = 0.495 kN/m
```

### **Example 3.2.9.3**

Softwood (7.8 kN/m³) purlins of 100x50 mm cross section at 3000 mm centres, spanning 4m, support Kliplok 406, 0.48 mm thick. Rafters at 4 m centres, spanning 6m, are steel 200x100x4 mm RHS. Find the line load G on (a) purlins, (b) rafters.

#### Solution

Sheeting: area load =  $5.3 \text{ kg/m}^2 = 5.3 \text{ x} 9.8 / 1000 \text{ kPa} = 0.052 \text{ kPa}$ Line load on purlin due to sheeting = area load x spacing = 0.052 x 3 = 0.156 kN/m

Purlins: Softwood 7.8 kN/m<sup>3</sup> Line load due to self weight =  $7.8 \text{ kN/m}^3 \times 0.1 \text{m} \times 0.05 \text{ m} = 0.039 \text{ kN/m}$ 

(a) : Total line load G on purlins = 0.156 + 0.039 = 0.195 kN/m

Line load on rafter due to sheeting = area load x rafter spacing =  $0.052 \times 4 = 0.208 \text{ kN/m}$ Line load on rafter due to purlins = purlin weight/m x rafter spacing / purlin spacing =  $0.039 \times 4 / 3 = 0.052 \text{ kN/m}$ 

Line load on rafter due to self weight = 17.9 kg/m = 17.9 x 9.8 / 1000 = 0.175 kg/m

(b) : Total line load G on rafters = 0.208 + 0.052 + 0.175 = 0.435 kN/m

# **Example 3.2.9.4**

19 mm thick hardwood flooring (11 kN/m³) is supported on 100x50 mm hardwood joists at 450 mm centres. The joists span 2 m between 200UB29.8 bearers, which span 3 m. Find line load G on (a) joists, (b) bearers. Find also the end reaction supported on the stumps.

### Solution

Area load due to 19 mm thick hardwood flooring (11 kN/m<sup>3</sup>) = 11 x 0.019 = 0.209 kPa : Line load on joists at 450 crs due to flooring = 0.209 x 0.45 = 0.094 kN/m

Line load due to self weight of 100x50 mm hardwood joists =  $11 \text{ kN/m}^3 \text{ x } 0.1 \text{ x } 0.05 = 0.055 \text{ kN/m}$ 

(a) Total line load G on joists = 0.094 + 0.055 = 0.149 kN/m

Line load on bearers at 2m crs due to flooring =  $0.209 \times 2 = 0.418 \text{ kN/m}$ . Line load on bearers due to joists at 450 crs =  $0.094 \times 2 / 0.45 = 0.418 \text{ kN/m}$  (*coincidence*) Line load due to bearer self-weight =  $29.8 \times 9.8/1000 = 0.292 \text{ kN/m}$ .

(b) Total line load G on bearers = 0.418 + 0.418 + 0.292 = 1.13 kN/m

End reaction supported on the stumps: Bearers span 3 m, so total load which must be supported by 2 stumps =  $1.13 \text{ kN/m} \times 3\text{m} = 3.39 \text{ kN}$ 

:. Weight supported by each bearer (for this span only) = 3.39 / 2 = 1.7 kN (However internal stumps would support bearers on each side, so would support 3.39 kN)

### **Example 3.2.9.5**

A 150 mm thick solid reinforced concrete slab (25 kN/m³) is supported on 360UB50.7 beams at 2 m centres, spanning 4 m. Find line load G on beams.

#### Solution

Area load due to 150 mm thick reinforced concrete slab  $(25 \text{ kN/m}^3) = 0.15 \text{x} 25 = 3.75 \text{ kN/m}^2$ Line load on beams at 2 m centres due to slab = 3.75 kN/m<sup>2</sup> x 2m = 7.5 kN/m Line load on beams due to self weight = 50.7 x 9.8 /1000 = 0.497 kN/m Hence total line load G on beams = 7.5 + 0.497 = 8.00 kN/m

# 3.3 ESTIMATING LIVE LOAD Q

It would be possible to estimate the maximum number of people that might be expected in a particular room, calculate their total weight and divide by the area of the room. For example you might expect about 30 people averaging 80 kg in a small lecture room 8 x 5 m in area, i.e. approximately 30 x 80 kg = 2400 kg in 40 m² =  $60 \text{ kg/m}^2 = 0.588 \text{ kPa}$ . But it is possible that many more might be in the room for some special occasion. It is physically possible to squeeze about 6 people into a 1 m square, i.e. about 6 x 80 kg/m² = 4.7 kPa. But it is most unlikely that there would ever be 6 people/m² in every part of a room. So what figure should we use? Fortunately the loading code AS/NZS 1170.1:2002 [1] gives guidelines for live loads on roofs and floors.

### 3.3.1 Live load Q on a roof

Live loads on "non-trafficable" roofs such as the roof of a portal frame building arrises mainly from maintenance loads where new or old roof sheeting may be stacked in concentrated areas. For purlins and rafters, the code provides for a distributed load of  $0.25 \text{ kN/m}^2$  where the supported area A is greater than or equal to  $14 \text{ m}^2$ , the area A being the plan projection of the inclined roof surface area. For areas A less than  $14 \text{ m}^2$ , the code specifies the distributed load of  $w_Q = [1.8/A + 0.12] \text{ kN/m}^2$  on the plan projection. In addition to the distributed live load, the loading code also specifies that portal frame rafters be designed for a concentrated load of 4.5 kN at any point, this concentrated load is usually assumed to act at the ridge.

# 3.3.2 Live load Q on a floor

This is very simple to calculate. Floor live loads are given in AS1170.1. For example a floor in a normal house must be designed for an area load Q = 1.5 kPa (i.e. approximately 150 kg per m<sup>2</sup> of floor area. In addition, it must be designed to take a "point" load of 1.8 kN on an area of 350 mm<sup>2</sup>. Usually the area load governs the design. Suppose a house has floor joists at 300 mm centres. These must be designed for a live load of 1.5 kN/m<sup>2</sup> x 0.3 m = 0.45 kN/m. If the bearers are at 2.4 m centres, the live load will be 1.5 x 2.4 = 3.6 kN/m.

### 3.3.3 Other live loads

These may include impact and inertia loads due to highly active crowds, vibrating machinery, braking and horizontal impact in car parks, cranes, hoists and lifts. AS 1170.1 gives guidance on design loads due to braking and horizontal impact in car parks. Other dynamic loads are treated in other standards such as the Crane and Hoist Code AS 1418. Some of these will be treated later in this chapter.

# 3.3.4 Worked Examples on Live Load Estimation

### **Example 3.3.4.1**

A 0.42 mm Custom orb steel roof sheeting is supported on Z15015 purlins at 1200 centres. Rafters of 200UB29.8 section are at 5 m centres and span 10 m. Find line load Q on (a) purlins, (b) rafters.

### Solution

- (a) Each purlin span supports an area  $A = 5 \times 1.2 = 6 \text{ m}^2$   $\therefore w_Q = [1.8/A + 0.12] = 0.42 \text{ kN/m}^2$  AS 1170.1 Cl 4.8.1.1 Line load Q on purlin = 0.42×1.2 = 0.504 kN/m
- (b) Each rafter span supports an area of 5 x10 = 50 m<sup>2</sup> > 14 m<sup>2</sup>  $\therefore w_Q = 0.25 \text{ kPa}$  AS 1170.1 Cl 4.8.1.1 Line load Q on rafter = 0.25 x 5 = 1.25 kN/m



Line Loads for Live Load Q (left) for Purlin, (right) for Rafter

# **Example 3.3.4.2**

19 mm thick hardwood flooring is supported on 100x50 mm hardwood joists at 450 mm centres. The joists span 2 m between 200UB29.8 bearers, which span 3 m. Find line load Q on (a) joists, (b) bearers. If it is a normal house floor (Q = 1.5 kPa).

#### Solution

Area load Q (normal house) = 1.5 kPa

- (a) Line load Q on joist =  $1.5 \times 0.45 = 0.675 \text{ kN/m}$
- (b) Line load Q on bearers =  $1.5 \times 2 = 3 \text{ kN/m}$

### **Example 3.3.4.3**

A 150 mm thick solid reinforced concrete slab is supported on 360UB50.7 beams at 2 m centres, spanning 4 m. Find line load Q on (a) a 1 m wide strip of floor, (b) beams if it is a library reading room (Q = 2.5 kPa).

### Solution

- (a) Line load Q on a 1 m wide strip of floor =  $2.5 \text{ kN/m}^2 \text{ x } 1\text{m} = 2.5 \text{ kN/m}$
- (b) Line load Q on beams at 2 m centres =  $2.5 \times 2 = 5 \text{ kN/m}$

### 3.4 WIND LOAD ESTIMATION

Relative motion between fluids (e.g. air and water) and solid bodies causes lift, drag and skin friction forces on the solid bodies. Examples include lift on an aeroplane wing, drag on a moving vehicle, force exerted by a flowing river on a bridge pylon, and wind loads on structures.

The estimation of wind loads is a complex problem because they vary greatly and are influenced by a large number of factors. The following introduction is intended only to illustrate the procedure for estimating wind loads on rectangular buildings and lattice towers. For a more complete treatment the reader should consult wind loading codes and specialist references.

### 3.4.1 Factors influencing wind loads

Wind forces increase with the square of the wind speed, and wind speed varies with geographical region, local terrain and height. Tropical coastal areas are subject to tropical cyclones, and structures in these areas must be designed for higher wind speeds than those in other areas. Wind speed generally increases with height above the ground, and winds are stronger in more exposed locations such as hilltops, foreshores and flat treeless plains than in sheltered inner city locations. Thus tall structures and structures in exposed locations must be designed for higher wind gusts than low structures and those in sheltered locations.

The pressure exerted by the wind on any part of a structure depends on the shape of the structure and the wind direction. Windward walls and upwind slopes of steeply pitched roofs experience a rise in pressure above atmospheric pressure, while side walls, leeward walls, leeward slopes of roofs and flat roofs experience suction on the outside. The greatest suction pressures tend to occur near the edges of roofs and walls. This is shown in Fig.3.11 below.

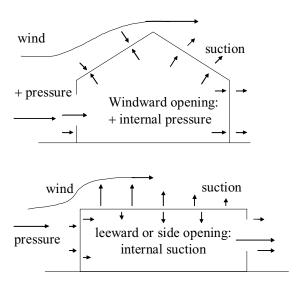


Figure 3.11 Internal Pressure can be either Positive or Negative, Depending on Location of Openings Relative to Wind Direction

Internal pressure is positive (pushing the walls and roof outwards) if there is a dominant opening on the windward side only, and negative (sucking inwards) if the openings are on a side or leeward wall.

The extreme storm wind gusts are the ones that do the damage, and these are very variable in time and space, as shown in Fig.3.12 below. Statistically it is most unlikely that the pressure from an extreme gust will act over a large area of a building all at the same time, so design pressures on small areas are greater than those on large areas. (Of course the total **force** is more on a large area). All of these factors are taken into account in design codes.

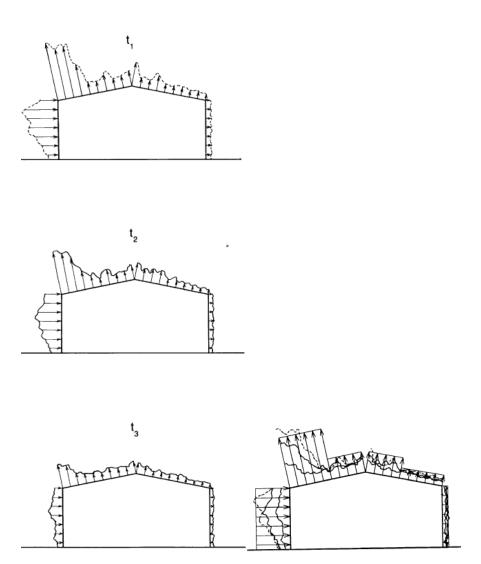


Figure 3.12 External Pressure Distribution Varies with Time due to Turbulence and Gustiness of the wind. Codes must use a simplified envelope. [2]

### 3.4.2 Design wind speeds

It is important to design structures so they will not collapse completely or disintegrate and allow sharp pieces to fly through the air. Most of the loss of life in the 1974 Darwin cyclone was caused by flying roof sheeting. At the same time it is uneconomic to design structures so strong that they will remain totally undamaged even in an extreme storm that may occur on average only once in hundreds of years. Thus it is normal to use two wind speeds in design: one for ultimate strength design, which is unlikely to occur during the structure's life but could occur at any time. In this wind some minor damage is acceptable but the structure must not totally collapse. A lower wind speed is used to design for serviceability, in which the structure should survive without damage.

The discussion below will follow the current Australian and New Zealand wind loading code, AS/NZS 1170.2:2002 [3], which represents a major revision of the previous AS1170.2, and will deal mainly with rectangular buildings, although lattice towers will also be treated.

The procedure is summarised in the flow charts in Figs. 3.13 and 3.14.

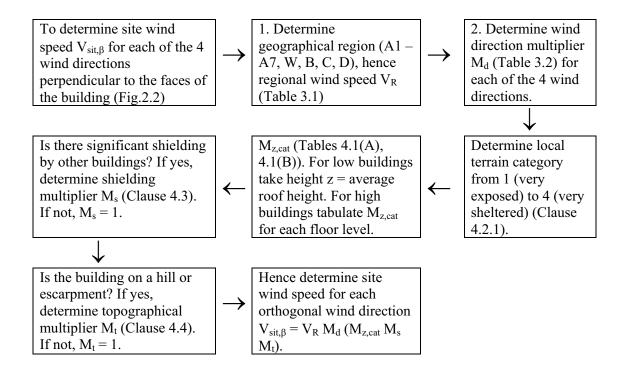


Figure 3.13 Flow Chart to Determine Site Wind speed V<sub>sit,B</sub>

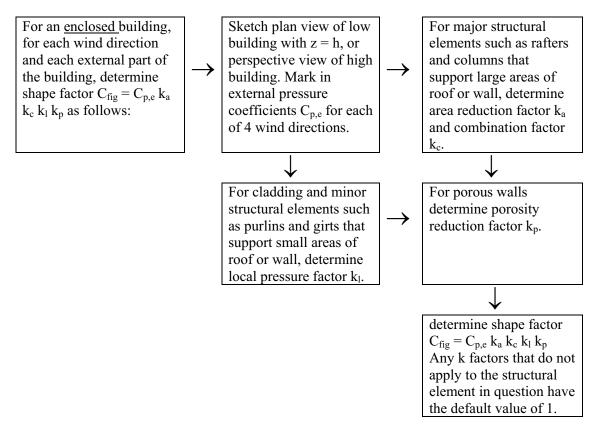


Figure 3.14 Flow Chart to Determine Shape factor Cfig

### 3.4.3 Site wind speed $V_{sit,\beta}$

The site wind speed  $V_{sit,\beta}$  used in design is a function of 5 variables:

- 1. The regional wind speed  $V_R$  shown in Table 3.1.
- 2. The wind direction multiplier  $M_d$  given in Table 3.2.
- 3. The terrain and height multiplier M<sub>z,cat</sub>
- 4. The shielding multiplier M<sub>s</sub>
- 5. The topographical multiplier M<sub>t</sub>.

These are discussed below.

### 3.4.3.1 Regional wind speed $V_R$

The maps of Australia and New Zealand are divided into 5 regions: A (most non-cyclonic areas), W (near Wellington), B (near cyclonic coastal areas of subtropical Australia),C (normal cyclonic) and D (extreme cyclonic). Region A is further subdivided in to 7 subregions. The designer should first check which region the structure will be located in. For example all Australian capitals are in Region A except Brisbane, which is in Region B.

The return period must then be decided. In the pre-2002 Australian wind code, ultimate strength limit state design was based on wind speed with a 5% chance of occurring once in 50 years, i.e. a 1000 year return period, while the wind speed for serviceability was based on a 5% chance of occurring once per year, i.e. a 20 year return period. Of course a 1000 year

return period does not imply that this wind speed will occur once every 1000 years. It should rather be considered as a 0.1% probability of occurring in any year. In the 2002 code the choice of return period is not specified, and regional wind speeds  $V_R$  are listed for each region and for return periods ranging from 5 to 2000 years (Table 3.1). The discussion which follows is based on  $V_{1000}$ , the same as the old strength limit state. For Brisbane  $V_{1000} = 60$  m/s and for all other Australian capital cities  $V_{1000} = 48$  m/s.

### 3.4.3.2 Wind direction multiplier M<sub>d</sub>

For regions A and W, stronger winds come from some directions, so wind direction multipliers  $M_d$  are provided for 8 wind directions (Table 3.2). Thus for example Sydney is in Region A2, where the strongest winds come from the west, where the direction multiplier  $M_d$  = 1, while Md for north, north east and east is only 0.8. So structures in Sydney must be designed for a regional wind speed of  $46 \times 1 = 46$  m/s from the west, but only  $46 \times 0.8 = 36.8$  m/s from the north or east. For regions B, C and D it is assumed that the maximum wind speed is the same for all directions.

# 3.4.3.3 The terrain and height multiplier $M_{z,cat}$

Terrain is classified into 4 categories, ranging from 1 for very open exposed terrain, through to 4 for well sheltered sites. Most built up areas are classified as Category 3.  $M_{z,cat}$  decreases from category 1 to 4, and increases with height z above ground level (Tables 4.1(A), 4.1(B)). For small buildings z is taken as the average height of the roof, while for tall buildings it is normal to calculate different values of  $M_{z,cat}$  at different heights up the building (Fig.2.1). Thus for example a house 5 m high in a Sydney suburb (Region A, z = 5m, cat 3) would have  $M_{z,cat} = 0.83$ , while the top of a 50 m high structure on a fairly exposed site in Sydney (Region A, z = 50m, cat 2) would have  $M_{z,cat} = 1.18$ .

### 3.4.3.4 Other multipliers

The shielding multiplier  $M_s$  allows for a reduced design wind speed on structures which are shielded by adjacent buildings, while the topographical multiplier  $M_t$  provides for increased wind speeds on hilltops and escarpments. A detailed discussion of these factors is outside the scope of this book and the reader is advised to study the code for further information.

When the above 5 variables have been evaluated for a particular site and wind direction in Regions A and W, the orthogonal design wind speeds  $V_{des,\theta}$  for the 4 faces of a rectangular building can be determined by interpolating between the wind speeds in the "cardinal wind directions". For example the top of the 50 m high building mentioned above is located in Region A2, with  $V_{1000} = 46$  m/s,  $M_d$  as shown below,  $M_{z,cat} = 1.18$ , assuming  $M_s$  and  $M_t = 1$ , would have a site wind speed for west wind given by

$$V_{sit,\beta} = V_R M_d (M_{z,cat} M_s M_t) = 46 \times 1 \times (1.18 \times 1 \times 1) = 54.3 \text{ m/s}.$$

Site wind speeds for the other cardinal directions are tabulated in Table 3.1 and graphed in Fig.3.15:

| Wind      | V <sub>sit,β</sub> , m/s                                |  |  |
|-----------|---|--|--|
| direction |   |  |  |
| N         | $46 \times 0.8 \times (1.18 \times 1 \times 1) = 43.4$  |  |  |
| NE        | $46 \times 0.8 \times (1.18 \times 1 \times 1) = 43.4$  |  |  |
| Е         | $46 \times 0.8 \times (1.18 \times 1 \times 1) = 43.4$  |  |  |
| SE        | $46 \times 0.95 \times (1.18 \times 1 \times 1) = 51.6$ |  |  |
| S         | $46 \times 0.9 \times (1.18 \times 1 \times 1) = 48.9$  |  |  |
| SW        | $46 \times 0.95 \times (1.18 \times 1 \times 1) = 51.6$ |  |  |
| W         | $46 \times 1 \times (1.18 \times 1 \times 1) = 54.3$    |  |  |
| NW        | $46 \times 0.05 \times (1.19 \times 1 \times 1) = 51.6$ |  |  |

Table 3.1 Site Wind speeds for the 8 Cardinal Directions for One Particular Site in Sydney

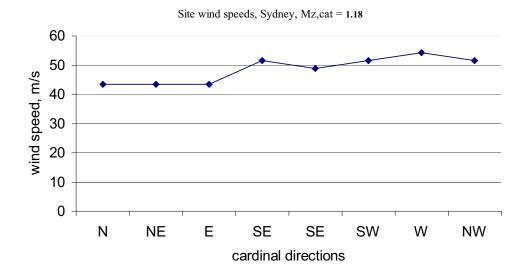


Figure 3.15 Variation of Site Wind Speed with Direction for one Particular Site in Sydney

These site wind speeds are shown in Fig.3.16 below on a plan view of a proposed building and the procedure for calculating the design wind speed for one face of that building is outlined (the wording in the code is hard to follow without an example). The East-North East face of the building has been selected to illustrate the procedure. Taking a 45° arc each side of the normal to the building face, and interpolating linearly between the wind speeds for the adjacent cardinal directions gives 43.4 m/s for NNE and 47.5 m/s for ESE. The greater of these is 47.5 m/s so this is taken as the design wind speed for this face of the building. Likewise the design wind speeds for the other 3 faces would be NNW 43.4 m/s, WSW 53.0 m/s, SSE 50.2 m/s.

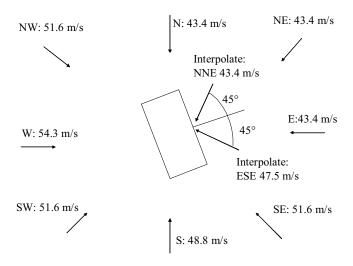


Figure 3.16 Site Wind Speeds Used for Interpolation, Building at 22.5° to Cardinal Directions

If the faces of the building form other angles with the cardinal directions the same procedure is followed, using linear interpolation. Thus for example a building face oriented 10° south of west would have the 45° lines oriented 35° north of west and 55° south of west, respectively, as shown in Fig.3.17 below. The values of site wind speed for these 45° directions would be obtained by interpolation between the adjacent cardinal points, and the maximum wind speed within that 90° would be used for design. In this case that maximum is 54.3 m/s for due west.

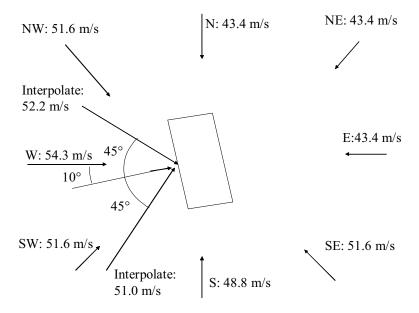


Figure 3.17 Site Wind Speeds Used tor Interpolation, Building at any Angle to Cardinal Directions

### 3.4.4 Aerodynamic shape factor

Having determined the design wind speed, two more variables must be evaluated before the design wind pressure on any surface of a structure can be determined. These are the aerodynamic shape factor  $C_{\rm fig}$ , which is itself a function of several variables, and the dynamic response factor  $C_{\rm dyn}$ .  $C_{\rm dyn}=1$  unless the structure is "wind sensitive", i.e. has a natural frequency less than 1 Hz.

The aerodynamic shape factor Cfig is defined in Clause 5.2 for enclosed buildings as

 $C_{\text{fig}} = C_{\text{p,e}} k_a k_c k_l k_p$  for external pressures

 $C_{fig} = C_{p,i} k_c$  for external pressures

 $C_{fig} = C_f k_c$  for frictional drag forces

For freestanding walls, hoardings, canopies and roofs

 $C_{fig} = C_{p,n} k_a k_l k_p$  for pressures normal to the surface

 $C_{fig} = C_f$  for frictional drag forces

#### where

 $C_{p,e}$  = external pressure coefficient  $C_{p,e}$ ,

 $k_a$  = area reduction factor

 $k_c = combination factor$ 

 $k_1$  = local pressure factor

 $k_p$  = porous cladding reduction factor

 $\hat{C}_{p,i}$  = internal pressure coefficient

 $C_f$  = frictional drag force coefficient

 $C_{p,n}$  = net pressure coefficient acting normal to the surface for canopies etc.

# 3.4.5 Calculating external pressures

Values of  $C_{p,e}$  vary over the external surface of the building and depend on the structure's shape and the orientation of the surface relative to the wind. For example the windward wall generally has  $C_{p,e} = +0.7$ , while the side walls near the windward end of a building have  $C_{p,e} = -0.65$ . Other values are given in Tables 5.2(A), (B), (C), 5.3(A), (B) and (C).

The design external pressure at a given point depends not only on the site wind speed and the external pressure coefficient, but also on the 4 factors  $k_a$ ,  $k_c$ ,  $k_l$  and  $k_p$ , which are explained below

The area reduction factor  $k_a$  allows for the fact that peak wind gusts will not hit the whole of a building at the same time, so the bigger the area supported, the lower the average pressure. Thus large structural elements such as rafters and columns that support a big area of roof or wall can be designed for a smaller average pressure than can small elements such as purlins or girts. If you leave  $k_a = 1$  you will get a safe design but it may be conservative and therefore not the most economical.

The combination factor  $k_c$ , illustrated in Table 5.5 of the wind loading code, allows a reduction in design pressures for some situations where pressures acting on two or more surfaces of an enclosed building contribute to forces in a particular structural member, and are unlikely to be at their maximum values simultaneously. As with  $k_a$ , the maximum allowable reduction is only 20% and a safe but conservative design is obtained by ignoring this factor.

In contrast to  $k_a$  and  $k_c$ , the local pressure factor  $k_l$  actually requires an increase in design pressure for small areas which may experience local extreme pressures. It applies only to cladding, purlins, girts and battens which support areas less than  $a^2$  where "a" is defined as the minimum of 0.2b, 0.2d and h, b and d being the plan dimensions and h the height of a rectangular building.

The porous cladding reduction factor  $k_p$ , as its name implies, allow for a reduction in design pressures if the cladding is porous, and ignoring it will produce a safe but conservative design.

We are now in a position to calculate the aerodynamic shape factor  $C_{\rm fig}$  and hence the design external pressures acting on each part of an enclosed building. Let us take as an example an industrial building 30 x 36 m in plan with eaves 5 m high and a ridge 7 m high, giving a roof pitch of 7.6°. Portal frames are at 6 m centres as shown in Fig.3.18 below, with purlins and girts at 1.2 m centres.

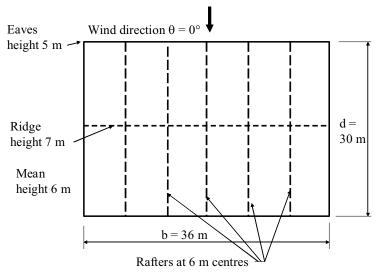


Figure 3.18 Plan View of Building Showing Dimensions

Values of  $C_{p,e}$  are shown in Fig.3.19 below for a cross wind (i.e. wind direction normal to long walls, defined as  $\theta = 0^{\circ}$ ).

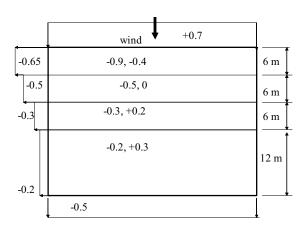


Figure 3.19 Plan View of Building Showing Values of C<sub>p,e</sub> for Cross Wind

### Area reduction factor ka

The rafters each support an area of  $30\times6=180~\text{m}^2$  of roof, so for the rafters  $k_a=0.8$  (Table 5.4). The columns each support  $30~\text{m}^2$  of wall, so for the columns  $k_a=0.9$  approximately (interpolating in Table 5.4). For purlins, girts and cladding,  $k_a=1$ .

### The combination factor $k_c$

Is tricky to evaluate, and it is argued below, always safe to ignore and best ignored in most cases. Clause 5.4.3 of AS 1170.2:2002 states that  $k_c$  may be applied "where wind pressures acting on two or more surfaces of an enclosed building contribute simultaneously to a structural action effect on a major structural element." Light portal frame buildings are the most common structures for which wind loads govern the design, and the action effect which determines member sizing is usually bending moment. The bending moment at any point in a statically indeterminate portal frame is influenced by loads on every member, i.e. it is true that "wind pressures acting on two or more surfaces ... contribute simultaneously" to the bending moment. Thus it would appear at fist sight that  $k_c$  should be evaluated.

To do this for the portal frame building shown in Fig.3.18, we must look at an end elevation of the building: see Fig.3.20 below. This loading case corresponds to case (b) in Table 5.5, so it would appear that  $k_c = 0.8$ , i.e. the design loads can be reduced by 20%. However this is not so.

According to Table 5.5,  $k_c = 1$  "where wind action from any single surface contributes 75% or more to an action effect." Thus to evaluate  $k_c$  it is necessary to perform an analysis of the portal frame with wind load applied to each single surface in turn to assess whether or not "wind action from any single surface contributes 75% or more" to the maximum bending moment in the frame. This was done for the frame shown in Fig.3.18 above for a typical but arbitrary wind loading, and as shown in Fig.3.21 below, it was found that

- 1. The bending moment due to wind load on the whole frame is 196 kNm at the windward rafter-column connection and 112 kNm at the leeward rafter-column connection,
- The bending moment due to wind load on the upwind slope of the roof alone is 94 kNm at the windward rafter-column connection and 153 kNm at the leeward rafter-column connection.

Thus wind action on the upwind roof slope produces 153/196 = 78% of the maximum bending moment in the frame due to the combined actions. In this situation it seems prudent to use the conservative value of 1 for  $k_c$ .

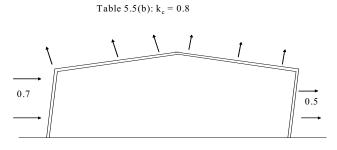


Figure 3.20

End Elevation Showing Wind Pressure Coefficients Contributing to Action Combination Factor k<sub>c</sub>

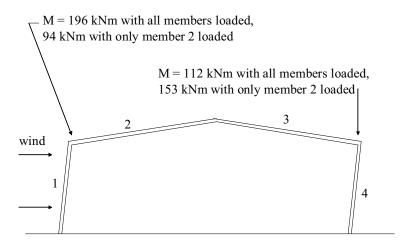


Figure 3.21 End Elevation Showing Bending Moments due to Loading on Upwind Roof Slope only, Compared with Moments due to Loading on Complete frame

In any case, Clause 5.4.3 of AS 1170.2:2002 states that "for all surfaces,  $k_c$  shall be not less than  $0.8/k_a$ ." But  $k_a=0.8$  for any rafter supporting more than  $100m^2$  of roof, so  $k_c$  shall be not less than 0.8/0.8=1, i.e.  $k_a$  does not apply. Rafters will generally support at least  $100m^2$  of roof on all but small buildings where the cost of the extra design time is likely to outweigh the savings achieved by applying  $k_c$ . Thus in most cases the designer is probably justified in using the default value of 1 for  $k_c$ , as this will always give a safe design and will save some design time and therefore money. In any case  $k_c$  is never less than 0.8 so the potential savings, especially on small structures, are fairly small.

### Local pressure coefficient $k_l$

For rafters and columns, the local pressure coefficient  $k_l$ , which allows for high local pressures acting over small areas, equals 1. For purlins and girts, the value of  $k_l$  will depend on (i) the location of the particular member on the building relative to the wind direction (see Fig.5.3), and (ii) the area supported by one span compared to the size of the building (see Table 5.6). From the note at the bottom of Fig.5.3, the value of the dimension "a" is the minimum of 0.2b, 0.2d and h.

For this particular building a = min [0.2(25), 0.2(30) and 6] = 5 m. Thus  $a^2 = 25 \text{ m}^2$  and  $0.25a^2 = 6.25\text{m}^2$ . Each purlin and girt span supports and area of roof  $A = span \times spacing = 6m \times 1.2m = 7.2 \text{ m}^2$ . Thus  $0.25a^2 < A < a^2$ . Referring now to Table 5.6 of AS 1170.2:2002,  $k_1 = 1.5$  for areas RA1 and SA 1 in fig.5.3, for a distance a = 5 m from the windward wall of the building. This means that the first 4 purlins in from the roof edge, which are within 5 m of the edge, must be designed for a 50% higher wind load than those purlins more than 5 m from the edge. The first 5 m of girts from the corner of the building must be similarly designed. In practice this will probably mean the first span, as shown in Fig.3.22.

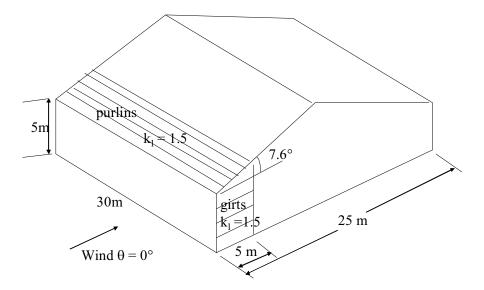


Figure 3.22 Local Pressure Coefficient  $k_l = 1.5$  for first 5 m back from Windward Wall on this particular building

When the wind strikes the opposite side of the building,  $k_l = 1.5$  for the first 5 m back from that windward wall, and when it strikes either end of the building ( $\theta = 90^{\circ}$ ),  $k_l = 1.5$  for the first 5 m back from each end of the building in turn. If the design wind speed is the same for all directions, these zones of  $k_l = 1.5$  can be combined on the same diagram as in Fig.3.23 below. If the roof pitch exceeds  $10^{\circ}$ , an additional area within a distance a from the ridge must also be designed with  $k_l = 1.5$ , and in many cases there will then be so little roof left with  $k_l = 1$  that it will probably be more economical to design the whole roof with  $k_l = 1.5$ , to save having to use different purlin sections on small areas.

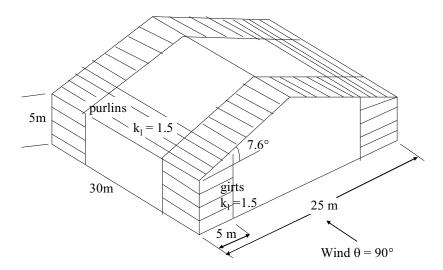


Figure 3.23 Local Pressure Coefficient  $k_l = 1.5$  for first "a" metres back from each wall to allow for different wind directions

# The permeable cladding reduction factor $k_p$

This reduction factor allows for a small reduction in negative external pressures acting on a permeable wall with a solidity ratio between 0.1% and 1%. The code gives no indication what sorts of walls might have such a permeability, and the design will be safe if  $k_p$  is taken as 1, i.e. ignored.

Thus  $C_{\rm fig}$  can now be calculated. For example for the columns in the windward side wall in Figs.3.21-3.25 above,  $C_{\rm fig} = C_{\rm p,e} \ k_a \ k_c \ k_l \ k_p = +0.7 \times 0.9 \times 1 \times 1 \times 1 = +0.63$  For the purlins within 5m of the roof edge,  $C_{\rm fig} = C_{\rm p,e} \ k_a \ k_c \ k_l \ k_p = -0.9 \times 1 \times 1 \times 1.5 \times 1 = -1.35$ 

### 3.4.6 Calculating internal pressures

The internal pressure coefficient  $C_{p,i}$ , as shown in Fig.3.14 above, may be either positive or negative, depending on the location of openings relative to the wind. For buildings with open interior plan,  $C_{p,i}$  is assumed to be the same on all internal surfaces of an enclosed building at any given time. Values of  $C_{p,I}$  are given in Table 5.1(A) and (B) of AS 1170.2:2002. If there is just one dominant opening,  $C_{p,i}$  corresponds to the external pressure coefficient  $C_{p,e}$  that would apply on the face where the opening is located. Thus for example a shed with one end open will experience  $C_{p,I} = +0.7$  when the wind blows directly into the open end, and  $C_{p,i} = -0.65$  for a cross wind, as shown in Fig.3.24 below. If the building is enclosed, or has openings distributed around faces which have both positive and negative external pressures, internal pressure coefficients are smaller.

To calculate internal pressures, look up internal pressure coefficients in Table 5.1 of AS 1170.2:2002 and multiply by the combination factor  $k_c$  if applicable, then multiply by the dynamic pressure  $q_z$ .

It can be difficult to estimate  $C_{p,i}$  because most industrial buildings have large roller doors that can constitute "dominant openings" if open. A dominant opening, especially on the windward side of a building, makes a huge difference to internal pressures and hence to design wind loads, which usually dominate the design of light steel buildings. Hence they greatly affect the cost of a building. So the designer must decide whether to assume roller doors are shut or whether to allow for the possibility that one or more could be open. In cyclonic areas there is plenty of warning of normal tropical cyclones, so it can be assumed that roller doors can be closed before the cyclone hits the building, and the same applies to strong winds in non-cyclonic areas. However so-called "mini-cyclones" frequently strike without warning and do significant damage in some areas, such as South Eastern Queensland. (These are really microbursts associated with thunderstorms). If we assume the roller doors could be open in a mini-cyclone, or could fail in a cyclone, we have to allow for dominant openings. If they are on the windward side of the building they result in large positive internal pressures, which combine with large external suction pressures near the windward edge of the roof to cause very large uplift loads.

However most engineers would not do this because it would result in an over-designed, uneconomical building. Statistically  $V_{1000}$  has a 0.1% chance of occurring in any one year, so it is unlikely that the building will ever experience it. But there is always a chance it will. So what do we do? One rationale for not allowing for full positive internal pressure is that the steel roof structure is ductile and will bend, distorting roof sheeting and allowing some equalisation of pressure between inside and outside before the roof literally flies away and starts killing people.

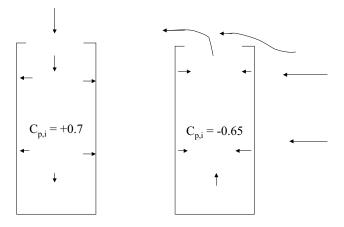


Figure 3.24 Internal Pressure Coefficients  $C_{p,i}$  for a building with a dominant opening (shown in plan view)

### 3.4.7 Frictional drag

The frictional drag force coefficient  $C_f$  ranges from 0.01 to 0.04 depending on the roughness of the surface. Unlike the pressure coefficients,  $C_f$  is used to calculate forces parallel to the building surface. Skin friction drag on buildings is easily calculated from Clause 5.5.

### Example

Wind blows parallel to the long walls of a building 6 m high, 50 m long and 20 m wide. Roof and wall sheeting have corrugations across the wind direction.  $q_z = 2$  kPa. Calculate the frictional drag.

### Solution

From Clause 5.5, 
$$h < b$$
, ∴ area  $A = (b + 2h)x(d - 4h) = (20 + 12) x (50 - 24)m^2 = 832 m^2$   
Drag force coefficient  $C_f = 0.04$   
∴  $F_f = C_f q_z A = 0.04 \times 2 \text{ kPa } \times 832m^2 = 66.6 \text{ kN}$ 

### 3.4.8 Net pressures

For enclosed buildings the design loads on structural elements are calculated from the most extreme net pressures, which are simply the algebraic sum of external and internal pressures. Care is necessary to get the signs right. For example positive pressure on both the outside and inside surfaces will tend to cancel out.

For plane structures such as free roofs, canopies, awnings, hoardings, free standing walls, etc., the net pressure coefficient  $C_{p,n}$  takes the place of  $C_{p,e}$  and  $C_{p,i}$ . These are dealt within Appendix D of the code.

### 3.4.9 Exposed structural members

Drag forces on exposed structural members such as lattice towers are covered on Appendix E of the code.

### 3.4.10 Worked Examples on Wind Load Estimation

#### **Example 3.4.10.1**

List the basic wind speeds V<sub>u</sub> for the following locations:

- (a) Perth
- (b) Cairns
- (c) Brisbane
- (d) Toowoomba.

#### Solution

The regional wind speeds  $V_{1000}$  for the following locations (see AS 1170.2 Clause 3.4):

- (a) Perth: 46 m/s (Region A1)
- (b) Cairns  $70F_c = 73.5 \text{ m/s}$  (Region C)
- (c) Brisbane 60 m/s (Region B)
- (d) Toowoomba. 46 m/s (Region A4)

### **Example 3.4.10.2**

Calculate the design wind speed  $V_{des,\theta}$  and the dynamic wind pressure  $q_z$  for the following sites, assuming  $M_s = M_t = 1$ :

- (a) A 7 m high portal frame building on an exposed site (cat 2) in Cairns, with b = 48 m and d = 15 m. Rafters at 6 m centres span 15 m. Purlins and girts are spaced at 1200 centres.
- (b) A 20 m high building on an exposed site in Pt Hedland, WA (Region D). For this flat roofed building, b = d = 15 m.
- (c) A 5 m high building in a built up area on the Gold Coast.
- (d) The upper portion of a 50 m high building on the Gold Coast without any significant shielding by adjacent structures. This building has a 30° roof pitch.

### Solution

- (a) A 7 m high portal frame building on an exposed site in Cairns.  $V_{1000} = 73.5$  m/s,  $M_{z,cat} = 0.97$  (Table 4.1(B)),
- $\therefore~V_{des,\theta} = 73.5~\text{x}~0.97 = 71.3~\text{m/s}$  and  $q_z = 0.6~V_{des,\theta}^{}^2~\text{x}~10^{-3}~\text{kPa} = 3.05~\text{kPa}$
- (b) A 20 m high building on an exposed site in Pt Hedland, WA.  $V_{1000} = 85F_D = 93.5$  m/s,  $M_{z,cat} = 1.13$

$$\therefore$$
 V <sub>des, $\theta$</sub>  = 93.5 x 1.13 = 105.7 m/s and q<sub>z</sub> = 0.6 V<sub>des, $\theta$</sub> <sup>2</sup> x 10<sup>-3</sup> kPa = 6.7 kPa

- (c) A 5 m high building in a built up area on the Gold Coast.  $V_u = 60 \text{ m/s}$ ,  $M_{z,cat} = 0.83$
- :. V  $_{des,\theta}$  = 60 x 0.83 = 49.8  $\approx$  50 m/s and  $q_z$  = 0.6 V  $_{des,\theta}$  2 x 10<sup>-3</sup> kPa = 1.5 kPa
- (d) The upper portion of a 50 m high building, with plan dimensions 30x30 m, roof pitch = 45°, wind direction  $\theta$  = 0°, on the Gold Coast, without any significant shielding by adjacent structures.  $V_u$  = 60 m/s,  $M_{z,cat}$  = 1.18

:. 
$$V_{des,\theta} = 60 \text{ x} 1.18 = 70.8 \text{ m/s} \text{ and } q_z = 0.6 V_{des,\theta}^2 \text{ x} 10^{-3} \text{ kPa} = 3.0 \text{ kPa}$$

### **Example 3.4.10.3**

Calculate the design external pressure  $p_e$  for the following, assuming  $k_a = k_l = k_p = 1$ :

- (a) The windward face of the building in Example 3.4.10.2(a).
- (b) The side wall near the windward face of the building in Example 3.4.10.2(a).
- (c) The windward portion of the roof of the building in Example 3.4.10.2(a) (roof pitch < 10°).
- (d) The leeward face of the building in Example 3.4.10.2(a).
- (e) The windward face of the building in Example 3.4.10.2(b).
- (f) The windward portion of the roof of the building in Example 3.4.10.2(b)(roof pitch < 10°).
- (g) The windward portion of the roof of the building in Example 3.4.10.2(d) (roof pitch =  $30^{\circ}$ ,  $\theta = 0$ ).

#### Solution

- (a) Referring to Table 5.2(A), height h < 25 m and this is not an elevated building. We are using a single value of  $q_{z,u} = q_h$ , the value at the top of the building, for the whole building.
- $\therefore$  C<sub>p,e</sub> = 0.7 for windward face of building q<sub>z</sub> = 3.05 kPa

$$p_e = C_{p,e}q_z = +0.7 \times 3.05 = 2.14 \text{ kPa}$$

(b) Referring to Table 5.2(C),  $C_{p,e} = -0.65$ 

$$\therefore p_e = C_{p,e}q_z = -0.65 \times 3.05 = -1.98 \text{ kPa}$$

(c) Referring to Table 5.3(A), in this case the ratio of height h to depth d (plan dimension in direction of wind, see Fig.3.4.3) is 7/15 < 0.5 so  $C_{p,e}$  can vary from -0.9 to -0.4. Uplift is usually the critical load on roofs, so the worst case will be -0.9.

$$\therefore p_e = C_{p,e}q_z = -0.9 \text{ x } 3.05 = -2.75 \text{ kPa}$$

- (d) Referring to Table 5.2(B),  $\theta = 0^{\circ}$  with  $\alpha < 10^{\circ}$ , so use left hand side of table. Also, d / b < 1 (wind is hitting long face of building), so  $C_{p,e} = -0.5$ .  $\therefore p_e = C_{p,e}q_z = -0.5 \times 3.05 = 1.53 \text{ kPa}$
- (e)  $q_z = 6.7$  kPa. Again this building is < 25 m high,  $q_z = q_h$  and it is not a highset building.  $\therefore p_e = C_{p,e}q_z = +0.7$  x 6.7 = 4.69 kPa
- (f) From part (e) above,  $q_z = 6.7$  kPa.

From Table 5.3(A), with h/d = 20/15 > 1 (extreme right side of table),  $C_{p,e}$  can range from -1.3 to -0.6. We will again take the maximum suction pressure.

$$\therefore p_e = C_{p,e}q_z = -1.3 \times 6.7 = -8.71 \text{ kPa}$$

(g) The windward portion of the roof of the building in Question 2(d)

(roof pitch = 
$$45^{\circ}$$
,  $\theta = 0$ ). $q_z = 3.0 \text{ kPa}$ 

From Table 5.3(B), with h / d = 50/30 > 1 and  $\alpha = 45^{\circ}$ ,  $C_{p,e} = 0$ ,  $0.8 \sin \alpha = 0$ , +0.57  $\therefore p_e = C_{p,e}q_z = 0$ , +1.7 kPa.

# **Example 3.4.10.4**

Calculate the design external pressure p<sub>e</sub> (see Clause 2.4.1, AS 1170.2) for the following:

- (a) A purlin within 2 m of the windward face of the building in Question 2(a). (Consider k<sub>1</sub>).
- (b) A rafter near the leeward end of the building in Question 2(a) for wind direction  $\theta = 90^{\circ}$  (hitting the end of the building). (Consider  $k_a$ ).

## Solution

- (a) From Fig.5.3 in AS1170.2, dimension "a" is min. of 0.2 x 15 and 0.2 x 48 and 7 = 3 m. Referring to Table 5.6 in AS1170.2, case WA1 applies only to windward walls, i.e. not applicable. Case RA1 does apply: Each span of the purlin we are considering supports an area of roof = 6 x 1.2 = 7.2 m<sup>2</sup> < 1.0a<sup>2</sup> = 9 m<sup>2</sup>, and is within 2 m i.e. < distance "a" = 3 m from a roof edge. So  $k_1$  = 1.5.
- :.  $P_e = 1.5 \text{ x } (-3.05) \text{ (from Example 3.4.10.3 (c))} = -4.6 \text{ kPa}$
- (b) A rafter near the leeward end of the building in Example 3.4.10.2(a) for wind direction  $\theta = 90^{\circ}$  (hitting the end of the building).

The area supported by one rafter =  $6 \times 15 = 90 \text{ m}^2$ . So from Table 3.4.4 in AS1170.2, we can interpolate and take  $k_a = 0.82$  approx.

Referring to Table 5.3(A) AS1170.2, with wind from end of building ( $\theta = 90^{\circ}$ ), and h/d = 7/48 < 1.0,  $C_{p,e}$  on downwind part of roof ranges from -0.2 to +0.2.

- : For max uplift,  $P_e = 0.82 \text{ x} (-0.2) \text{ x} (3.05)$  (Example 3.4.10.3 (a)) = -0.50 kPa, or
- :. For max downward force,  $P_e = 0.82 \text{ x } (+0.2) \text{ x } (-3.05) = +0.50 \text{ kPa}$ .

# **Example 3.4.10.5**

For the building in Example 3.4.10.2, a roller door on the windward side is open and this is estimated to constitute a "dominant opening" 3 times bigger than the total of openings on other walls and roofs subject to external suction (Table 3.4.7 in AS1170.2). Calculate:

- (a) the internal pressure
- (b) the net pressure difference acting on the sheeting supported by the purlin in Example 3.4.10.4 (a)
- (c) the line load W<sub>u</sub> on this purlin for ultimate strength design
- (d) the net pressure difference acting on the sheeting supported by the rafter in Example 3.4.10.4 (b)
- (e) the line load W<sub>u</sub> on this rafter for ultimate strength design.

#### Solution

- (a) For the building in Example 3.4.10.2 (a), a roller door on the windward side is open and this is estimated to constitute a "dominant opening" 3 times bigger than the total of openings on other walls and roofs subject to external suction (Table 5.1(A) in AS1170.2).
- $\therefore C_{p,i} = 0.6$
- ∴(a) the internal pressure =  $C_{p,i}$   $q_z$  = + 0.6 x 3.05 (from Example 3.4.10.3(a)) = + 1.83 kPa

(b) the net pressure difference acting on the sheeting supported by the purlin in Question 4(a) is the difference between the answers in Example 3.4.10.4 (a) and 5(a),

i.e. 
$$-4.6 - (+1.83) = -6.43$$
 kPa.

- (c) the line load W<sub>u</sub> on this purlin for ultimate strength design = area load x purlin spacing =  $-6.43 \text{ kN/m}^2 \text{ x } 1.2 \text{m} = -7.72 \text{ kN/m}$
- (d) the net pressure difference acting on the sheeting supported by the rafter in Example 3.4.10.4 (b) is the difference between the answers in Example 3.4.10.4 (b) and 5(a), i.e. either

$$-0.5 - (+1.83) = -2.33$$
 kPa or  $+0.5 - (+1.83) = -1.33$  kPa

Clearly the first case is gives a larger wind load, but we would also have to consider the combination of wind and dead load.

(e) the line load W<sub>u</sub> on this rafter for ultimate strength design = area load x rafter spacing =  $-2.33 \text{ kN/m}^2 \text{ x } 6\text{m} = -14 \text{ kN/m}$ 

# **Example 3.4.10.6**

A building is to be constructed in a built-up industrial area at Ernest on the Gold Coast. It will be 5 m high, 30m long and 12 m wide, with a flat roof. Rafters spanning 12 m, at 5m centres, are to support purlins at 1 m centres. There will be 2 roller doors, each 4.5m wide and 4.5 m high, in the 2 middle bays on one long side of the building. Calculate the design wind load for ultimate strength on (a) the rafters (b) the purlins.

#### Solution

- 1. Region B, assume importance level = 2, so return period = 500 years and regional wind speed  $V_R = V_{500} = 57 \text{ m/s}$  (Table 3.1, AS/NZS 1170.2:2002).
- 2. Terrain category 3 (normal built up area).
- 3. Height = 5 m.
- 4. Terrain and structure height multiplier  $M_{z,cat} = 0.83$  (Table 4.1(A)).
- 5. Assume Shielding multiplier & Topographic multiplier = 1.
- 6. Calculate Design wind speed  $V_{des,\theta} = V_R \ M_{(z,cat)} \ M_s \ M_t = 57 \ x \ 0.83 \ x \ 1 \ x \ 1 = 47.3 \ m/s$ 7. Dynamic wind pressure  $q_z = 0.6 V_{des,\theta}^2 \ x \ 10^{-3} = 0.6 \ x \ 47.3^2 \ x \ 10^{-3} = 1.34 \ kPa$
- 8. External pressure coefficients. Sketch the building and mark in external pressure coefficients  $C_{p,e}$  from Tables 5.2 and 5.3 on the roof of the building for 2 wind directions: (i)  $\theta = 0^{\circ}$  (wind normal to long side of building), and (ii)  $\theta = 90^{\circ}$  (wind parallel to long side of building). (The angle  $\theta$  is defined in the 1998 wind code but not in the 2002 code. It is retained in the present work as the authors consider it to be useful). Since we are only designing the rafters and purlins in this example, we need not worry about the walls.

Refer to Table 5.3(a) since roof slope  $\alpha < 10^{\circ}$ . For this building, h = 5 m, and for  $\theta = 0^{\circ}$ , d = 12 m, so h/d < 0.5, so we use the left hand column. On the roof,  $C_{p,e}$  can range from -0.9 to -0.4 for the first 5 m (0 to h/2 and h/2 to h). For the next 5 m (1h to 2h), C<sub>p,e</sub> can range from – 0.5 to 0, and from there to the leeward edge it can range from -0.3 to +0.1 (+0.2 in the 1998 code) So the external pressure coefficients that will give maximum upward load effect (which is usually the critical loading on flat or slightly sloping roofs) will be -0.9 for the first 5m, -0.5 for the next 5m and then -0.3. Those that will give maximum downward load effect will be -0.4 for the first 5m, 0 for the next 5m and then +0.1.

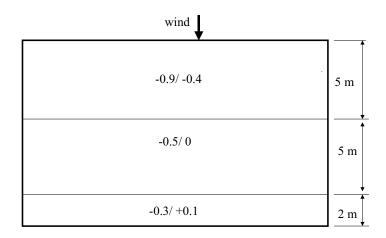


Figure 3.25  $C_{p,e}$  for Maximum Upward and Downward Wind Load,  $\theta = 0^{\circ}$ 

Repeat for  $\theta = 90^{\circ}$  (wind parallel to long side of building). The same procedure, but now d = 30m, h/d still < 0.5, and the only difference is that the building goes on to 6h before we reach the leeward wall. External pressure coefficients  $C_{p,e}$  are shown below.

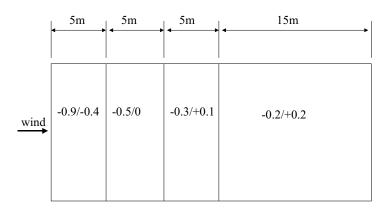


Figure 3.26  $C_{p,e}$  for Maximum Upward and Downward Wind Load,  $\theta = 90^{\circ}$ 

9. External pressures on rafters. The pressure varies across a rafter span for  $\theta=0^\circ$ , but is the same for all rafters. For  $\theta=90^\circ$ , the pressure is constant across one rafter, but changes from one rafter to another. To calculate external pressures we need the area reduction factor on rafters. Each internal rafter supports 12 x 5 = 60 m² of roof. Thus  $k_a$  interpolated from Table 3.4.4 = 0.87 approx. We do not need to consider local pressure factors  $k_l$  or porosity factors for rafters, so we can now calculate external pressures:

 $p_e = C_{p,e} \; k_a \; k_l \; k_p \; q_z$  (Clause 3.4.2) = -0.9 x 0.87 x 1 x 1 x 1.34 = 1.166  $C_{p,e}$ .  $p_e$  values for rafters are now calculated and marked in on the building outline as shown below (note that this must be done separately for rafters and purlins, as different pressure factors apply).

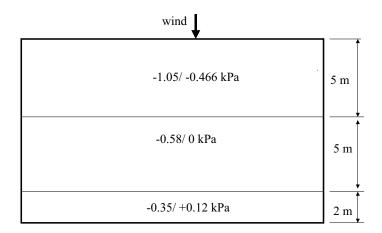


Figure 3.27  $p_e$  for Maximum Upward and Downward Wind Load,  $\theta = 0^{\circ}$ 

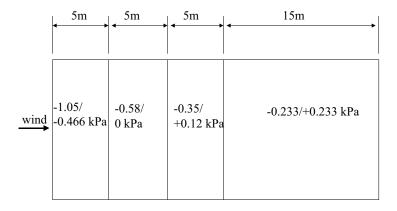


Figure 3.28  $p_e$  for Maximum Upward and Downward Wind Load,  $\theta = 90^{\circ}$ 

10. Design net pressures and line loads on rafters. Internal pressures must now be calculated. If the building is effectively sealed, internal pressure coefficients  $C_{pi}$  are taken as -0.2 or 0 (Table 5.1(A)), so in the present design the internal pressure can range from -0.233 kPa to 0. However if the roller doors are assumed to be open, they would constitute a dominant opening and the internal pressure coefficient would be equal to the external pressure coefficient on the wall containing the roller doors: +0.7 or -0.5 for  $\theta = 0^{\circ}$ , depending whether the roller doors are on the windward or the leeward side, and -0.3 for  $\theta = 90^{\circ}$ . Most designers would assume the roller doors would be shut during an extreme wind event. A negative internal pressure will tend to offset a negative external pressure. Thus the internal pressure will not increase the net pressure for maximum uplift. Taking a typical internal rafter when  $\theta = 0^{\circ}$  gives a maximum uplift on the windward 5 m of 1.05 kPa over the 5 m wide tributary area, or 5.25 kN/m line load on the rafter, as shown in Fig.3.30.

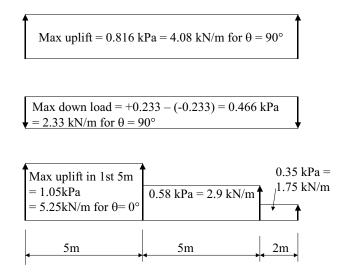


Figure 3.29 Maximum Upward and Downward Wind Load on Rafters

10. External pressures on purlins. Purlins must be designed for higher external pressures than those used for rafters because extreme pressure spikes can occur over small areas, which are too small to have a significant effect on a whole rafter, but can affect purlins. To calculate local pressure factors  $k_l$  for this building, refer to Table 5.6. The dimension "a" = minimum of 0.2b, 0.2d and h (Fig.5.3, footnote) =  $\min(0.2 \times 12, 0.2 \times 30, 5) = 2.4$  m.  $a^2 = 5.75$  m<sup>2</sup> > tributary area of purlins = 5m<sup>2</sup>, so  $k_l = 1.5$  applies to the first 2.4 m from each edge, since the wind can come from any side so that any side may be the windward side on which  $k_l$  applies. Thus the design maximum upward external wind pressure on purlins within 2.4 m of any edge is given by

Pe =  $C_{pe}$  k<sub>1</sub> q<sub>z</sub> = -0.9×1.5×1.34 = -1.81 kPa. This will also be the maximum net pressure in this case, since the maximum positive internal pressure adding to the effect of internal pressure = 0. Other design net pressures are as shown in Fig.3.31 below. Because the purlin spacing is 1m, the line loads are numerically equal to the pressures.

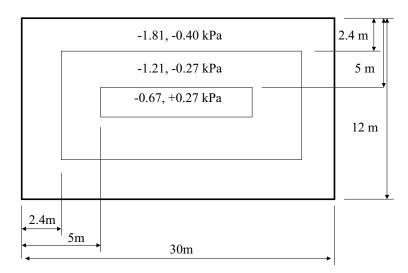


Figure 3.30 Net Pressures on Purlins for Maximum Uplift and Maximum Downward Force for Wind from any Direction,  $C_{p,i} = 0$ , -0.2

### 3.5 SNOW LOADS

The Australian/New Zealand standard AS/NZS 1170.3:2003[4] describe requirements for snow and ice actions. Only a small part of Australia is affected, but Australian engineers should still know something about this type of loading. The following is only intended as a very brief introduction, and the reader should consult the code for a more complete explanation.

Australian regions requiring design for snow and ice loads are set out in Clause 2.2. They are divided into Alpine (above 1200 m on the mainland and above 900 m in Tasmania), and subalpine (600-1200 m on the mainland and 300-900 m in Tasmania). Corresponding elevations for NZ are set out in Clause 2.3.

Section 3 deals with ice action, which must be considered on masts, towers, antennas etc. Section 4 deals with snow action. Snow loads s are given by

$$s = s_g C_e \mu_i$$

Where  $s_g$  = characteristic value of snow load on ground, in kPa, for the particular site, as listed in Table 5.2. For example Mount Kosciuszko has a 1 in 20 chance of 23.8 to 44.1 kPa, depending on the exact location, and a 1 in 150 chance of 35.7 to 66.2 kPa snow load on the ground in any given year. The corresponding figures are 6 and 9 kPa for Thredbo Village and 0.7 and 1.1 kPa for Guyra. For a 1 in 1000 chance, the 1 in 20 figures are multiplied by 2.

 $C_e$  is an exposure reduction coefficient, which allows the design snow loads on "semi-sheltered" roofs in alpine regions to be reduced to 75% of the values for sheltered roofs, and loads on "windswept" roofs to be reduced to 60%, provided there is no possibility that future construction may shelter the roof. Thus for most buildings  $C_e$  can be assumed = 1. The reader should study the code for further details.

 $\mu_i$  is a shape coefficient which depends on the slope and shape of the roof. For example in alpine regions a simple sloping roof has  $\mu_i$  ranging from 0.7 on slopes  $\alpha$  less than 10°, decreasing linearly to zero on slopes from 10 to 60°, i.e.  $\mu_i=0.7(60$  -  $\alpha)/50$ . Thus for example a roof with a 35° pitch would have  $\mu_i=0.7(60$  - 35)/50 = 0.35. For other roof shapes and for sub-alpine regions consult the code.

### 3.5.1 Example on Snow load Estimation

What is the design snow load for a simple pitched roof in a sheltered location at Thredbo Village with a 30° pitch?

Thredbo Village has an elevation of 1500 m, i.e. > 1200 m, so it is classed as alpine. If a 1 in 150 probability of failure is acceptable, the design snow load s would be given by

$$s = s_g C_e \mu_i = 9 \times 1 \times 0.7(60 - 30)/50 = 3.78 \text{ kPa}$$

### 3.6 DYNAMIC LOADS AND RESONANCE

Dynamic loads are caused by acceleration: F = ma. A constant acceleration will give a constant force, which can be treated as a static load. But a varying acceleration will give a varying force, which can result in resonant vibrations of a structure.

# Examples include:

- 1. vehicles accelerating, braking or crashing into barriers in car parks
- 2. cranes and other machinery in factories
- 3. earthquakes in which the ground, and hence the whole building, moves
- 4. unsteady flow of air or water around a structure.

Strictly wind loads are caused by acceleration of air moving past buildings, but the basic provisions of the Wind Code treat these as static loads because steady flow results in steady forces. This is a simplification which is good enough for most purposes, but will not always work. So we extend our treatment of wind loads to include a brief examination of unsteady effects.

### 3.6.1 Live loads due to vehicles in car parks

The loading code treats these in Clause 4.5 under "Live loads." It uses a simple formula based on: Work done to stop vehicle = force times distance = FS = kinetic energy lost =  $\frac{1}{2}$  m  $v^2$  Hence Force F = m  $V^2/2\Delta$ , where  $\Delta$  = distance.

# Example:

A crash barrier is to be designed to stop a 2000 kg vehicle travelling at 10 m/s in a distance of 0.15m (including deflection of vehicle). What force must the barrier be designed for?

#### Solution

 $F = m V^2/2\Delta = 2000 \times 10^2 / (2 \times 0.15) N = 667 kN$  (note that this treated as a live load Q, so limit state design requires a load factor of 1.5, i.e the design load = 1.5Q = 1000 kN.

#### 3.6.2 Crane, hoist and lift loads

Industrial buildings are commonly equipped with overhead travelling cranes, which run on rails supported by the building structure. The loads imposed by these cranes must therefore be allowed for. However the design of cranes is specialised field and structural engineers designing buildings containing overhead cranes do not normally design the cranes themselves. The procedure for estimating loads due to an overhead travelling crane can be found in many factories. For other types of cranes and for hoists and lifts, see AS1418.

# 3.6.3 Unbalanced Rotating Machinery

As far as practicable, machine designers try to balance rotating masses in machinery to avoid dynamic loading. But sometimes it is impossible or impracticable to avoid, e.g. in crushing and screening plant, and even supposedly balanced machinery can give rise to vibrations.

The graph below shows the magnification ratio M, which is the ratio of dynamic to static displacement, plotted against  $\omega/\omega_n$ , the ratio of actual frequency to natural frequency.  $\zeta(\text{zeta})$  is the damping ratio. Thus for example the dynamic displacement will beat least 3 times the static displacement for frequencies within 10% of the natural frequency, unless the damping ratio is well over 0.1.

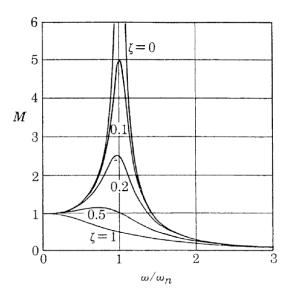


Figure 3.31 Magnification of Static Displacements and Stresses due to Dynamic Resonance

The structural engineer must be aware of the possibility of resonance and must be able to design structures so as to avoid large resonant vibrations. The basic equations are very simple, and you will probably recall them from physics:

- (i) Force = mass x acceleration
- (ii) Natural period of vibration  $T = 2\pi\sqrt{(m/k)}$ , or natural circular frequency  $\omega_n = \sqrt{(k/m)}$ , or natural frequency f = 1/T, where m is the vibrating mass and k is the stiffness, or restoring force per unit displacement.

**Example:** A 10 tonne machine is mounted centrally on two parallel simply supported beams, each 3m long, of 310UB32 section. It is rotating at 600 RPM. Calculate the period of resonant vibration of this system and suggest how the structure can be modified to avoid resonance.

### Solution

Stiffness of beams is given by the formula for deflection of a simply supported beam carrying a point load at mid span, i.e.  $y = PL^3 / (48EI)$ 

Or k = force per unit deflection  $P/y = 48EI/L^3$ 

=  $48 \times 200,000 \text{N/mm}^2 \times (2 \times 63.2 \times 10^6 \text{ mm}^4) / 3000^3 \text{ mm}^3$ 

 $= 44,800 \text{ N/mm} = 44.8 \times 10^6 \text{ N/m}$ 

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{10000 kg}{44.8 \times 10^6 kg/s^2}} = 0.0939 \text{ sec} \approx 0.094 \text{ sec.}$$

Thus this machine will tend to experience problems with resonant vibration.

Either the beams must be made stiffer, to increase the natural frequency, or less stiff to reduce it, or the structure must be modified in some other way, and/or damping must be introduced.

Note that multiples of the natural frequency should also be avoided.

### 3.6.4 Vortex shedding

When a fluid (e.g. air or water) flows past a bluff (i.e. not streamlined) body such as a building, a chimney, a bridge deck or a bridge pier, vortices are shed alternately on either side. If the frequency of vortex shedding corresponds to the natural frequency of a structure with little damping, dangerous vibrations may result.

Vortex shedding caused the famous failure of the Tacoma Narrows suspension bridge in Washington.

As shown above, natural frequencies of point masses on elastic beam type supports are easily calculated. When the mass is distributed along the elastic member it is not quite so simple. However you can get Spacegass to calculate it.

Vortex shedding frequencies are also simple to calculate approximately from the following formula [5]:

Frequency  $n = S V_o / d$ 

where

S = Strouhal number = 0.2-0.3 approx (it varies to some extent with Reynolds number),  $V_0 = \text{flow velocity}$  and d = diameter.

**Example:** Calculate the vortex shedding frequency of flow past a 1 m diameter chimney if the wind velocity is 5 m/s.

#### Solution

$$n = S V_0 / d = 0.2 \times 5/1 = 1 \text{ Hz}$$

There is one cycle of force causing oscillation in the direction of the wind for every vortex shed, and 1 cycle at right angles to the wind direction for every pair of vortices. Thus for the above example there would be 2 critical frequencies, 1 and 0.5 Hz.

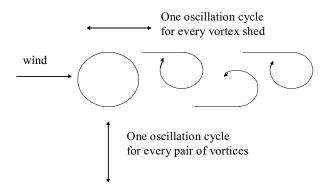


Figure 3.26 Oscillations Induced by Vortex Shedding

# 3.6.5 Worked Examples on Dynamic Loading

### **Example 3.6.5.1 Acceleration loads**

A 1 tonne car rounds a 10 m radius bend on a roller coaster ride at 20 m/s. What radial load (1.5Q) must be designed for? (Centripetal acceleration  $a = v^2/r$ ).

#### Solution

Force = 
$$ma = mv^2/r = 1000 \text{ kg} \text{ x} (20 \text{ m/s})^2 / 10 \text{ m} = 40,000 \text{ N} = 40 \text{ kN}$$

## Example 3.6.5.2 Crane loads

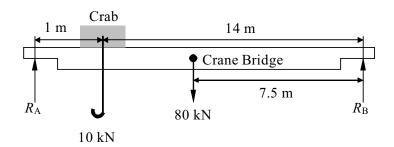
For an overhead crane with the details listed below calculate the factored design wheel load.

#### *Client requirements:*

- Maximum safe lifted weight (SWL) = 150 kN
- Crane rail AS41
- Crane span  $\approx$  portal frame span = 15 m
- Intended purpose of crane: Frequent light loads, occasional loads near SWL
- Required hoisting speed  $V_H = 0.42 \text{ m/s}$

### Data supplied by crane manufacturer:

- Crane frame self weight 80 kN (8 Ton)
- Trolley (crab) + hook = 10 kN (1 Ton)
- Maximum safe lifted weight (SWL) = 150 kN, Class 3
- Nearest approach of hook to rail (end carriage return) = 1 m
- Wheel base SG or C = 2500 mm (wheel distance)
- Load due to oblique travelling  $P_{ot} = 13.8 \text{ kN}$
- Lateral inertia loads P<sub>bb</sub> = 20.1 kN
- Lateral inertia loads P<sub>hc</sub> = 1.8 kN
- Longitudinal inertia  $P_{ht} = 12 \text{ kN}$



#### Solution

### Static vertical loads

Maximum static vertical wheel reaction =  $P_{GH} + P_{H}$  where

 $P_{GH}$  = component of wheel load due to frame plus trolley, i.e. those parts that don't move vertically

 $P_H$  = component of wheel load due to SWL carried by hook (ignoring weight of hook itself) i.e. the part that moves vertically.

We will calculate these two components of the wheel reaction separately because we must apply and extra dynamic factor to the vertically moving load to account for upwards acceleration and braking while lowering the load.

Looking at the crane in end elevation and taking moments about B, the "far side" rail, for the effect of parts that don't move vertically:

$$15R_A = 7.5 \times 80 + 14 \times 10$$
  
 $R_A = 49.33 \text{ kN}$   
 $P_{GH} = \frac{1}{2} R_A (2 \text{ wheels}) = 49.33 / 2 = 24.67 \text{ kN}$ 

Now for parts that do move vertically:

$$15R_A = 14 \times 150$$
  
 $R_A = 140 \text{ kN}$   
 $P_H = \frac{1}{2} R_A = 70 \text{ kN}$ 

# Dynamic magnification factors

Duty factor  $K_D = 1.1$  for class 3 cranes (this is applied to all moving loads, i.e. crane + hook load).

```
Hoisting factor K_H = 1.15 + 0.35V_H where V_H = hoisting speed in m/s = 1.3 for this particular crane (applies only to SWL) 
 \therefore Estimated maximum vertical wheel load = P_{GH}K_D + P_HK_DK_H
```

 $= 24.67 \times 1.1 + 70 \times 1.1 \times 1.3 = 127.2 \text{ kN}$ 

This estimated actual load is treated as a live load Q on the runway beam, since it comes and goes.

Thus the factored design vertical wheel load  $1.5Q = 1.5 \times 127.2 = 190.8 \text{ kN}$ 

There will of course be 2 of these wheel loads on one rail, spaced 2500 mm apart in this case.

Lateral loads (horizontal loads at right angles to the runway beam)

$$\begin{aligned} &{P_{ot}}^* = 1.5 \text{ x } 13.8 = 20.7 \text{ kN} \\ &{P_{hb}}^* = 1.5 \text{ x } 20.1 = 30.15 \text{ kN} \\ &{P_{hc}}^* = 1.5 \text{ x } 1.8 = 2.7 \text{ kN} \end{aligned}$$

Longitudinal loads (parallel to the runway beam)

$$P_{ht}^* = 1.5 \times 12 = 18 \text{ kN}$$

This load will have very little effect on the crane runway beam but the bracing between the portal frames will have to be designed to withstand this load.

# **Example 3.6.5.3 Unbalanced machines**

A machine weighing 20 tonnes is to be supported on a grid of 4 simply supported beams spanning 10 m.

- (i) Select a suitable beam section such that the maximum stress under static loading does not exceed 100 MPa.
- (ii) Calculate the critical or resonant frequency of the system, treating the machine as a point mass and ignoring the self weight of the beams.
- (iii) Suggest an alternative structure to increase the natural frequency.

#### Solution

Assuming each beam takes 20/4 = 5 tonnes  $= 5 \times 9.8 = 49$  kN as a point load at midspan,  $M^* = PL / 4 = 49 \times 10 / 4 = 122.5$  kNm

Max stress  $\sigma_{max} = M/Z \le 100 \text{ MPa}$ 

Elastic section modulus  $Z \ge 122.5 \times 10^6$  N mm / 100 N/mm<sup>2</sup> =  $1225 \times 10^3$  mm<sup>3</sup> Select 460UB67.1 with  $Z_x = 1300 \times 10^3$  mm<sup>3</sup>

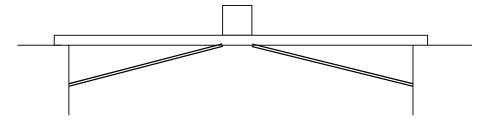
Mid span deflection under static central point load  $y = PL^3/48EI = 17.2 \text{ mm}$ 

- ∴ Stiffness of one beam  $k = P/y = 48EI/L^3$
- =  $(48 \times 200,000 \text{ N/mm}^2 \times 296 \times 10^6 \text{ mm}^4)/(10,000 \text{ mm})^3 = 2842 \text{ N/mm} = 2.84 \times 10^6 \text{ N/m}$
- :. Natural period of vibration  $T = 2\pi \sqrt{(m/k)} = 2\pi \sqrt{(5000 \text{kg}/(2.84 \text{x} 10^6 \text{kg.m.s}^{-2}.\text{m}^{-1})} = 0.263 \text{ sec}$

To increase the natural frequency we need to make the structure stiffer. We could do this be using two deeper beams with the required total  $Z \ge 4x1225 = 4900x10^3 \text{ mm}^3$  For example two of 610UB101 would give almost the same Z ( $2x2530 = 5060x10^3 \text{ mm}^3$  instead of  $5200x10^3 \text{ mm}^3$ , but total I would increase from  $4 \times 296 = 1184x10^6 \text{ mm}^4$  to  $2x761 = 1522x10^6 \text{ mm}^4$ . This would increase stiffness a bit, but maybe not enough. (A single I section would not be stable, as it would not have enough torsional stiffness).

Or we could go to 2 of 800WB122 (2 x 122 = 244 kg/m) without increasing weight and therefore at about the same cost as the original 4 of 460UB67.1 (4 x 67.1 = 268 kg/m). This would increase I to 2 x 1570 = 3140 x  $10^6$  mm<sup>4</sup>, increasing k by a ratio of 3140/1184 = 2.65, and hence natural frequency by a ratio of  $\sqrt{2.65} = 1.63$ . This may be enough.

If not, a much more effective way to increase stiffness drastically is to create a triangulated structure by adding diagonals, possibly as shown below, although this will depend on the particular situation.



#### Example 3.6.5.4 Vortex shedding

A 1 m/s current flows past a series of jetty piers 0.4 m in diameter. Assuming S = 0.2, calculate the frequency of vortex shedding.

### Solution

Frequency  $n = S V_0 / d = 0.2 \times 1 / 0.4 = 0.5 \text{ Hz}$ , i.e. period of 2 sec

# 3.7 EARTHQUAKE LOADS

Earthquake loads are treated as a separate load category. However they are in fact a form of dynamic loading because they are caused by the acceleration of the ground, the inertia and the elastic behaviour of a structure. Earthquakes are of concern mainly with buildings of large mass and low ductility, e.g. unreinforced masonry, because large forces are needed to accelerate large masses and low ductility means there is little "give," i.e. little capacity in the structure for energy absorption.

Steel structures tend to be less susceptible to earthquakes than masonry buildings, but a steel framed structure such as a car park with heavy floor slabs and large live loads, or a structure supporting heavy machinery or other mass such as the tank stand, may be susceptible. Part 4 of the Australian loading code, AS1170.4[6], deals with earthquake loading. A brief introduction to earthquake load estimation is provided below. For a more comprehensive general introduction to the topic, the reader should study articles such as Hutchinson et al [7], which includes worked examples, and McBean [8], which discusses a specific example of an eccentrically braced frame.

# 3.7.1 Basic concepts

In order to start to understand how structures are affected by earthquakes, it is necessary to understand a few basic concepts. These are introduced below.

- 1. **Ground acceleration**. Earthquakes can cause movement in any direction with accelerations up to about 0.2g or more. Early codes assumed an acceleration coefficient a  $= 0.1g = 0.98 \text{ m/s}^2$ . Thus a rigid building on a rigid foundation would be designed to resist a force F = ma = m approx. (note this is mass. not weight). So a building with a mass of 1000 kg would have a downward gravitational force of mg = 9.8 kN plus an earthquake load of approx 1 kN in any direction. Being only 10% of the gravity load its effects are negligible in the vertical direction, but they may be significant in any horizontal direction. So we must design for horizontal shear forces at each level depending on the mass of the structure above the level we are considering.
- 2. **Periods of vibration.** Various periods are present but as shown in the earthquake accelerogram, Fig.4 of Hutchinson et al's article[7], they are mainly much less than 1 sec. Since earthquakes contain most of their energy in the range of 1 to 10 Hz generally squat buildings don't do well in an earthquake and tall buildings often do.
- 3. In the Australian code, AS 1170.4, horizontal shear forces are calculated from:

 $V = I(CS/R_f)G_g$ 

where,

V = total horizontal earthquake base shear.

I = importance factor

AS1170.4 Table 2.5

```
\begin{array}{l} C = \text{earthquake design coefficient} = 1.25 \text{a}/\text{T}^{2/3} \\ \text{a} = \text{acceleration coefficient} \\ T = \text{period of vibration of building} = \text{h} \ / \ 46 \ \text{approx, where h} = \text{height in m.} \\ S = \text{site factor} \\ R_f = \text{structural response factor} \\ G_g = \text{gravity load (the weight G} + \psi_c \ Q \ \text{above the plane considered)} \end{array}
```

Codes allow for different acceleration coefficients in different regions depending on their history of seismic activity. Information in Australia is sketchy due to our short history, sparse population and relative geological stability - the contours in an area are often based on a single event - see for example Meckering in WA, which has a = 0.22, based on one event in the 1960s, and Tennant Creek in NT with a = 0.15 based on another event in the 1980s (see AS 1170.4, Figs.2.3 (a) and (e)). The Meckering and Tennant Creek earthquakes were major seismic events comparable in intensity to events which have killed thousands of people in more densely populated areas. They occurred in areas hitherto assumed to be seismically inactive and did little damage only because there was not much there to be damaged. They show that a major earthquake could hit anywhere anytime, and although the probability is low, the potential for loss of life and damage to infrastructure is high, so we need to take the possibility seriously.

### 3.7.2 Design procedure

- 1. Assuming it is not a structural alteration to an existing building and AS 1170.4 is applicable (check Clause 1.1), decide if is a domestic structure (< 8.5 m high, < 16 m wide) or a general structure.
- 2. If a "general structure," decide if it is "type III" (essential to post earthquake recovery). If so, importance factor I = 1.25, otherwise I = 1.
- 3. Determine acceleration coefficient "a" from map (ranging from 0.22 for Meckering, 0.15 Tennant Creek, 0.12 Bundaberg, down to 0.03 around Longreach and on the Nullarbor.
- 4. Determine period T = h / 46 approx, where h = height in m, for regular shaped buildings.
- 5. Hence earthquake design category from structure type, site factor and earthquake design coefficient  $C = 1.25a/T^{2/3}$ .
- 6. Calculate gravity load  $G_g = G + \psi_c Q$  (see AS 1170.1) at each floor level.
- 7. Determine Site factor S from Table 2.4(a) or (b). S ranges from 0.67 (best) for a rock base to 2 (worst) for loose sand to 12 m or more depth.
- 8. Determine Structural response factor R<sub>f</sub> (Table 6.2.6(a) or (b)). This depends on how brittle or ductile the structure is. It ranges from 8 (best) for special moment resisting frames of steel or reinforced concrete (e.g. eccentrically braced frames), through ordinary moment resisting frames of steel or reinforced concrete, down to 1.5 (worst) for unreinforced masonry shear walls.
- 9. Put them all together in  $V = I(CS/R_f)G_g$  to get shear force V.
- 10. Design building to take design shears at each storey level.

# 3.7.3 Worked Examples on Earthquake Load Estimation

#### Example 3.7.3.1 Earthquake Loading on a Tank Stand

Calculate the earthquake loading on a steel tower used as a water tank stand. The tank stand has 4 legs of 100x100x6 EA, on a 3m square plan, 15 m high. Cross diagonal bracing is 50x50x5 EA at  $45^{\circ}$  in both directions. The tank is 5m in diameter, 2.4m high, weighing 5 kN empty. The tank stand is located in Bundaberg, take S = 1.

#### Solution

$$V=I(CS/R_f)G_g$$

Importance factor I = 1 for normal buildings

Acceleration coefficient a = 0.12 for Bundaberg

 $T \approx h / 46 = 17.4/46 = 0.378$  sec (this can be checked later, using the stiffness of the tower, since the tank is effectively a point mass on an elastic cantilever. Clearly the stiffness will depend on the sections used for legs.)

: Earthquake design coefficient C = 1.25a /  $T^{2/3}$  = 1.25x0.12 / 0.378 $^{2/3}$  = 0.287

Structural response factor  $R_f = 2.1$ 

AS1170.4 Table 6.2.6(b)

G = self weight of tank = 5 kN

Q = weight of water in the tank when it's full =  $(\pi D^2/4)$  x h x  $\gamma_{water}$ 

$$= (\pi \times 5^2/4) \times 2.4 \times 10$$

= 470 kN

 $G_g = G + Q$  (since it is likely that the tank will be full)

$$= 5 + 470 = 475 \text{ kN}$$

Since there is only one level at which there is significant mass, we need only calculate one shear force V

$$\therefore$$
 V= I (CS/R<sub>f</sub>) G<sub>g</sub> = 1(0.287x1/2.1) x 475 kN = 64.92 kN

# **Example 3.7.3.1 Earthquake Loading on a Multistorey Building**

4 storey steel framed office block located in Newcastle, NSW, has floor heights at 4, 8 and 12 m above ground level and the roof at 16 m. The structural frame is an ordinary moment resisting frame. The total weight at each floor level is 1600 kN, including slab, walls and live load, and 1000 kN at roof level. Geotechnical investigations indicate a site factor S = 1.5. Calculate earthquake loads at each storey level.

# Solutions

 $V = I (CS/R_f) G_{\sigma}$ 

Importance factor I = 1 for normal buildings

Acceleration coefficient a = 0.12 for Newcastle

 $T \approx h/46 = 16/46 = 0.348 \text{ sec}$ 

: Earthquake design coefficient C =  $1.25a / T^{2/3} = 1.25 \times 0.12 / 0.348^{2/3} = 0.303$ 

Structural response factor  $R_f = 4.5$ 

AS1170.4 Table 6.2.6(b)

 $G_g = G + \psi_c Q = 1000$  kN at roof, plus 1600 kN at each of 3 floor levels, i.e. 1000 + 3x1600 = 5800 kN

Base shear 
$$V = I(CS/R_f)G_g = 1(0.303x1.5/4.5)x5800 \text{ kN} = 586 \text{ kN}$$

This is balanced by horizontal earthquake forces at each floor. These forces are calculated using the following formula:

$$F_{x} = C_{vx}V = \frac{G_{gx}h_{x}^{k}}{\Sigma G_{gi}h_{i}^{k}}V$$

i.e. the force at each level = the base shear V times the coefficient  $C_{vx}$  for that floor. In this case, k = 1 since T < 0.5 sec. Thus the equation simplifies to:

$$F_{x} = C_{vx}V = \frac{G_{gx}h_{x}}{\Sigma G_{gi}h_{i}}V$$

 $C_{vx}$  = (floor weight times height) divided by the sum of all (floor weight times height) (i.e.  $\Sigma G_{gi}h_i = 1600x4+1600x8+1600x12+1000x16 = 54400 \text{ kNm}$ ). The forces at each floor height are shown in the Table 3.2

Note that the cumulative shear  $V_x$  which the columns and bracing (if any) must withstand at each storey level is made up of the sum of the inertial earthquake forces acting on all floors above. This shear will cause bending moments in columns in a moment resisting frame (no diagonal bracing), or axial forces in a braced frame.

The cumulative shear  $V_x$  will be shared between the two columns at the ground level  $V_x/\text{col.} = 586/2 = 293 \text{ kN}$ 

The moment acting on each column at ground level  $M_x^* = 293 \text{ x } 4 = 1172 \text{ kNm}$ 

The moment  $M_{\rm x}^*$  will cause tension in the windward columns and compression in leeward columns.

Taking the free body diagram of the whole structure and then taking the summation of moment about any of the two base supports we can get the axial forces in the lower two columns.

$$R \times 8 = 69 \times 4 + 138 \times 8 + 207 \times 12 + 172 \times 16$$
  
 $R = 827 \text{ kN}$ 

Table 3.2

| Storey | G <sub>gx</sub> (kN) | h <sub>x</sub> (m) | $C_{vx} = G_{gx}hx / \sum G_{gi}h_i$ | F <sub>x</sub> (kN) | V <sub>x</sub> (kN) |
|--------|----------------------|--------------------|--------------------------------------|---------------------|---------------------|
| Roof   | 1000                 | 16                 | 16000/54400 = 0.294                  | 0.294x586 = 172     | 172                 |
| 3      | 1600                 | 12                 | 19200/54400 = 0.353                  | 0.353x586 = 207     | 379                 |
| 2      | 1600                 | 8                  | 12800/54400 = 0.235                  | 0.235x586 = 138     | 517                 |
| 1      | 1600                 | 4                  | 6400/54400 = 0.118                   | 0.118x586 = 69      | 586                 |
| Ground | -                    | 0                  | -                                    | -                   | 586                 |
| Σ      |                      |                    |                                      | 586                 |                     |

### 3.8 LOAD COMBINATIONS

# 3.8.1 Application

The purpose of load combinations is to obtain the most critical condition for which the structure must be designed.

#### 3.8.2 Strength Design Load Combinations

The following load combinations must be considered when designing for strength

### (a) Dead load

1.35G

- (b) Live load
  - (i) 1.2G + 1.5Q
  - (ii) 0.9G + 1.5Q
- (c) Wind load
  - (i)  $1.2G + W_u + \psi_c Q$
  - (ii)  $0.9G + W_u$
- (d) Earthquake
  - (i)  $G + \psi_c Q + E_u$
- (e) Snow, liquid pressure, earth pressure and ground water pressure
  - (i)  $1.2G + S_u + \psi_c Q$
  - (ii)  $0.9G + S_u$

where  $S_u$  is the snow load or liquid pressure or earth and/or ground water pressure.

# 3.8.3 Serviceability Design Load Combinations

The following load combinations must be considered when designing for serviceability.

- (a) Short-term effects
  - (i) G
  - (ii)  $G + W_s$
  - (iii)  $G + \psi_s Q$
- (b) Long-term effects
  - (i) G
  - (ii)  $G + \psi_l Q$

 $\psi_s$ ,  $\psi_l$  and  $\psi_c$ , varies from 0.0 to 1.0 depending on the usage of the structure to be designed. the values for  $\psi_c$ ,  $\psi_s$  and  $\psi_l$  are given in Tables 3.3 and 3.4

| Type of Live Load | Combination factor (ψ <sub>c</sub> ) |  |  |  |  |
|-------------------|--------------------------------------|--|--|--|--|
| <u>Floors</u>     |                                      |  |  |  |  |
| Domestic          | 0.4                                  |  |  |  |  |
| Office            | 0.4                                  |  |  |  |  |
| Parking area      | 0.4                                  |  |  |  |  |
| Storage area      | 0.6                                  |  |  |  |  |
| Other             | 0.6, unless otherwise assessed       |  |  |  |  |
| Roofs             |                                      |  |  |  |  |
| Trafficable       | 0.4                                  |  |  |  |  |
| Non-trafficable   | 0.0                                  |  |  |  |  |

Table 3.3 Live Load Combination Factor for Strength Design

Table 3.4 Live Load Factors for Serviceability Design

| Type of Live Load | Short-term factor (ψ <sub>s</sub> ) | Long-term factor $(\psi_l)$ |
|-------------------|-------------------------------------|-----------------------------|
| Floor             |                                     |                             |
| Domestic          | 0.7                                 | 0.4                         |
| Offices           | 0.7                                 | 0.4                         |
| Parking area      | 0.7                                 | 0.4                         |
| Retail store      | 0.7                                 | 0.4                         |
| Storage           | 1.0                                 | 0.6                         |
| Other             | As for storage, unless              | otherwise assessed          |
| Roofs             |                                     |                             |
| Trafficable       | 0.7                                 | 0.4                         |
| Non-trafficable   | 0.7                                 | 0.0                         |

## 3.9 REFERENCES

- 1. Australian/New Zealand standard .AS/NZS 1170.1:2002 Permanent, imposed and other actions.
- 2. Holmes, J.D. and Syme, M.J. (1994), *Wind loads on Steel-Framed Low-Rise Buildings. Steel Construction* Vol.28, No.4, Dec., pp.2-12).
- 3. Australian/New Zealand standard. AS/NZS 1170.2:2002 Wind Actions.
- 4. Australian/New Zealand standard. AS/NZS 1170.3:2002 Snow and Ice Actions.
- 5. Roberson, J.A. and Crowe, C.T. (1997). *Engineering Fluid Mechanics*, *Article* 11.3).
- 6. Standards Association of Australia (1993). AS 1170.4 Earthquake Loads.
- 7. Hutchinson, G.L., Pham, L. and Wilson, J.L. (1994). Earthquake Resistant Design of Steel Structures-an Introduction for the Practising engineer. Steel Construction, Vol.28 No.2 June, pp.6-22.
- 8. PC McBean. (1997). Steel Construction, Vol.31 No.1, March, pp.2-11.

# 4 METHODS OF STRUCTURAL ANALYSIS

### 4.1 INTRODUCTION

Although analysis is an essential stage in design, some textbooks on the design of steel structures pay little attention to this stage. Textbooks on structural analysis, on the other hand, can be rather divorced from the practicalities of design. Section 4 of AS 4100[1] is entitled "methods of structural analysis," and it contains some guidelines and rules, but on its own it is not enough to guide the designer through the analysis process. AS 4100 Supplement – 1999, the Steel Structures Commentary [2], contains further explanation and lists a large number of references on analysis. The present chapter does not attempt to duplicate existing references but provides some brief explanatory material including diagrams and examples where it is felt that these will help to clarify the provisions of AS 4100[1].

#### 4.2 METHODS OF DETERMINING ACTION EFFECTS

AS 4100[1] Clause 4.1.1 refers to three methods of analysis: elastic, plastic and "advanced." Most analysis is now done using commercial software packages such as Spacegass[3], Microstran[4] and Multiframe[5], using elastic methods, and this chapter will focus mainly on this approach, with a brief section on plastic analysis. Neither the code nor the commentary give any guidance as to how "advanced" analysis may be carried out.

Clause 4.1.2 of AS 4100[1] defines braced members, in which transverse displacement of one end of the member relative to the other is prevented, and sway members, in which such displacement is allowed. Examples of braced members include all the members in a braced frame Fig.4.1 (b) and the horizontal members in a rectangular sway frame Fig.4.1 (a). Examples of sway members include the vertical members in a rectangular sway frame Fig.4.1 (a) and the columns and rafters in a pitched roof portal frame.

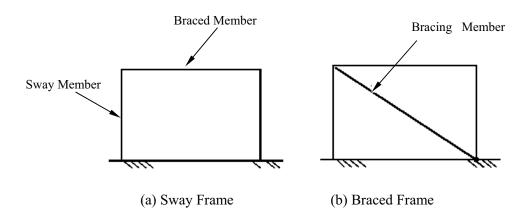
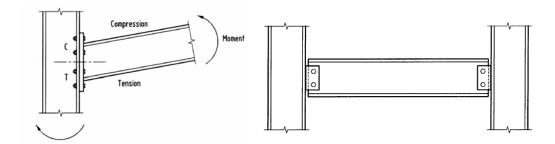


Figure 4.1 Sway and Braced Frame

### 4.3 FORMS OF CONSTRUCTION ASSUMED FOR STRUCTURAL ANALYSIS

Clause 4.2 of AS 4100[1] distinguishes between "rigid," "semi-rigid" and "simple" construction. Most designers assume either rigid construction in which the angles between members do not change, or simple construction in which connections are assumed not to develop bending moments. Examples are shown in Fig.4.2.

Semi-rigid construction, in which connections provide some flexural restraint but may not maintain original angles, may exist in reality but is more difficult to analyse and this analysis is usually avoided in practice.



- (a) Rigid Rafter-Column Connection
- (b) Flexible Beam-Column Connection

Figure 4.2 Rigid and Flexible Connections

## 4.4 ASSUMPTIONS FOR ANALYSIS

Clause 4.3 deals with some assumptions that can be made to simplify structural analysis in some situations. For example the analysis of regular shaped structures with a large number of members can be simplified by treating sub-structures in isolation from the rest of the structure.

#### 4.4.1 Treating sub-structures in isolation from the rest of the structure

For example the regular three-dimensional structure shown in Fig.4.3(a) below has 3 storeys, 4 bays in the X direction and 1 bay in the Z direction. According to Clause 4.3.1(a), this could be treated as a series of two-dimensional frames of 3 storeys and 4 bays in the x-y plane, and a series of two-dimensional frames of 3 storeys and 1 bay in the y-z plane as long as loads and stiffness do not vary markedly from one bay to another.

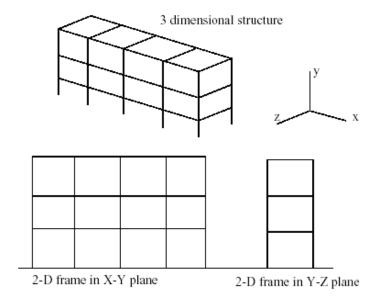


Figure 4.3(a) A Regular Building Structure treated as a Series of Parallel Two-Dimensional Sub-Structures

Clause 4.3.1(b) deals with vertical loads on braced multi-storey building frames, in which a floor level plus the columns above and below can be treated as a sub-structure in isolation from the rest of the structure, as shown in Fig.4.3(b) below.

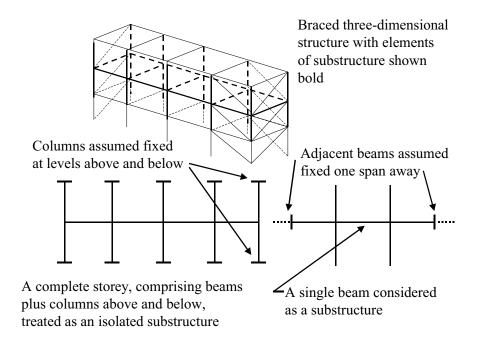


Figure 4.3(b) Assumptions for Individual Levels of Multi-Storey Buildings and Floor Beams considered as Substructures

#### 4.4.2 The effect of finite width of members

Structural analysis of skeletal structures treats structural members as "line elements," ignoring the fact that they actually have a finite width. Because of this finite width, it is usually impracticable to make connections at the centroid of each member. For example floor beams in simple construction multi-storey buildings must be joined to the sides of columns, and the weight supported by the beam does not act through the centroid of the column. The column must therefore carry not only axial force but bending moment also, as shown in Fig.4.3(c) below (unless there is another beam on the other side that exerts an equal and opposite moment on the column). They must therefore be designed as beam-columns, i.e. members under combined axial and bending loads, using Section 8 of AS 4100.

Clause 4.3.2 of AS 4100 defines the span length as the centre to centre distance between supports, not the actual length of the beam, as shown in Fig.4.3(c) below. Clause 4.3.4 of AS4100 specifies the minimum eccentricity e of the load R from a simply supported beam acting on a column, as shown in Fig.4.3(c): "A beam reaction or a similar load on a column shall be taken as acting at a minimum distance of 100mm from the face of the column towards the span or at the centre of the bearing, whichever gives the greater eccentricity."

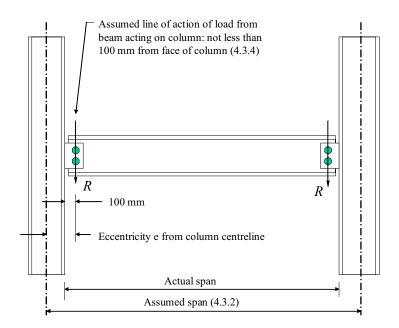


Figure 4.3(c) Actual Span and Span Assumed in AS4100

The same clause also specifies the distribution of the resulting moment eR in the column as shown in Fig.4.3(d).

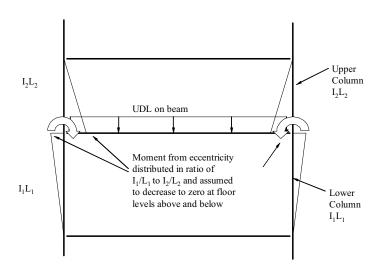


Figure 4.3(d) Distribution of Bending Moment from Eccentricity of Supports of Simply Supported Beam

# Illustrative Example: Bending moments in columns due to eccentric vertical loads

In the detail shown in Fig.4.3(e) below, the beam is simply supported on the angle, which is bolted to the face of the column. It is not clear exactly where the end reaction acts, so in accordance with Clause 4.3.4, it is taken as either the middle of the support (75 mm from column face) or 100 mm, whichever is greater, i.e. 100 mm.

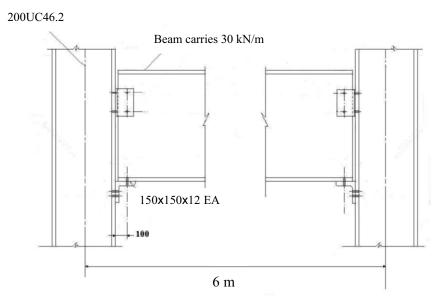


Figure 4.3(e) A typical Beam-Column Connection showing Eccentricity of Load on Column

To calculate the bending moment caused in the column by the eccentricity in the diagram above,

- 1. Calculate the total load on the beam as 30 kN/m x 6 m = 180 kN (ignoring the fact that 6 m is the distance between centres of columns and the actual beam length is a bit less. We will also assume the 30 kN/m includes the self-weight of the beam.
- 2. End reaction to support beam =180/2 = 90 kN.
- 3. Assume that this reaction force acts at a distance of 100 mm from the face of the column, i.e. in this case 200 mm from the centreline of the column, since the column section is approximately 200 mm deep. Thus it will exert a moment of 90 kN x 0.2 m = 18 kNm. This moment must be balanced by moments in the columns above and below the connection.
- 4. Next the distribution factors between connecting members are calculated. Because a joint must be in equilibrium, the sum of the bending moments in the members connected at any joint must be zero (taking clockwise as positive and anticlockwise as negative). The 18 kNm moment exerted by the beam must be balanced by moments at the ends of the columns above and below, where they connect to the beam. For example if the storey height below is 4 m and that above is 3 m, the moments are distributed in the ratio 3 below to 4 above, i.e. the column below takes  $18 \times 3/7 = 7.7$  kNm at the connection and the column below takes  $18 \times 4/7 = 10.3$  kNm. These moments in the columns are assumed to decrease linearly to zero at the floor levels above and below, as shown in Fig.4.3(d). If there were another beam to the left of the column, the moment in it at this connection would also have to be taken into account.

### 4.5 ELASTIC ANALYSIS

Most analysis of steel structures is done using elastic theory, although in practice some local yielding and plastic behavior is acceptable. Methods of analysis vary from approximate analysis using simplifying assumptions, through to highly sophisticated finite element analysis. Manual methods of analysis have now been largely replaced by faster and more accurate computer methods, but the designer should be aware of the existence of the older manual methods such as moment amplification and moment distribution.

## 4.5.1 First and second order elastic analysis

Clauses 4.4.1 and 4.4.2 of AS 4100 deal with first order analysis, in which deflections in members are not taken into account in calculating moments and forces, and second order analysis in which they are. Second order effects are illustrated in the examples below.

### **Illustrative Example 1:**

A 6 m high signpost of 150x150x6 mm SHS section in Grade 350 steel carries a 50kN vertical load at an eccentricity of 0.5m, as shown in Fig.4.4(a). First order analysis predicts a uniform bending moment of 25 kNm, which corresponds to a maximum stress of 247 MPa, well below yield. The deflection at the top due to this uniform bending moment would be approximately 200mm.

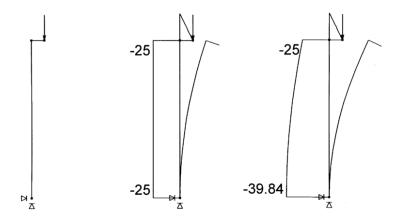


Figure 4.4(a) Bending moment diagram and deflected shape of a signpost carrying an eccentric point load at the top, showing moment amplification effect. Centre: result of first order analysis. Right: result of second order analysis.

However this deflection increases the moment arm of the eccentric load about the column base from 500 to 700 mm, thereby increasing the deflection, which in turn increases the moment arm, and so on. Second order or non-linear elastic analysis predicts 297 mm deflection at the top and a maximum bending moment of 39.84 kNm at the bottom, as shown below. This is a 59% increase, enough to cause the column to yield. Although this is an extreme example it illustrates the importance of second order effects.

# **Illustrative Example 2:**

The moment amplification effect is further illustrated in Fig.4.4(b) below. A two storey frame carries a wind load from the left and gravity loads on the two beams. First order analysis predicts a lateral movement of 215 mm at the top floor due to the wind load, but this movement creates an eccentricity which increases the bending moments in the columns and increases the lateral movement.

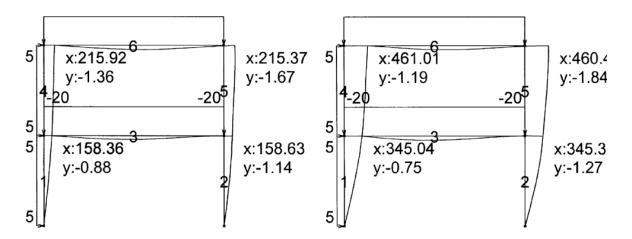


Figure 4.4(b) Moment Amplification in a Sway Frame: Left, First Order Analysis, Right, Second Order Analysis

## 4.5.2 Moment amplification

AS4100 provides equations for calculating moment amplification effects in braced members in Clause 4.4.2.2 and for sway members in Clause 4.4.2.3. These equations tend to give conservative predictions for normal cases and do not accurately predict extreme cases such as those shown above. They are intended for use as a part of a manual analysis process, but most analysis is now done using software packages which make the moment amplification equations obsolete.

For a braced member subjected to axial compression, i.e. a member in a frame with diagonal bracing that effectively prevents sidesway such as a tankstand, clause 4.4.2.2 of AS4100 requires an increase in the design moment by a moment amplification factor  $\delta_b$  which is given by:

$$\delta_b = \frac{c_m}{1 - \left(\frac{N^*}{N_{omb}}\right)} \ge 1$$
AS 4100 Cl. 4.4.2.2

where the factor  $c_m = 0.6 - 0.4 \ \beta_m \le 1$  allows a reduction in  $\delta_b$  where the moment shape factor  $\beta_m$ , given in Table 4.4.2.2, is favorable. N \* is the design axial force and  $N_{omb}$  is the Euler buckling load for the axis of bending, i.e.normally the major or x axis (the code does not make this clear)

$$N_{omb} = P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

#### Illustrative Example 1: Moment Amplification Factor on a Braced Frame

An edge column in a building of "simple" construction is of 150UB14 section, effective length 4m. It has an axial compressive force  $N^* = 50$  kN and a uniform bending moment  $M_m^* = 20$  kNm calculated by first order analysis. Calculate  $\delta_b$  and hence the amplified moment that should be used for design.

#### Solution

$$c_{m} = 0.6 - 0.4 \ \beta_{m} = 1.0 \ \text{since} \ \beta_{m} = -1.0$$

$$N_{omb} = P_{cr} = \frac{\pi^{2} \times 200,000 \times 6.66 \times 10^{6}}{4000^{2}} = 821 \text{kN}$$

$$\delta_{b} = \frac{1}{1 - \frac{50}{821}} = 1.065$$

 $M^* = \delta_b M_m^* = 1.065 \times 20 = 21.3 \text{ kNm}$ 

## Illustrative Example 2: Moment Amplification Factor on a Sway Frame

For a moment resisting frame without bracing, i.e. a moment resisting frame such as the one shown in Fig.4.4(c) below, AS4100 Clause 4.4.2.3 gives several equations. Only the simplest, applicable to rectangular frames, will be used here. In 4.4.2.3(a)(ii) the moment amplification factor  $\delta_s$  is given by:

$$\delta_s = \frac{1}{1 - \left(\frac{1}{\lambda_{ms}}\right)}$$

where  $\lambda_{ms}$  is the elastic buckling load factor for the storey under consideration, given in Clause 4.7.2.2 of AS 4100 by

$$\lambda_{ms} = \frac{\sum \left(\frac{N_{oms}}{l}\right)}{\sum \left(\frac{N*}{l}\right)}$$

where  $N^*$  is the member design axial force, with tension taken as negative and  $N_{\rm oms}$  is the Euler elastic buckling load.

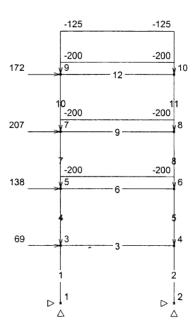


Figure 4.4(c) Four-Storey Frame with Gravity Loads and Wind Loads

We will check the bottom storey of the frame. Column length l = storey height = 4000 mm. We will assume the lower columns are pinned at the base and both columns and beams are 800WB168 sections with  $I_x = 2480x10^6$  mm<sup>4</sup>.  $\sum N^* = 5800$  kN

To calculate  $N_{\rm oms}$  we must estimate the effective length from AS 4100 Fig. 4.6.3.3(b). The stiffness ratio  $\gamma$  is theoretically infinite for the pinned lower ends of the columns, i.e. the column is infinitely stiff compared to an ideal pin. However Clause 4.6.3.4(a) allows us to take  $\gamma_1$  as 10 (taking the lower end as end 1).

For  $\gamma_2$  at the top of the columns we must calculate the ratio of stiffness of the columns to the beams joined at the top of the column under consideration:

$$\gamma = \frac{\sum \left(\frac{\mathbf{I}}{l}\right)_{c}}{\sum \beta_{e} \left(\frac{\mathbf{I}}{l}\right)_{b}}$$

where  $\beta_e$  is determined from Table 4.6.3.4. In this case  $\beta_e = 1$  since the beam restraining the column under consideration is rigidly connected to a column at its other end. I is the same for the beam and the column, and l for the beam is 8 m, twice that for the column, so  $\gamma_2 = 2$ . To find the effective length factor  $k_e$  we go up the left hand side of Fig.4.6.3.3(b) of AS4100 to  $\gamma_1 = 10$  and across to  $\gamma_2 = 2$ . Thus  $k_e = 2.15$  and

$$N_{oms} = \frac{\pi^2 EI}{(k_e l)^2} = 66189 \text{ kN}$$

Hence

$$\lambda_{ms} = \frac{\left(\frac{2 \times 66189}{4000}\right)}{\left(\frac{5800}{4000}\right)} = 22.8$$

Thus

$$\delta_s = \frac{1}{1 - \left(\frac{1}{22.8}\right)} = 1.046$$

i.e. we must increase the design moment by 4.6%.

## Illustrative Example 3: Second Order Elastic Analysis of a Sway Frame

It is just as easy to run a complete non-linear analysis using a computer package, if available, as it is to calculate  $\lambda_s$  manually. The results of first and second order analysis using Spacegass are shown in Fig.4.4(d) below. These show that the actual moment amplification on the bottom left hand column is 910/815 = 1.17, and the actual moment amplification on the bottom left hand column is 1620/1529 = 1.06. The latter figure is the important one, since both lower columns must be designed for the larger moment.

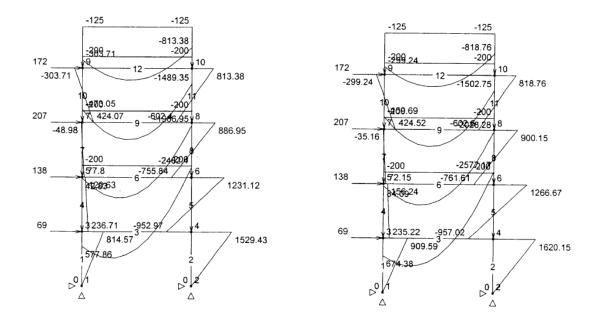


Figure 4.4(d) Bending Moments for Frame and Loads shown in Fig.4.4(c) Determined by First Order Analysis (left) and Second Order Analysis (right), using SpaceGass [3]

#### 4.5.3 Moment distribution

This is a manual analysis process involving successive approximations, which was invented by Professor Hardy Cross in 1932. It was the normal method of analysing statically indeterminate frames before computers and analysis software became cheap and powerful enough to be generally available and practicable for use by practitioners. Moment distribution is very laborious for any but the simplest structures, but may still be useful when structural analysis software is not available.

#### 4.5.4 Frame analysis software

Frame analysis software packages are now available that can analyse structural frames quickly and easily - provided the designer understands how to use them intelligently. Spacegass [3], Microstran [4] and Multiframe [5], are examples. There are also finite element analysis (FEA) packages, which can analyse and predict stresses and displacements in 2 or 3 dimensional continua. ANSYS [6] and STRAND [7] are examples. The present discussion is limited to frame analysis.

Before using a frame analysis package the designer must understand the terminology and the way structures under load behave. The first thing to be aware of is that frame analysis packages treat structures as a series of lines of negligible thickness, joining nodes. For many types of structure this is quite accurate enough, provided allowance is made for eccentric forces as shown above. But structures such as walls, plates, shells and complex 3 dimensional shapes such as corbels require a more rigorous approach.

## 4.5.5 Finite element analysis

Finite element analysis or FEA involves representing a real object as a "mesh" – a series of small, regularly shaped, connected elements, as shown in Fig.4.4(c), and then setting up and solving huge arrays of simultaneous equations. The finer the mesh of elements, the more accurate the results but the more computing power is required.

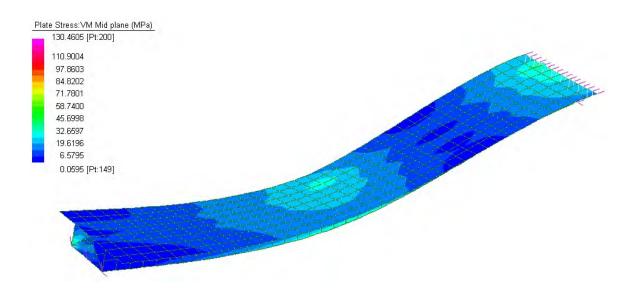


Figure 4.4(c) FEA Mesh and Stress Contours for a Box Girder Bridge

# 4.6 PLASTIC METHOD OF STRUCTURAL ANALYSIS

This is an alternative to the elastic method. Because it is not widely used it will not be treated in detail here. However the designer should be aware that it exists and know something of the principles involved.

The basis of the method is that a ductile steel structure will not collapse as soon as yield occurs: one or more "plastic hinges" must form. Plastic hinges are sections where the full section has reached the yield stress. Once the plastic moment has been reached it is assumed that the section can offer no additional resistance to rotation, behaving as a hinge but with constant resistance  $M_p$ , a condition known as a plastic hinge (in fact there will be a slight increase in moment due to strain hardening as the rotation at a plastic hinge increases). In general, any combination of three hinges, real or plastic, in a span will create an unstable situation known as a collapse mechanism.

Because we design structures to survive extreme loads, it makes sense to examine what loading will cause actual collapse of a structure, and then apply suitable load factors to ensure that we have an acceptable margin of safety before collapse.

Thus for example a simply supported beam with a point load at midspan or a UDL (uniformly distributed load) will collapse when one plastic hinge forms at midspan. The "plastic moment"  $M_P$ , i.e. the moment necessary to form a plastic hinge, is given by:

where  $f_y$  = yield stress, and S = plastic section modulus (listed in tables of section dimensions and properties. Note that in Australia Z = elastic section modulus and S = plastic section modulus. In the USA the reverse notation is used. The elastic and plastic section modulus for built up sections with one axis of symmetry can be calculated using the principles of stress analysis.

### **Illustrative Example 1:**

Find the yield moment  $M_y$  and the plastic moment  $M_p$  for a 610UB125 section of Grade 300 steel. Hence find the UDL w (in kN/m) on a simply supported beam of this section, spanning 16m, that would cause (a) first yield, (b) collapse.

#### Solution

$$M_y = f_y Z = 300 \times 3230 \times 10^3 = 969 \text{ kNm}$$
  
 $M_p = f_y S = 300 \times 3680 \times 10^3 = 1104 \text{ kNm}$ 

(a) for first yield and including the capacity factor  $\phi$ 

$$\phi M_{\rm y} = \frac{w_{\rm y}L^2}{8}$$

$$w_{\rm y} = \frac{8 \times 0.9 \times 969}{16^2} = 27.25 \text{ kN/m}$$

(b) for collapse and including the capacity factor  $\phi$ 

$$\phi M_{\rm p} = \frac{w_c L^2}{8}$$

$$w_c = \frac{8 \times 0.9 \times 1104}{16^2} = 31.05 \text{ kN/m, which is } 14\% \text{ higher than the UDL for first yield.}$$

A beam with fixed ends will require three plastic hinges, one at midspan and one at each end, before it will collapse. For a fixed end beam with a point load at midspan, the midspan moment = end moments = PL/8 = half the midspan moment for a simply supported beam. Thus the fixed end beam can take double the load before first yield, and yield will occur simultaneously at all 3 potential plastic hinge points.

However for a UDL, the end moments are  $wL^2/12$ , double the midspan moment, so as the load increases, yield will occur first at the ends. Once plastic hinges form at the ends, the ends will rotate so the beam behaves as if the ends are partially fixed, and the midspan moment increases until yield occurs there, and finally a plastic hinge, at which point the beam collapses.

## **Illustrative Example 2:**

Find the UDL w (in kN/m) on a fixed end beam of Grade 300, 610UB125 section, spanning 16m, that would cause (a) first yield (b) collapse.

### Solution

As in the example above,  $M_y = 969 \text{ kNm}$  $M_p = 1104 \text{ kNm}$  Under elastic conditions, end moments  $M_{\text{end}} = \text{wL}^2/12$ 

(a) for first yield and including the capacity factor  $\phi$ 

$$\phi M_y = \frac{w_y L^2}{12}$$

$$w_y = \frac{12 \times 0.9 \times 969}{16^2} = 40.88 \text{ kN/m}$$

(b) for collapse and including the capacity factor  $\phi$ , the sum of the end moment plus the midspan moment (i.e. the total statical moment) is equal to wL<sup>2</sup>/8 i.e.

$$2\phi M_{\rm p} = \frac{w_c L^2}{8}$$

$$w_c = \frac{16 \times 0.9 \times 1104}{16^2} = 62.1 \text{ kN/m which is 52\% above the UDL for first yield.}$$

For more complex structures it is less obvious where the plastic hinges will form in order for collapse to occur. For example a pinned base portal frame subjected to non-symmetrical loading under cross wind will need two plastic hinges to collapse. One will form at the eaves, and the other may form any where within the rafter, depending on the geometry and loading pattern.

### 4.7 MEMBER BUCKLING ANALYSIS

AS4100 Clauses 4.6.1 and 4.6.2 deal with calculation of the Euler elastic buckling load

$$N_{om} = \frac{\pi^2 EI}{\left(k_e l\right)^2}$$

where  $k_e$  is the member effective length factor, determined in accordance with Clause 4.6.3. This clause gives four methods of determining  $k_e$ . The first assumes idealised end restraints as shown in Fig.4.6(a) below, reproduced from AS4100 Fig.4.6.3.2.

|                              | Bı      | aced memb  | per | S   | way memb | er  |
|------------------------------|---------|--|-----|-----|----------|-----|
| Buckled shape                |         |  |     | - P |          | - P |
| Effective length factor (ke) | 0.7     | 0.85   | 1.0 | 1.2 | 2.2      | 2.2 |
| Theoritical k value          | 0.5     | 0.7  | 1.0 | 1.0 | 2.0      | 2.0 |
| End condition code           | <i></i> | = Rotation fixed, translation fixed = Rotation free, translation fixed   = Rotation free, translation fixed   o = Rotation free, translation free free |     |     |          |     |

Figure 4.6(a) AS4100 Effective Length Factors for Members with Idealised End Restraints

These values of  $k_e$  are conservative compared to the theoretical design values given in the US steel code LRFD [8], shown in Fig.4.6(b) below, to allow for non-ideal end conditions. Also there is a typographical error in the Fig.4.6(a) – the upper right hand symbol represents "rotation fixed translation free."

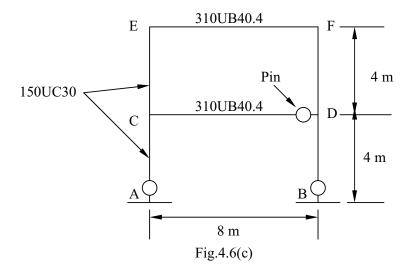
|                              | Br      | aced meml   | oer | Sway member |                   |     |  |
|------------------------------|---------|-------------|-----|-------------|-------------------|-----|--|
| Buckled shape                |         |             |     |             |                   |     |  |
| Effective length factor (ke) | 0.65    | 0.80        | 1.0 | 1.2         | 2.1               | 2.0 |  |
| Theoritical k value          | 0.5     | 0.7         | 1.0 | 1.0         | 2.0               | 2.0 |  |
| End condition code           | <i></i> | fixed T fix |     |             | totation free, tr |     |  |

Figure 4.6(b) LRFD [8] Theoretical and Recommended Effective Length Factors

The second method of determining the effective length factor  $k_e$  involves estimating the ratios  $\gamma_1$  and  $\gamma_2$  of the compression member stiffness to the end restraint stiffness at each end. The stiffer the end restraints the closer  $k_e$  approaches 0.5 for a braced member and 1 for a sway member. Having estimated the ratios  $\gamma_1$  and  $\gamma_2$ , the designer reads off  $k_e$  from a chart.

### **Illustrative Example:**

A frame as shown in Fig.4.6(c) below comprises two columns of 150UC30 section, 8m high with pinned bases, connected to two beams of 310UB40.4 section at 4m and 8m height, spanning 8 m. Beam CD is rigidly connected at end C and pin connected at end D while beam EF is rigidly connected at both ends, the columns are rigidly connected at ends C,D,E and F. Determine the effective length factor  $k_e$ , the effective buckling length  $k_e$  l and the member elastic buckling load  $N_{om}$  of the columns AC and CE, (i) if the frame is braced, (ii) if it is unbraced (i.e. a sway frame).



#### Solution

To find  $k_e$ , we must first find the stiffness ratios  $\gamma$  at each end of each column. From AS4100 Clause 4.6.3.4,

$$\gamma = \frac{\sum \left(\frac{I}{l}\right)_{c}}{\sum \beta_{e} \left(\frac{I}{l}\right)_{b}}$$

where  $\gamma$  is the stiffness ratio, the subscript c refers to the compression members rigidly connected to the node under consideration, the subscript b refers to the beams connecting to the same node in the plane of buckling, and  $\beta_e$  is a modifying factor given in Table 4.6.3.4 of AS4100 to allow for fixity at the far end of the beam. At first sight it might seem that all other members besides the member under consideration should contribute to the end restraint stiffness. However it is conservatively assumed that other compression members will also tend to buckle and will therefore contribute to the tendency of the member under consideration to buckle, rather than restraining it. This is true only if all compression members are on the point of buckling under the same loading case (i.e. all columns buckle simultaneously when the frame reaches its buckling load).

#### (i) Braced frame

For member AC at node C,  $I_c = 17.6 \times 10^6 \text{ mm}^4$ ,  $l_c = 4000 \text{ mm}$ ,  $I_b = 86.4 \times 10^6 \text{ mm}^4$ ,  $l_b = 8000 \text{ mm}$ .

AS4100 Table 4.6.3.4

Since beam CD is pin connected at far end D we have

$$\beta_e = 1.5$$

$$\gamma_1 = \frac{\frac{2 \times 17.6 \times 10^6}{4000}}{1.5 \times \frac{86.4 \times 10^6}{8000}} = 0.543$$

Member AC is nominally pinned at A, so theoretically  $\gamma_2 = \infty$  (the choice of subscripts 1 and 2 is arbitrary). In fact there is some restraint and Clause 4.6.3.4 allows a  $\gamma$  value not less than

10. Taking the values 0.543 and 10 and reading from Fig.4.6.3.3(a) of AS4100 for braced members gives:

$$k_e = 0.82$$
 and  $k_e l = 0.82 \times 4 = 3.28 \text{ m}$ 

The value of  $k_e$  is close to the value of 0.85 for a braced member fixed at one end and pinned at the other

Hence

$$N_{\rm om} = \frac{\pi^2 \times 200000 \times 17.6 \times 10^6}{3280^2} = 3229.2 \text{ kN}$$

For member CE

At End C  $\gamma_1 = 0.543$  same as before

At End E beam EF is rigidly connected to a column at far end F and therefore  $\beta_e = 1.0$ AS4100 Table 4.6.3.4

$$\gamma_2 = \frac{\frac{17.6 \times 10^6}{4000}}{1.0 \times \frac{86.4 \times 10^6}{8000}} = 0.407$$

Taking the values 0.543 and 0.407 and reading from Fig.4.6.3.3(a) for braced members gives  $k_e = 0.68$  and  $k_e l = 2.72$  m. The value of  $k_e$  is close to the value of 0.7 given in AS4100 for a braced member fixed at both ends, and greater than the theoretical value of 0.5, reflecting the fact that the ends are not fully prevented from rotating.

Hence

$$N_{\rm om} = \frac{\pi^2 \times 200000 \times 17.6 \times 10^6}{2720^2} = 4695.75 \text{ kN}$$

## ii) Sway frame

## Member AC

At end C beam CD is pin connected at far end D and therefore:

$$\beta_e = 0.5$$
 AS4100 Table 4.6.3.4

$$\gamma_1 = \frac{\frac{2 \times 17.6 \times 10^6}{4000}}{0.5 \times \frac{86.4 \times 10^6}{8000}} = 1.63$$

Member AC is nominally pinned at A, so theoretically  $\gamma_2 = \infty$  (the choice of subscripts 1 and 2 is arbitrary). In fact there is some restraint and Clause 4.6.3.4 allows a  $\gamma$  value not less than 10. Taking the values 1.63 and 10 and reading from Fig.4.6.3.3(b) of AS4100 for sway members gives:

$$k_e = 2.05$$
 and  $k_e l = 2.05 \times 4 = 8.2 \text{ m}$ 

The value of  $k_e$  is close to the ideal value of 2.0 for a sway member fixed against rotation at one end and pinned at the other.

Hence

$$N_{\rm om} = \frac{\pi^2 \times 200000 \times 17.6 \times 10^6}{8200^2} = 516 \text{ kN}$$

#### Member CE

At End C  $\gamma_1 = 1.63$ 

At End E  $\gamma_2 = 0.407$ 

Taking the values 1.63 and 0.407 and reading from Fig.4.6.3.3(b) of AS4100 for sway members gives:

$$k_e = 1.30$$
 and  $k_e l = 1.30 \times 4 = 5.2 \text{ m}$ 

The value of  $k_e$  is close to the value of 1.2 given in AS4100 for a sway member fixed against rotation at both ends, and greater than the theoretical value of 1, reflecting the fact that the ends are not fully prevented from rotating.

Hence

$$N_{\rm om} = \frac{\pi^2 \times 200000 \times 17.6 \times 10^6}{5202^2} = 1283.81 \text{ kN}$$

#### 4.8 FRAME BUCKLING ANALYSIS

It will be apparent from the foregoing discussion that members of a frame interact in such a way that its buckling behavior depends on the geometry, section properties and loads acting on all members. In order to predict whether a given frame will buckle under a given load condition it is necessary to calculate the elastic buckling load factor  $\lambda_c$  for the frame. The elastic buckling load factor for any load case is the factor by which the axial forces in all the members in a frame must be multiplied to cause the frame to become unstable. The factor  $\lambda_c$  is best determined using elastic critical load computer packages such as Space Gass and Microstrand. A  $\lambda_c$  value less than 1 indicate that the frame will buckle under a load less than the design load.

Compression member effective lengths can be easily determined using buckling analysis. Once the axial force distribution throughout the structure is established, using either first order elastic analysis or second order elastic analysis, the designer can easily run buckling analysis using Space Gass or Microstrand to determine the elastic buckling load factor  $\lambda_c$ ,. Multiplying this factor by the design axial compression force we obtain Euler elastic buckling load for the compression member in question. The effective length is then back calculated from the buckling load using the following expression:

$$L_e = \pi \times \sqrt{\frac{E \ \mathrm{I}}{\lambda_c N^*}}$$

Since the elastic buckling load factor depends on the axial force distribution throughout the structure, which varies depending on the load arrangement, a compression member in a frame will have a different effective length in each load case. If the compression member in question contributes to a large degree to the buckling mode of the frame (i.e. the frame will collapse as result of this member buckling) the effective length determined using buckling analysis will be close to that obtained using Fig.4.6.3.3 (a) or (b) of AS4100[1]. On the other hand if the compression member caries a small axial force when the frame collapse (i.e. the member does not contribute to the buckling mode) it will have a huge effective length. Because of this the

capacity of the compression member evaluated, using an effective length associated with a given load case, must be compared with the design axial compression force in the member under the same load case. Alternatively investigate all load cases and choose a single effective length by comparing the buckled shape of the compression member in question with the simple buckled shapes given in Figure 4.6(a). An acceptable single effective length can also be obtained by constructing a special load case in which the compression member in question will have a profound effect on the buckling mode.

The use of Fig.4.6.3.3 (a) for braced members and Fig.4.6.3.3 (b) for sway members will always give a safe design provided that the conditions of their use are met. For cases beyond the scope of these figures buckling analysis must be used. A common example where AS 4100[1] approach can't be used is a pitched roof portal frame, to determine the compression member effective length for the columns and rafters in a portal frame the designer should perform a buckling analysis using elastic critical load computer packages such as Space Gass [3] or Microstran [4], in the absence of these computer packages simple approximate expressions for determining  $\lambda_c$  for pinned and fixed base portal frames may be found in Reference [9]. These expressions ignore the stiffening effect of any haunches and the nominal base restraint (i.e. a  $\gamma$  value of 10 for a pinned base when using AS 4100[1] Figures to determine the effective length) and therefore should be conservative.

• For pinned base portal frames:

$$\lambda_{c} = \frac{3E \ I_{r}}{l_{r} \times \left(N_{c}^{*} h_{e} + 0.3 N_{r}^{*} l_{r}\right)}$$

• For fixed base portal frames:

$$\lambda_{c} = \frac{3E \left(10 + R\right)}{\frac{5 N_{r}^{*} l_{r}^{2}}{I_{r}} + \frac{2 R N_{c}^{*} h_{e}^{2}}{I_{c}}}$$

in which

$$R = \frac{I_c \ l_r}{I_r \ h_e}$$

and E is Young's modulus,

 $N_{\rm c}^*$  is the axial compression force in the column,

 $N_{\rm r}^*$  is the axial compression force in the rafter,

I<sub>c</sub> is the second moment of area of the column,

 $I_{r}$  is the second moment of area of the rafter,

 $h_{\rm e}$  is the height to the eaves, and

 $l_r$  is the length of rafter between the centre of the column and the apex.

Additional Examples on cases where AS 4100 approach ceased to be valid are given in Chapter 6.

According to AS 4100 Commentary [2] Clause C4.7.2.1,  $\lambda_c$  may be approximated "for regular braced frames with regular loading and negligible axial forces in the beams" "from the lowest of the member buckling load factors ( $\lambda_m$ ) calculated from the member buckling loads ( $N_{om}$ ) determined using Clauses 4.6.2 and 4.6.3."  $\lambda_m$  is simply the ratio of the member buckling load  $N_{om}$  to the design load  $N^*$ . For other braced frames this method may be too conservative,

according to the Commentary [2], and an iterative method or a frame analysis package should be used.

The commentary goes on to say that for rectangular frames with sway members,  $\lambda_c$  may be approximated by the lowest storey buckling load factor  $\lambda_{ms}$ , given in clause 4.7.2.2 as

$$\lambda_{ms} = \frac{\sum \left(\frac{N_{oms}}{l}\right)}{\sum \left(\frac{N*}{l}\right)}$$

Provided  $\lambda_m > 1$  for all columns, the frame is safe.

## **Illustrative Example:**

Determine the in plane compression member effective length for the columns and rafters in a 30 m span,  $14.93^{\circ}$  pitched roof portal frame with 800WB 122 columns 8 m high and 530UB 92.4 rafters, if the design axial compression force in the columns and rafters obtained using second order elastic analysis is  $N_c^* = 355.27$  kN and  $N_r^* = 188.24$  kN. All steel is grade 300.

### Solution

$$l_r = 15 \times 10^3 / \cos 14.93^\circ = 15.52 \text{ m}, l_c = 8.0 \text{ m}$$

$$\lambda_c = \frac{3 \times 200000 \times 554 \times 10^6}{15.52 \times 10^3 \times \left(355.27 \times 10^3 \times 8 \times 10^3 + 0.3 \times 188.24 \times 10^3 \times 15.52 \times 10^3\right)} = 5.76$$

$$(L_{ex})_{column} = \pi \times \sqrt{\frac{E \ I_{xc}}{\lambda_c N_c^*}} = \pi \times \sqrt{\frac{2 \times 10^5 \times 1570 \times 10^6}{5.76 \times 355.27 \times 10^3}} \ x10^{-3} = 38.92 \ m$$

## 4.9 REFERENCES

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# 5 DESIGN OF TENSION MEMBERS

### **5.1 INTRODUCTION**

A member that supports axial tension loads is defined as a tension member. Steel tension members are covered in section (7) of AS 4100 [1], while members subjected to bending and compression are treated in Sections 5 and 6 respectively. These three topics are treated in the reverse order this book, in increasing order of complexity. Tension members are simple structural elements to design, with perhaps the simplest being concentrically loaded uniform tension members, as they are nominally in a state of uniform axial stress. However, a tension member is not always connected concentrically. In many cases the fabrication of tension members is simplified by making their end connections eccentric, but this will induce bending moments which interact with the tensile loads leading to a reduction in the ultimate strength. The effect of the bending action caused by eccentric connections is dealt with in AS 4100 [1] by introducing a correction factor  $k_l$ .

AS 4100 provides two criteria which a tension member must meet:

- (i) yield
- (ii) ultimate strength.

The logic behind these criteria is explained below.

A ductile steel member loaded in axial tension can be expected to yield at a load  $N^* = f_y$   $A_g$  where  $f_y$  is the yield stress and  $A_g$  is the gross cross sectional area. Although it will not fracture at this load because of strain hardening, it is unlikely to serve its purpose in the structure if it elongates excessively. Hence the yield criterion  $N_t = A_g f_y$ .

The ultimate strength criterion is a little more complex. A tension member with bolted end connections will tend to yield first at a cross section containing one or more bolt holes, but this limited local yielding does not constitute failure because the overall increase in length of the member is negligible. However the member may fail by fracture through the bolt holes at a load smaller than that required to cause general yielding on the gross area along the member length. Hence the second criterion,  $N_{\rm t}=0.85~k_{\rm t}A_{\rm n}f_{\rm u}$ , where  $A_{\rm n}$  is the net cross sectional area,  $f_{\rm u}$  is the ultimate tensile strength,  $k_{\rm t}$  as explained above is a factor to account for eccentric loading and 0.85 is a further safety or capacity factor.

The gross area and the tensile stress area for some common merchant round bars are given in Table 5.1. The properties of some steel wire ropes are given in Table 5.2.

| Bar diameter (mm)                      | 12   | 14  | 16  | 18  | 20  | 22  | 24  | 27  | 30  | 33  | 36   | 39   |
|--|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|
| Shank area (mm²)                       | 113  | 154 | 200 | 254 | 314 | 380 | 452 | 573 | 706 | 855 | 1016 | 1194 |
| Tensile stress area (mm <sup>2</sup> ) | 84.3 | 115 | 157 | 192 | 245 | 303 | 353 | 459 | 561 | 694 | 817  | 976  |

Table 5.1 Design Areas –Round Bars

Nominal 12 14 22 16 18 20 24 26 28 dia.(mm) Mass(kg/m) 0.73 0.99 1.29 1.63 2.12 2.55 2.99 3.48 4.27 Effective area 119 254 305 491 87.2 145 183 357 400 (mm<sup>2</sup>)Min. breaking 997 210 265 442 713 126 172 368 518 strength (kN)

Table 5.2 Properties of Steel Wire Ropes ( $f_u = 1570 \text{ MPa}$ )

*Note*: Ultimate tensile strength is taken as the minimum breaking strength in the table [2]

## **5.2 DESIGN OF TENSION MEMBERS TO AS 4100**

A member subject to a design axial tension force  $N^*$  shall satisfy  $-N^* \le \phi N_t$ 

where,

 $\phi$  = the capacity factor, (see Table 3.4 of AS 4100)

 $N_t$  = is the nominal section capacity in tension taken as the lesser of –

 $N_{\rm t} = A_{\rm g} f_{\rm v}$ ; and

 $N_{\rm t} = 0.85 \, k_{\rm t} A_{\rm n} f_{\rm u}$ 

Where,

 $A_{\rm g}$  = the gross area of the cross-section

 $f_{\rm y}$  = the yield stress used in design

 $k_t$  = the correction factor for distribution of forces determined in accordance with Clause 7.3 of AS 4100.

 $A_{\rm n}$  = the net area of the cross-section, obtained by deducting from the gross area the sectional area of all penetrations and holes, including fastener holes. The deduction for all fastener holes shall be made in accordance with Clause 9.1.10 of AS 4100.

For threaded rods, the net area shall be taken as the tensile stress area of the threaded portion, as defined in AS 1275.

 $f_{\rm u}$  = the tensile strength used in design

The 0.85 is an additional safety factor to allow for the fact that this equation is dealing with actual fracture, not yield.

### 5.3 WORKED EXAMPLES

## **Example 5.3.1 Truss Member in Tension**

An equal angle section of Grade 250 steel is to be used in a truss such as the one shown in Fig.5.1 below. It is to be connected at the ends by welding one leg of the angle to a plate. Select a suitable section for a factored design axial tension force  $N^* = 100 \text{ kN}$ .



Figure 5.1 Truss structures spanning a clarifier in a water purification plant

#### Solution

The design of a tension member is carried out using the following design procedure:

- 1. Determine  $N^*$  from load estimation and structural analysis. Already done for this example.
- 2. Determine minimum  $A_g$  to meet yield requirement, i.e  $A_g \ge N^*/0.9 f_y = 100,000/(0.9x260) = 427 \text{ mm}^2$  (assuming the thickness will be <12 mm)
- 3. Choose a member to satisfy  $A_g \ge N^* / 0.9 f_y$ . From AISC design capacity tables [1] choose 50x50x5 mm EA with  $A_g = 443 \text{ mm}^2$ .

$$\phi N_t = 0.9 \text{ x } A_g f_y = 0.9 \text{ x } 443 \text{ x } 260 = 103.6 \text{ kN} > N^* = 100 \text{ kN}$$
 OK

4. Check member capacity using ultimate strength (i.e. check section fracture at the connection)

#### **Example 5.3.2 Checking a Compound Tension Member with Staggered Holes**

Fig.5.2 below shows a compound tension member made up of 2L150x100x12 UA in Grade 300 steel ( $f_u = 440$  MPa,  $f_y = 300$  MPa). Determine the maximum design force  $N^*$  that can be transmitted by the angles. Assume bolt holes are 2 mm larger than bolt diameter to allow for misalignment. Do not consider shear on the bolts, bearing or tearing at the holes. These possible failure mechanisms will be considered in Chapter 8, Connections. All bolts are M30 Grade 8.8/S.

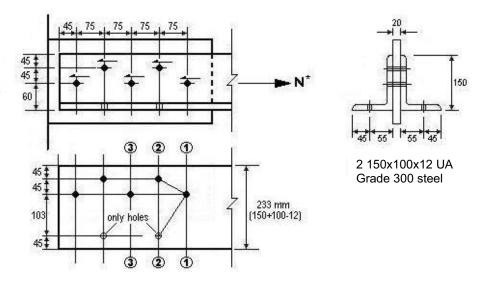


Figure 5.2 Compound Tension Member with Staggered Holes

#### Solution

## Member yield

$$\phi N_{\rm t} = 0.9 A_{\rm g} f_{\rm v} = 0.9 \text{ x } (2 \text{ x } 2870) \text{ x } 300 = 1549.8 \text{ kN}$$

AS 4100 Cl.7.2

# Section fracture

$$\phi N_t = 0.9 [0.85 k_t A_n f_u]$$

AS 4100 Cl.7.2

Consider failure path (1-1) along non-staggered holes

$$A_{\rm n} = A_{\rm g} - nt \, (D_{\rm b} + 2)$$

$$A_{\text{n (1-1)}} = (2 \times 2870) - 2 [1 \times 12 (30 + 2)] = 4972 \text{ mm}^2 \text{ (takes } 100\% \text{ N}^*\text{)}$$

Consider failure path (2-1-2) along staggered holes

$$A_{\rm n} = A_{\rm g} - nt (D_{\rm b} + 2) + t \Sigma (S_{\rm p}^2 / 4S_{\rm g})$$

$$A_n = (2 \times 2870) - 2 \times 3 \times 12 (30 + 2) + 2 \times 12 \times [75^2 / (4 \times 45) + 75^2 / (4 \times 103)]$$

$$A_{\text{n (2-1-2)}} = 4513.67 \text{ mm}^2 \quad (takes 100\% N^*)$$

$$\phi N_t = 0.9 \text{ x} 0.85 \text{x} 1 \text{x} 4513.67 \text{x} 440 \text{x} 10^{-3} = 1519.30 \text{ kN } (govern)$$

$$N^* \le 1519.30 \text{ kN}$$

Check failure path (2-2) along non-staggered holes

$$A_{\text{n (2-2)}} = 2 \times 2870 - 2 \times 2 \times 12 (30 + 2) = 4204 \text{ mm}^2$$

This net section caries only 80% of  $N^*$ 

$$\phi N_t = 0.9 \times 0.85 \times 1 \times 4204 \times 440 \times 10^{-3} = 1415.06 \text{ kN}$$

$$0.8 N^* \le 1415.06 \text{ kN}$$

$$N^* \le 1768.83 \text{ kN}$$

*Answer*  $N^* \le 1519.30 \text{ kN}$ 

#### Comment

The term  $t \Sigma (S_p^2/4S_g)$  is carried out for all staggered paths along the failure line. This term indirectly accounts for the existence of a combined stress state (tensile and shear) along the inclined failure path associated with staggered holes.

# **Example 5.3.3 Checking a Threaded Rod with Turnbuckles**

A 20 mm diameter threaded round bar in grade 300 steel is used as a tension tie in the roof and wall bracing system of an industrial building. Determine the maximum design tension force  $N^*$  that can be transmitted.

#### Solution

 $f_y = 300 \text{ MPa}, f_u = 440 \text{ MPa}$ 

From Table 5.1 of this book

Shank area =  $314 \text{ mm}^2$ 

Tensile stress area  $A_s = 245 \text{ mm}^2$ 

Unthreaded Shank Yield Load / Member Yield along the Member Length

$$\phi N_t = 0.9 A_g f_v = 0.9 \times 314 \times 300 = 84.8 \text{ kN}$$

AS 4100 Cl.7.2

Threaded Rod Tensile Capacity / Section Fracture at the Connection

$$\phi N_t = \phi \ 0.85 \ k_t A_n f_u = 0.9 \times 0.85 \times 245 \times 440 = 82.5 \ kN \ (governs)$$

AS 4100 Cl.7.2

 $N^* \le 82.5 \text{ kN}$  Answer

#### **Comment**

Yielding on the tensile stress area is not a limit state, as the capacity of any tension member will be controlled by either yielding in the body of the member or by section fracture at the connection.

# **Example 5.3.4 Designing a Single Angle Bracing**

An angle section is to be used as a diagonal bracing in a wall bracing system of an industrial building. The bracing member is subjected to a design tension force  $N_t^* = 172.78$  kN. Choose a section to satisfy yielding requirements, and then check the section capacity based on the ultimate strength (i.e. check fracture limit state at the connection).

## Solution

## Member yield

$$N_{\rm t}^* \le \phi N_{\rm t} \Longrightarrow N_{\rm t}^* \le 0.9 A_{\rm g} f_{\rm y}$$

$$A_{\rm g} \ge N_{\rm t}^* / 0.9 \, f_{\rm y} \Rightarrow A_{\rm g} \ge [(172.78 \, \text{x} 10^3) / (0.9 \, \text{x} \, 260)] = 738.38 \, \text{mm}^2$$

Try equal angle (75 x 75x 6 EA,  $A_g$  = 867 mm<sup>2</sup>, actual thickness t = 6 mm), in Grade 250 ( $f_y$  = 260 MPa,  $f_u$  = 410 MPa) connected to the gusset plate by one row of M20 bolts Grade 8.8/S,  $f_{uf}$  = 830 MPa.

## Check section fracture

$$N^* \leq \phi N_t$$

$$\phi N_{\rm t} = 0.9 \text{ x } 0.85 \ k_{\rm t} A_{\rm n} f_{\rm u}$$

$$A_{\rm n} = A_{\rm g} - n \ t \ (D_{\rm Bolt} + 2 {\rm mm})$$
 
$$A_{\rm n} = 867 - 1 \ {\rm x} \ 6 \ {\rm x} \ (20 + 2) = 735 \ {\rm mm}^2$$
 
$$k_{\rm t} = 0.85 \qquad \qquad AS4100 \ Table \ 7.3.2$$
 
$$\phi N_{\rm t} = 0.9 \ {\rm x} \ 0.85 \ {\rm x} \ 7.35 \ {\rm x} \ 410 = 195.95 \ {\rm kN} > N_{\rm t}^* = 172.78 \ {\rm kN} \qquad OK$$
 
$$Hence \ Adopt \ 75 {\rm x} 75 {\rm x} 6 \ {\rm EA} \ in \ Grade \ 250 \ steel$$

## Example 5.3.5 Designing a Steel Wire Rope Tie

A steel wire rope used as a diagonal bracing in a roof bracing system is subjected to a design tension force  $N_t^* = 60$  kN. What is the minimum diameter of the cable that can be used?

#### Solution

High tensile steel wire ropes are not covered in AS 4100[1]. However they are commonly used as tension-only bracing in steel industrial buildings, being more convenient to transport and install than threaded rods. These wire ropes are made up of many strands of small diameter wire which are prone to corrosion and having little ductility, may be weakened by the clamps used to secure their ends. For reasons such as these, they should not be loaded to more than 1/4 - 1/3 of their ultimate tensile capacity. Gorenc et al [2] recommend 0.3 of the ultimate capacity for limit state design, and this factor will be used in the present example.

From Table 5.2 a steel wire rope of 16 mm diameter has a breaking strength equal to 210 kN Hence,  $\phi N_t = 0.3 \times 210 = 63 \text{ kN} > N_t^* = 60 \text{ kN}$ 

# 5.4 REFERENCES

- 1. Standards Australia (1998). AS 4100 Steel Structures.
- 2. Gorenc, B., Tinyou, R. and Syam, A.(1996). *Steel Designer's Handbook*. 6<sup>th</sup> Edition, NSW University Press.

# 6 DESIGN OF COMPRESSION MEMBERS

### 6.1 INTRODUCTION

A member that supports axial compressive loads is defined as a compression member. The most common compression members in structures are truss chord members (Fig.5.1), struts and columns (Fig.6.1). Steel compression members are covered in Section 6 of AS 4100 [1]. However columns are commonly subjected to both compression and bending, in which case they are known as "beam columns" and are treated in Section 8 of AS 4100 and Chapter 8 of this book.



Figure 6.1 Compression Members include the Vertical Columns and the Horizontal Struts running across the picture

Compression members are a bit more complex to design than tension members, since they can fail in any of 3 ways: (i) yielding, (ii) inelastic buckling, or (iii) elastic buckling, depending on the slenderness ratio  $L_e/r$  where  $L_e$  is the effective buckling length and r is the radius of gyration of the cross section. For slender members where  $L_e/r$  is large, failure is a result of elastic buckling and is closely predicted by Euler's formula. At the other extreme, very short thick members fail essentially as a result of yield. Columns of intermediate slenderness fail by inelastic buckling (i.e. both yield and buckling occur together). In practice most compression members have intermediate slenderness ratio and therefore the most common mode of failure is inelastic buckling.

#### Overall buckling modes

Depending on the geometry of the member's cross-sectional area, compression members may buckle in one of the three following modes:

- Flexural
- Flexural-torsional
- Torsional

For most practical cases it is only necessary to consider simple flexural buckling, where the whole cross section moves laterally without twisting.

Flexural buckling occurs in members with two axes of symmetry, for example I-sections (UB and UC sections), rectangular hollow sections (RHS), and in members with doubly antisymmetric cross sections, for example Z-sections. Flexural buckling also occurs in members with one axis of symmetry such as C, channel, T, equal legged angle and double angles when such sections buckle about the axis perpendicular to the axis of symmetry, as shown in Fig.6.2.

Singly symmetric sections: flexural buckling (without twisting). Movement is in direction of axis of symmetry, i.e. buckling about axis perpendicular to axis of symmetry

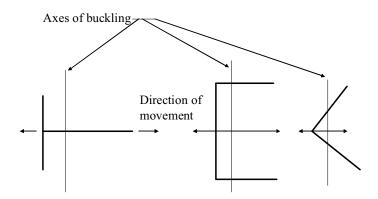


Figure 6.2 Flexural Buckling of Singly Symmetric Sections

Flexural-torsional buckling, as shown in Fig.6.3, occurs in members with one axis of symmetry when such members are buckled about the axis of symmetry. Flexural torsional buckling also occurs in members with no axis of symmetry such as unequal-legged angle.

Singly symmetric sections: flexural-torsional buckling (with twisting).

Movement is in direction perpendicular to axis of symmetry, i.e. buckling about axis of symmetry

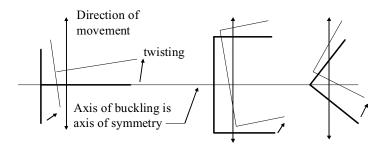


Figure 6.3 Flexural-Torsional Buckling

Normally flexural-torsional buckling is not important in the design of compression members since the buckling strength for flexural torsional buckling does not differ very much from the minor axis flexural buckling strength. Torsional buckling (Fig.6.4) occurs in doubly

symmetric sections such as cruciform (+) section and in some built up sections with very thin walls. For most doubly symmetric sections the minor axis critical load for flexural buckling is less than the critical load at which the compression member may twist about the longitudinal axis and therefore the possibility of torsional buckling can be ignored.

Torsional buckling: twisting only, no translational movement of whole section

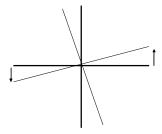


Figure 6.4 Torsional Buckling

#### Plate element slenderness

The strength of compression members is reduced if the "plate element slenderness"  $\lambda_e$  is too large.  $\lambda_e$  is the ratio of width to thickness of the plate elements that make up the cross section (web and flanges of I beam, legs of angle etc). If the component plate element slenderness is less than the plate element yield slenderness limit  $\lambda_{ey}$  given by table 6.2.4 of AS 4100, local buckling of the component element will not occur, as the plate element will yield or strainharden before it buckles locally. Therefore local buckling does not affect the strength of the compression member, so that a very short column may reach the full squash load  $N_s = A_g f_y$ . However if the component plate element slenderness exceeds  $\lambda_{ey}$  the component plate element will buckle locally before the yield stress is reached, as shown in Fig.6.5 below (i.e. the buckling of the plate element component is elastic).

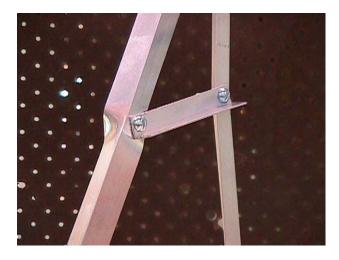


Figure 6.5 Local Buckling Caused Failure of this Slender Aluminium Angle

This elastic local buckling will reduce the capacity of the compression member, so that a very short column will squash at a load less than the full squash load, and a compression member with intermediate slenderness will fail by inelastic buckling at a reduced axial load (inelastic buckling with local buckling). A long slender compression member will fail by elastic buckling at a reduced axial load (elastic buckling with local buckling).

The effects of local buckling are allowed for in the design of compression members by using an effective width approximation for the post buckling resistance. For example in the I section shown below (assume HW), the flange slenderness is greater than the flange yield slenderness limit (i.e. the flange is slender) and therefore flange local buckling under axial compression will occur. The web slenderness exceeds the web yield slenderness limit (i.e. the web is slender) and therefore the web will also buckle locally under axial compression. AS 4100 allows for this local buckling effect by reducing the width of the plate element to an effective width. Thus in Fig.6.6 below the actual 400 mm wide flange is reduced to an effective width of  $(14/17.8) \times 195 \times 2 + 10 = 316.74$  mm, i.e the unshaded portion away form the supported edge of the flange is omitted from the effective cross section. The actual 760 mm wide web is also reduced to an effective width of  $(35/83.3) \times 760 = 319.5$  mm, i.e. the unshaded central web portion is omitted from the effective cross sectional area. The "form factor"  $k_{\rm f}$  is defined as the ratio of the effective area  $A_{\rm e}$  to the gross cross sectional area  $A_{\rm g}$ . Thus for the section shown below,

$$k_f = \frac{2 \times 316.74 \times 12 + 319.5 \times 10}{2 \times 400 \times 12 + 760 \times 10} = 0.628$$

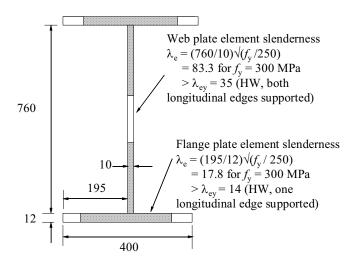


Figure 6.6 Effective Portion of Steel Column with Slender Web and Flange

The load capacity of a compression member can be predicted to some degree by theory. The following equation, adapted from [Trahair], takes into account any initial out of straightness:

$$N_{\rm L} = N_{\rm s} \left[ (1 + (1 + \eta) N_{\rm cr} / N_{\rm s}) / 2 \right] - \left\{ \left[ (1 + (1 + \eta) N_{\rm cr} / N_{\rm s}) / 2 \right]^2 - N_{\rm cr} / N_{\rm s} \right\}^{1/2}$$
(6.1)

where

 $N_{\rm L}$  is the limiting load at which compressive yielding starts in a compression member of intermediate slenderness.

 $N_s = A_n f_v$  is the nominal section capacity (squash load)

 $N_{cr} = \pi^2 \text{EI} / \text{L}_e^2$  is the elastic critical load.

 $\eta$  = is the imperfection parameter

However  $\eta$  is difficult to quantify, the equation is already cumbersome and it does not take into account other imperfections such as eccentricity of loading and residual stresses. Thus equations and curves used for practical design are semi-empirical, i.e. analytical equations like (6.1) above are adjusted to agree with experimental data.

## Practical design

The procedure given in AS 4100[1] involves calculating the following:

- (i) the section capacity  $N_s$ , i.e. the squash load at which a very short column of the cross section under consideration will fail, and
- (ii) the member capacity  $N_c$ , which is the section capacity reduced by a member slenderness reduction factor  $\alpha_c$ .

For a compact section with no unfilled holes,

 $N_s = A_g f_y$  i.e. cross sectional area times yield strength.

However the code allows for unfilled holes by using the net area  $A_n$ , and for local buckling by the form factor  $k_f$  defined above.

Thus 
$$N_s = k_f A_n f_y$$
  
And  $N_c = \alpha_c N_s$ 

The slenderness reduction factor  $\alpha_c$  is calculated, or more commonly obtained by interpolation from a table of  $\alpha_c$  as a function of "modified slenderness"  $\lambda_n$ .

The modified slenderness is given by

$$\lambda_n = \frac{L_e}{r} \sqrt{k_f} \sqrt{\frac{f_y}{250}}$$

where  $k_{\rm f}$  is the form factor which includes the effect of local buckling of the plate elements that constitute the cross section. The slenderness must also be modified according to the yield strength, since elastic buckling does not depend on yield strength and yield strength is used in calculating  $N_{\rm s}$ .

Having calculated  $\lambda_n$ , a further constant  $\alpha_b$  is introduced which shifts the  $\alpha_c$  value for different cross-section types. This constant is given in tables 6.3.3(1) and (2) of AS 4100, and  $\alpha_c$  is given in Table 6.3.3(3).

After the value of  $\alpha_c$  has been determined, the nominal member capacity  $N_c$  is obtained from the modification of equation (6.1) by using:

$$N_{\rm c} = \alpha_c k_{\rm f} A_{\rm n} f_{\rm y} \le N_{\rm s} = k_{\rm f} A_{\rm n} f_{\rm y}$$

#### 6.2 EFFECTIVE LENGTHS OF COMPRESSION MEMBERS

The effective length concept is one method of estimating the interaction effect of the total frame on a compression element being considered. This concept uses  $k_e$  factors to equate the strength of a framed compression element of length L to an equivalent pin-ended member of length  $k_e$  L subject to an axial load only. In simple words we say that the effective length factor  $k_e$  is a factor which, when multiplied by the actual unbraced length L of an endrestrained compression member, will yield an equivalent pin-ended member whose buckling strength is the same as that of the original end-restrained member.

The value of the effective length factor depends on the rotational and translational restraints at the ends of the member. Table 6.1 gives both theoretical and recommended values of  $k_e$  for columns with idealized conditions of end restraint.

|                                      | Bı              | raced mem  | ber | Sw   | vay membe        | er  |
|--------------------------------------|-----------------|--|-----|------|------------------|-----|
| Buckled shape                        |                 |  |     |      |                  |     |
| Effective length factor $k_{\rm e}$  | 0.7             | 0.85   | 1.0 | 1.2  | 2.2              | 2.2 |
| Theoretical k <sub>e</sub>           | 0.5             | 0.7  | 1.0 | 1.0  | 2.0              | 2.0 |
| Symbols for end restraint conditions | <i>!!!!!!!!</i> | = Rotation fixe<br>fixe<br>= Rotation free<br>fixe |     | fixe | otation free, tr |     |

Table 6.1  $k_e$  for columns with idealized conditions of end restraint

In Table 6.1 the values of  $k_e$  recommended for design are slightly higher than their theoretical equivalents. This is due to the fact that joint fixity is seldom fully realized (i.e. the ends will always have some flexibility in them).

The values of  $k_e$  given in Table 6.1 can be used if the support conditions of the compression member can be closely represented by those shown.

For members in frames where the support conditions cannot be represented by those shown in Table 6.1, the use of Fig.4.6.3.3(a) of AS 4100 for braced members or Fig.4.6.3.3(b) of AS 4100 for sway members gives a fairly rapid method for determining adequate  $k_e$  values. It is important to remember that these figures were developed based on a number of simplifying assumptions. As a result, they do not always give accurate results, especially for members in frames for which the parameter  $L\sqrt{(N^*/EI)}$  varies significantly from column to column in a given story. The alignment charts also fail to give accurate results for frames that contain leaner columns. For such situations, the following method for determining  $k_e$  is recommended:

## Lui's Method [2]

A simple and straightforward method for determining the effective length factors for framed columns without the use of Fig. 4.6.3.3(b) of AS 4100 was proposed by Lui [2].Lui's formula takes into account both the member instability and frame instability effects explicitly. The  $k_{\rm e}$  factor for the i-th column in a story was obtained in a simple form

$$k_{ei} = \sqrt{\left[\left(\frac{\pi^2 E I_i}{N_i * L_i^2}\right) \times \Sigma\left(\frac{N *}{L}\right) \times \left(\frac{1}{5\eta} + \frac{\Delta_I}{\Sigma H}\right)\right]}$$
(6.2)

Where  $\sum N^*/L$  is the sum of the factored axial force to length ratios of all members in a story, and  $\sum \eta$  is the sum of the member stiffness indexes of all members in the story. The member stiffness index  $\eta$  is given by:

$$\eta = [3 + 4.8 \ \beta_{\rm m} + 4.2 \ \beta_{\rm m}^{2}] \times EI / L^{3}$$
(6.3)

In the foregoing equation  $\beta_{\rm m} = M_{\rm A}/M_{\rm B}$  is the ratio of the smaller to larger end moments of the member, taken as positive if the member bends in reverse curvature and negative if the member bends in single curvature. Values for  $M_{\rm A}$  and  $M_{\rm B}$  are to be obtained from a first-order analysis of the frame subjected to a set of fictitious lateral forces applied at each story in proportion to the story factored gravity loads.  $\Delta_{\rm I}$  in Eq. (6.2) is the interstory deflection produced by these fictitious lateral forces, and  $\Sigma_{\rm H}$  is the sum of the fictitious lateral forces at and above the story under consideration.

In the event that both  $M_A$  and  $M_B$  are zero, as in the case for a pinned-pinned leaner column, the ratio  $(\theta_A / \theta_B)$ , where  $\theta$  is the member end rotation with respect to its chord, should be used in place of  $(M_A / M_B)$  when calculating  $\beta_m$ . For instance, if the leaner column buckles in reverse curvature,  $\eta$  should be taken as  $12EI / L^3$  (i.e. Taking  $\beta_m = 1$ ), but if the leaner column bends in single curvature,  $\eta$  should be taken as  $2.4EI / L^3$  (i.e. Taking  $\beta_m = -1$ ).

## **Example 6.2.1**

Determine the effective length factors for the rigid jointed frame shown below.

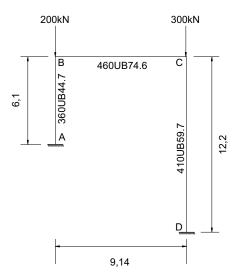


Figure 6.7 Frame with Columns of Unequal Height

#### Discussion

The example illustrates the determining of  $k_e$  for members in frames for which the parameter  $L\sqrt{(N^*/EI)}$  varies significantly from column to column in a given storey.

#### Solution

## Using Fig.4.6.3.3 (b) of AS 4100:

```
Column AB
```

$$\gamma_{A}$$
= N0. /  $\infty$  = 0  $\Rightarrow \gamma_{A}$  = 1 (fixed support)  
 $\gamma_{B}$  =  $\Sigma$ (EI /  $L$ ) column /  $\Sigma \beta_{e}$  (I / $L$ ) beam  
= [((121 x 10<sup>6</sup>) / (6.1 x 10<sup>3</sup>)) / (1x 335 x 10<sup>6</sup> / 9.14 x 10<sup>3</sup>)]  
= 0.54  
 $\Rightarrow$  Using Figure 4.6.3.3 (b) of AS 4100 we obtain  $k_{e}$  = 1.25

#### Column CD

Column CD  

$$\gamma_{\rm C} = \left[ \left( (216 \times 10^6) / (12.2 \times 10^3) \right) / (335 \times 10^6 / 9.14 \times 10^3) \right]$$

$$= 0.48$$

$$\gamma_{\rm D} = 1 \text{ (fixed support)}$$

$$\Rightarrow \text{Using Figure 4.6.3.3 (b) of AS 4100 we obtain } k_{\rm e} = 1.24$$

## Lui's Approach:

Apply a fictitious lateral force and perform a first order analysis on the frame. Determine the member end moments and the story deflection. The magnitude of this fictitious lateral force is quite arbitrary as long as it is proportional to the applied story gravity loads. In this example the magnitude of the fictitious lateral force is taken as 0.05 of the applied story gravity loads, see Fig.6.8.

First order Elastic Analysis output

| Node | $M^*$ (kNm) |
|------|-------------|
| A    | 66.54       |
| В    | 53.06       |
| С    | 31.972      |
| D    | 33.821      |

 $\Delta_{\rm I} = 21$  mm (Relative displacement)

$$\eta_{AB} = \left[ 3 + 4.8 \text{ x} \left( 53.06 \, / \, 66.54 \right) + 4.2 \text{ x} \left( 53.06 \, / \, 66.54 \right)^2 \right] \text{ x} \left( 200,\!000 \text{ x} \, 121 \text{ x} \, 10^6 \right) / \left( 6100 \right)^3 \\ = 1012.674 \text{ N/mm}$$

$$\eta_{CD} = [3 + 4.8 \text{ x} (31.972/33.821) + 4.2 \text{ x} (31.972/33.821)^2] \text{ x} (200\ 000 \text{ x} 216 \text{ x} 10^6) / (1220)^3 = 268.616 \text{ N/mm}$$

$$\Sigma \eta = \eta_{AB} + \eta_{CD} = 1281.3 \text{ N/mm}$$

$$\Delta_{\rm I} / \Sigma_{\rm H} = 21 / (25 \times 10^3) = 0.00084 \text{ mm/N}$$

#### Column AB

$$k_e = \sqrt{\left[ (\pi^2 \times 200,000 \times 121 \times 10^6 / (200 \times 10^3 \times 6100^2)) \times (200 \times 10^3 / 6100 + 300 \times 10^3 / 12200) \times (1 / (5 \times 1281.3) + 0.00084) \right]}$$

 $k_{\rm e} = 1.35$ 

## Column CD

 $k_e = \sqrt{[(\pi^2 \times 200,000 \times 216 \times 10^6 / (300 \times 10^3 \times 12200^2)) \times (200 \times 10^3 / 6100 + 300 \times 10^3 / 12200)} \times (1/(5 \times 1281.3) + 0.00084)]$  $k_e = 0.75$ 

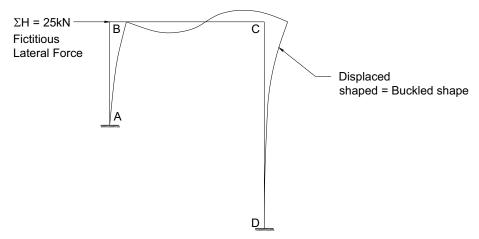


Figure 6.8 Fictitious lateral force applied to frame with columns of unequal height

# Theoretical $k_e$ factors

The theoretical  $k_e$  factors obtained using buckling analysis [3] are 1.347 and 0.71 for column AB and CD, respectively. Thus, it can be seen that figure 4.6.3.3 (b) of AS 4100 [1] give incorrect results when used for columns in frames for which the parameter  $L\sqrt{(N^*/EI)}$  varies significantly from column to column.

## Example 6.2.2

Determine the effective length factors  $k_e$  for columns AB and CD in the frame shown below.

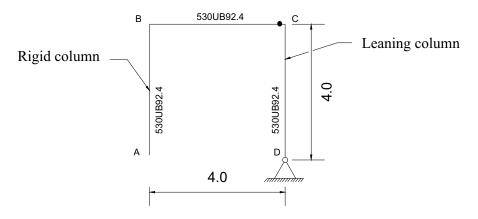


Figure 6.9 Frame with Rigid Column and "Leaning" Column

#### Discussion

The example illustrates the determination of  $k_e$  for members in a frame containing one or more "leaning" columns (i.e. pin-ended columns which do not contribute to the lateral stability of the frame).

#### Solution

### Effective length factor for column AB:

• Figure 4.6.3.3 (b) of AS 4100 [1] Approach

```
\gamma_{A} = \text{No.} / 0 = \infty \text{ (pinned support)}

\gamma_{B} = \Sigma(I / L) \text{ column } / \Sigma \beta_{e} (I / L) \text{ beam}

\gamma_{B} = \left[ ((554 \times 10^{6}) / (4 \times 10^{3})) / (0.5 \times 554 \times 10^{6} / 4 \times 10^{3}) \right] = 2

\Rightarrow \text{Using Figure 4.6.3.3 (b) of AS 4100 we obtain } k_{e} = 2.6
```

- Lui's Method [2]
- (a) Apply a fictitious lateral force, say H = 50 kN
- (b) Perform a first order analysis and find  $\Delta_I = 19.42$  mm
- (c) Calculate  $\eta$  factors from equation 5.3

Since column CD buckles in single curvature  $\beta_{\rm m}$  = -1

$$\eta_{CD} = 2.4 \text{EI} / L^3 = 2.4 \times 200 \times 554 \times 10^6 / 4000^3 = 4.155 \text{ kN/mm}$$

For column AB,  $\beta_{\rm m} = 0$ 

$$\eta_{AB} = 3EI/L^3 = 3 \times 200 \times 554 \times 10^6/4000^3 = 5.194 \text{ kN/mm}$$

$$\Sigma \eta = \eta_{AB} + \eta_{CD} = 9.349 \ kN/mm$$

(d) Calculate  $k_e$  using equation 5.2

$$k_e = \sqrt{\left[ (\pi^2 \times 200 \times 554 \times 10^6 / (400 \times 4000^2)) \times (2 \times 400 / 4000) \times (1 / (5 \times 9.349) + 19.42 / 50) \right]}$$
  
 $k_e = 3.74$ 

### • Theoretical $k_e$ factors

The theoretical  $k_e$  factor for column AB obtained using buckling analysis [3] is 3.69. Thus, it can be seen that figure 4.6.3.3 (b) of AS 4100 [1] significantly underestimates  $k_e$  for this frame, which overestimates the buckling load a by a factor of  $(3.69/2.6)^2 = 2$ , and leads to an unsafe design, while Lui's Method gives good results.

### Effective length factor for column CD:

Since column CD is framed with simple connections it has no lateral stiffness or sidesway resistance. Such a column is often referred to as a leaning column. Recognizing that rigid columns are bracing a leaning column, Lui [2] proposed a model for the leaning column, as shown in the figure below. Rigid columns provide lateral stability to the whole structure and are represented by a translation spring with spring stiffness,  $S_k$ . The  $k_e$  factor for a leaning

column can be obtained as 
$$k_e = \text{larger of 1}, \sqrt{\frac{\pi^2 EI}{S.L^3}}$$

The term  $\pi^2 EI/L^3$  represents the minimum spring stiffness needed to ensure that the effective length  $L_e$  is equal to and not greater than the unbraced length (i.e. the system length) L of the leaning column. For most commonly framed structures the minimum spring stiffness is satisfied and  $k_e = 1$  often governs.

Designers can readily determine the spring stiffness  $S_k$  by analyzing a special load case with a single fictitious lateral force at the top of the leaning column.

- (a) Apply a fictitious lateral force, say H = 50 kN
- (b) Perform a first order analysis and find  $\Delta_I$  = 19.42 mm

$$S_k = 50 / 19.42 = 2.57 \text{ kN/mm}$$

$$k_e = \text{larger of 1}, \ \sqrt{\frac{\pi^2 EI}{S_k L^3}} = \sqrt{\frac{\pi^2 \times 200 \times 554 \times 10^6}{2.57 \times 4000^3}} = 2.58$$

$$k_{\rm e} = 2.58$$

Note: the effective length for buckling of a pinned – pinned central column in a portal frame building in the plane of the frame, is determined using the method outlined above.

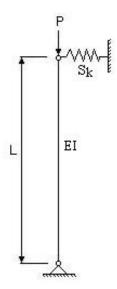


Figure 6.10 Model for a Leaning Column

## 6.3 DESIGN OF COMPRESSION MEMBERS TO AS 4100

A member subject to a concentric design axial compressive force N\* shall satisfy both-

$$N^* \leq \phi N_s$$
, and

$$N^* \leq \phi N_c$$

where

 $\phi$  = the capacity factor (see table 3.4 of AS 4100)

 $N_s$  = is the nominal section capacity which is calculated from  $N_s = k_f A_n f_y$  where

 $k_{\rm f} = A_{\rm e}/A_{\rm g}$  is the form factor

 $A_{\rm n}$  = the net area of the cross-section, except that for sections with penetrations or unfilled holes that reduce the section area by less than  $100\{1-[f_{\rm y}/(0.85f_{\rm u})]\}\%$ , the gross area may be used.

 $N_c$  = is the nominal member capacity, which is given by  $N_c = \alpha_c k_f A_n f_y \le N_s = k_f A_n f_y$  where

 $\alpha_c$  = is the slenderness reduction factor determined in accordance with Clause 6.3.3 of AS 4100.

The design of a compression member can be carried out using the following design procedure:

- 1. Calculate  $N^*$  from load estimation and structural analysis.
- 2. Determine  $L_{\rm e}$ .
- 3. Choose a trial section assuming  $A_n = A_g$  by one of two methods:
- A. Read off a section with a suitable value of  $\phi N_c$  directly from AISC tables [4], or
- B. If tables are not available,
- (i) Assume initially that the form factor  $k_f = 1$  (it will be between 0.88 and 1 so this is close enough for a first try).
- (ii) Assume  $\alpha_c \approx 0.8$  for a fairly short, stubby, heavily loaded member such as a lower column in a multi-storey building. This will have to be checked. For a longer, more lightly loaded members such as wind bracing designed to take both tension and compression in a factory,  $\alpha_c$  may be much lower,  $\approx 0.2\text{-}0.5$ .
- (iii) Calculate the minimum gross area  $A_g$  (assumed =  $A_n$ ) required from  $N^* \le \phi N_c = \phi \alpha_c N_s$ =  $\phi \alpha_c k_f A_n f_v$  (AS 4100 Cl. 6.1 and 6.2.1).
- (iv) Then choose a suitable section with the required  $A_g$  and check  $k_f$ . You can either read  $k_f$  off tables if available, or else calculate it from the plate element slenderness  $\lambda_e$  for the section chosen (AS 4100 Cl. 6.2.2-6.2.4).
- 4. If there are unfilled holes, calculate  $A_n$  (AS4100 Cl.6.2.1) and hence calculate  $N_s = k_f A_n f_v$
- 5. Calculate or look up  $r_x$  and  $r_y$  and hence calculate the modified slenderness  $\lambda_n$  for buckling about both principal axes from  $\lambda_n = \left(\frac{L_e}{r}\right) \sqrt{k_f} \sqrt{\frac{f_y}{250}}$ .
- 6. Next find  $\alpha_b$  and  $\alpha_c$ , and calculate  $N_c = \alpha_c N_s$

To calculate  $\alpha_c$  use one of the following ways:

- (1) Follow all the calculations in AS 4100 Clause 6.3.3. These are complicated and not recommended unless a spread sheet program is used.
- (2) Use Clause 6.3.3 in AS 4100 to get the modified slenderness  $\lambda_n$ , i.e.

$$\lambda_n = \left(\frac{L_e}{r}\right) \sqrt{k_f} \sqrt{\frac{f_y}{250}}$$

Then use Table 6.3.3(1) or (2) in AS 4100 to read off  $\alpha_b$ , the "member section constant", which takes into account the amount of residual stress in the section, for example stress relieved RHS and CHS have less residual stress than welded H and I sections. Having found  $\alpha_b$ , you can use table 6.3.3(3) in AS 4100 to read off  $\alpha_c$ , interpolating if necessary.

7. Check that  $N^* \le \phi N_s$  and  $N^* \le \phi N_c$ 

If  $N_c$  is not large enough, choose a stronger member, i.e. increase  $A_g$ , or if  $N_c$  is large enough but  $N_s$  is not large enough because  $\alpha_c \ll 1$ , look for ways to reduce  $L_e$  or increase r for the critical buckling mode.

### **6.4 WORKED EXAMPLES**

### **Example 6.4.1 Slender Bracing**

Suppose the bracing member in example 5.3.4 on tension members is also required to take  $N^* = 141$  kN compression under some loading conditions. It is 6 m long, pinned at both ends, so  $L_{\rm e} = 6000$  mm. Choose a suitable section (a) using a Grade 350 CHS, (b) using a Grade 300 equal angle.

#### Solution

- (a) Steps 1 and 2 are given. Step 3: From AISC design capacity tables [4], choose 139.7x3.5 CHS giving  $\phi N_c = 154$  kN.
- (b) Step 3: The member is long in relation to the load, so its modified slenderness  $\lambda_n$  is likely to be large, so assume  $\alpha_c = 0.3$ .

$$N^* \le \phi N_c = \phi \alpha_c N_s = \phi \alpha_c k_f A_n f_y$$
   
  $\therefore 141000 \le 0.9 \times 0.3 \times 1 \times A_n \times 300$   
 $\therefore A_n \ge 1741 \text{ mm}^2$   
Try 100x100x10 EA with  $A_g = 1810 \text{ mm}^2$ .  $k_f = 1$ , so  $A_e = A_g$ 

Step 4: There are no unfilled holes so  $A_n = A_g$ 

Step 5:  $r_x = 38.6$  mm,  $r_y = 19.6$  mm. Because the effective buckling length is the same about both axes, the slenderness is greater for y -axis buckling, so only check y -axis.

$$\lambda_n = \left(\frac{L_e}{r}\right) \sqrt{k_f} \sqrt{\frac{f_y}{250}} = (6000/19.6) \times \sqrt{1} \times \sqrt{(300/250)} = 335.3$$

Step 6:  $\alpha_b$ = 0.5 (AS 4100 Table 6.3.3(1) so  $\alpha_c$ < 0.071 (AS 4100 Table 6.3.3(3)). i.e. much lower than the 0.3 we assumed. Thus this section will clearly not be adequate, and we must either reduce  $L_e$  or choose a more efficient section such as a CHS (see above), or a larger EA section assuming a lower value of  $\alpha_c$ , say 0.1 (since a larger section will have larger  $r_y$  and therefore lower  $\lambda_n$ ,  $\alpha_c$  will be higher than it was for the first trial).

Second trial section. Return to step 3:

$$N^* \le \phi N_c = \phi \alpha_c N_s = \phi \alpha_c k_f A_n f_y$$
  
141000 \le 0.9 \times 0.1 \times 1 \times A\_n \times 300  
 $A_n \ge 5223 \text{ mm}^2$ 

Try 150x150x19 EA in Grade 300 steel with  $A_g = 5360 \text{ mm}^2$ .  $k_f = 1$ , so  $A_e = A_g$ 

Step 4: There are no unfilled holes so  $A_n = A_g$ 

Step 5: 
$$r_y = 29.3 \text{ mm.}$$
  $\lambda_n = \left(\frac{L_e}{r}\right) \sqrt{k_f} \sqrt{\frac{f_y}{250}} = (6000/29.3) \text{ x} \sqrt{1} \text{ x} \sqrt{(280/250)} = 216.72$ 

Step 6:  $\alpha_b = 0.5$  (AS 4100 Table 6.3.3(1)),  $\alpha_c = 0.146$  (AS 4100 Table 6.3.3(3))

$$\phi N_c = \phi \alpha_c N_s = 0.9 \times 0.146 \times 1 \times 5360 \times 280 = 197.2 \text{ kN} > N^* = 141 \text{ kN}$$
 OK

Hence, Adopt 150x150x19 EA in Grade 300 steel

#### Comment

The 150x150x19 EA section weighs 42.1 kg/m, which is much more than the 139.7 x 3.5 mm CHS which weighs only 11.8 kg/m. Weight is a rough guide to cost. Although the CHS is a bit more expensive per kg than angle and a bit more difficult to join, it would be a better choice for this long slender compression member.

### **Example 6.4.2 Bracing Strut**

A 139.7 x 5.4 CHS in Grade 250 steel is used as a bracing strut in the roof bracing system of an industrial building. The strut is 7.2m long with both ends pin connected. Determine the maximum design compressive force  $N^*$  that can be transmitted. The CHS is cold formed.

#### Solution

$$A_{\rm g} = \pi(r_{\rm o}^2 - r_{\rm i}^2) = \pi(70^2 - 64.6^2) = 2283 \ {\rm mm}^2$$
  
For CHS,  $\lambda_e = (d_{\rm o}/t)(f_{\rm y}/250) = (140/5.4)(1) = 25.93 < \lambda_{\rm ey} = 82$   
 $\therefore$  No reduction in  $A_{\rm e} = A_{\rm g}$  and  $k_{\rm f} = 1$   
 $N_s = k_{\rm f} A_{\rm n} f_{\rm y} = 1 \times 2283 \times 250 = 570.7 \ {\rm kN}$   
 $L_e = L = 7.2 \ {\rm m}$  (pinned ends).  
 $r = \sqrt{({\rm I}/A)} = \sqrt{(\pi/4)(r_{\rm o}^4 - r_{\rm i}^4)} / \sqrt{\pi(r_{\rm o}^2 - r_{\rm i}^2)} = 47.6 \ {\rm mm}$   
 $\lambda_{\rm n} = (L_e/r)\sqrt{k_{\rm f}} \sqrt{(f_{\rm y}/250)} = 7200/47.6 = 151$   
 $\alpha_{\rm b} = -0.5$   
 $\alpha_{\rm c} = 0.313$   
 $N_{\rm c} = \alpha_{\rm c} N_{\rm s} = 0.313 \times 570.7 = 178.6 \ {\rm kN}$   
 $\phi N_{\rm c} = 0.9 \times 178.6 = 160.7 \ {\rm kN}$   
 $Answer N^* = 160.7 \ {\rm kN}$ 

### Example 6.4.3 Sizing an Intermediate Column in a Multistory Building

Size a typical intermediate column at the ground level of a 6-storey office building, each storey is 4m in height giving a total height of 24m, the building will have a regular rectangular shape with braced frames designed to take the effect of the horizontal wind and earthquake forces these braced frames are hidden in the stairwells at the corners of the building. Composite construction is to be used where the concrete slabs act as compression flanges for the steel beams, all beam to beam and beam to column connections are flexible connections (i.e. pinned connections), all columns will have a pinned base plate. The slabs dead weight is 4.5 kN/m² including profiled steel sheeting, steel beams self weight and lightweight timber partitions weight, the live load acting on a typical floor including the roof floor is 3 kN/m². The column is in Grade 300 steel supporting a floor area of 56.25m², assume 1 kN/m. for the column's self- weight.

#### Solution

The suggested design procedure given below is based on the assumption that the designer does not have access to the design charts [4].

### Design procedure:

1. Determine the design axial load acting on the column.

$$N^* = 1.2G + 1.5Q = 1.2 \text{ x} [56.25\text{x}6\text{x}4 + 24\text{x}1] + 1.5 \text{ x} [56.25\text{x}6\text{x}3] = 3167.3 \text{ kN}$$

2. Choose a column section that satisfies the section capacity.  $\phi N_s$ 

$$N^* \le \phi N_s = k_f A_n f_y$$
, assume  $k_f = 1$ , there is no unfilled holes therefore  $A_n = A_g$   
 $N^* \le \phi N_s = \phi A_g f_y \implies A_g \ge N^* / \phi f_y$   
 $A_g \ge 3167.3 \times 10^3 / (0.9 \times 300)$   
 $A_g \ge 11730.6 \text{ mm}^2$ 

Try using 310UC137 in Grade 300 steel (
$$f_{\rm yf}$$
 = 280 MPa ,  $f_{\rm yw}$  = 300 MPa)  $A_{\rm g}$  = 17500 mm<sup>2</sup>  $\geq$  11730.6 mm<sup>2</sup>  $OK$ 

Note: since the effective length for buckling is the same for both principle axes using a universal column section will be more economical than using a universal beam section.

3. For the chosen section calculate  $k_f$ , if  $k_f$  is equal to 1 go to step 4, if  $k_f < 1$  calculate the section capacity and compare it with  $N^*$  to make sure that the section capacity after the reduction due to local buckling have been made, is still ok

To calculate  $k_{\rm f}$ , the plate element slenderness values are compared with the plate element yield slenderness limit  $\lambda_{\rm ev}$  given by table 6.2.4 of AS 4100.

### Flange slenderness

$$\lambda_{\text{ef}} = ((b_{\text{f}} - t_{\text{w}}) / 2t_{\text{f}}) \sqrt{(f_{\text{yf}} / 250)} \Rightarrow \lambda_{\text{ef}} = ((309-13.8) / (2x21.7)) \times \sqrt{(280/250)} = 7.2$$
 $\lambda_{\text{ey}} = 16$ 
 $\lambda_{\text{ef}} = 7.2 < \lambda_{\text{ey}} = 16$ 

## Web slenderness

$$\lambda_{\text{ew}} = b_{\text{w}} / t_{\text{w}} \sqrt{(f_{\text{yw}} / 250)} = (277/13.8) \times \sqrt{(280 / 250)} = 21.24$$
  
 $\lambda_{\text{ew}} = 7.2 < \lambda_{\text{ey}} = 45$  *OK AS*4100 *Table* 6.2.4

There will be no reduction in the gross sectional area, thus  $A_e = A_g$  and the form factor  $k_f = 1$ 

4. Check the member capacity for buckling about both principal axes.

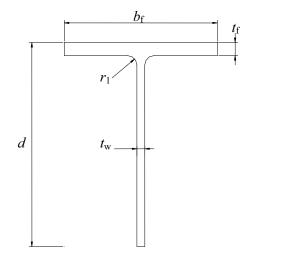
The beam column connections are flexible therefore the columns end at the beam column connection is restrained in position free in direction (i.e. effectively pinned), the column base is also pinned therefore the compression member is pin-ended for buckling about the x and the y axis, thus  $k_e = 1$  for x-axis buckling and for y-axis buckling, since  $L_{ey} = L_{ex} = 4$ m the least radius of gyration will govern the buckling which is  $r_y$  so we only need to check the member capacity for buckling about the y-axis.

For 
$$L_{\rm ey} = 4 \, {\rm m}$$
,  $\lambda_{\rm n} = (4 \times 10^3 / \ 78.2) \, {\rm x} \, \sqrt{(1)} \, {\rm x} \, \sqrt{(280/250)} = 54.1$    
  $\alpha_{\rm b} = 0$    
  $\alpha_{\rm c} = 0.8405$    
  $\phi N_{\rm c} = \phi \alpha_{\rm c} \, k_{\rm f} \, A_{\rm n} \, f_{\rm y}$    
  $\phi N_{\rm c} = 0.9 \, {\rm x} \, 0.8405 \, {\rm x} \, 1 \, {\rm x} \, 17500 \, {\rm x} \, 280 \, {\rm x} \, 10^{-3}$    
  $\phi N_{\rm c} = 3706.61 \, {\rm kN} > N^* = 3167.3 \, {\rm kN}$    
  $OK$ 

Hence Adopt 310 UC137 in Grade 300 steel

### **Example 6.4.4 Checking a Tee Section**

Determine the maximum design load  $N^*$  of concentrically loaded 305BT62.5 compression member of Grade 300 steel ( $f_{yf}$  = 280 MPa,  $f_{yw}$  = 300 MPa), if the effective length for buckling about each axis is 2m.



305BT 62.5

d = 305 mm  $b_f = 229$  mm  $t_f = 19.6$  mm  $t_w = 11.9$  mm  $r_l = 14$  mm  $A_g = 7970$  mm<sup>2</sup>  $r_x = 92.6$  mm  $r_y = 49.7$  mm

Figure 6.11 Dimensions of Tee Section

### Solution

Since the compression member have the same effective length for buckling about both axis, minor axis buckling governs, and the maximum design axial compression force  $N^*$  the member can possibly withstand is the minor axis compression capacity  $\phi N_{\rm cy.}$ 

$$\phi N_{\rm cy} = \phi \alpha_{\rm c} N_{\rm s} = \phi \alpha_{\rm c} k_{\rm f} A_{\rm n} f_{\rm y}$$
AS 4100 Cl. 6.3.3

To determine the value of  $k_{\rm f}$  (form factor) the plate element slenderness values are compared with the plate element yield slenderness limits in table 6.2.4 of AS 4100.

Note: For sections where the web and flange yield stresses ( $f_{yw}$ ,  $f_{yf}$ ) are different the lower of the two is applied to both the web and flange to determine the slenderness of these elements. Flange slenderness

$$\lambda_{\rm e} = b/t \sqrt{(f_{\rm y} / 250)}$$
  
 $b/t = (b_{\rm f} - t_{\rm w}) / 2t_{\rm f} = (229 - 11.9) / (2x19.6) = 5.54$   
 $\lambda_{\rm e} = 5.54 \times \sqrt{(280 / 250)} = 5.86$ 

The value of the plate element yield slenderness limit  $\lambda_{ey}$  for a flat element, which is, unstiffened (i.e. one longitudinal edge only supported) which is a part of a hot rolled section is  $\lambda_{ey} = 16$ AS 4100 Table 6.2.4

$$\lambda_{\rm e} = 5.86 < \lambda_{\rm ev} = 16$$
 OK

... There will be no reduction of the clear flange width outstanding from the face of the supporting plate element (i.e. stem)

### Web (Stem) slenderness

$$b/t = d_{\text{w}}/t_{\text{w}} = 285 / 11.9 = 24$$
  
 $\lambda_{\text{e}} = b/t \sqrt{(f_{\text{v}}/250)} = 24 \times \sqrt{(280 / 250)} = 25.4$ 

The value of the plate element yield slenderness limit  $\lambda_{ey}$  for a flat element, which is, unstiffened (i.e. one longitudinal edge only supported) which is a part of a cut hot rolled section is  $\lambda_{ey} = 16$ AS 4100 Table 6.2.4

```
\lambda_e = 25.4 > \lambda_{ey} = 16 and therefore the cross section is not fully effective under axial compression and the member capacity will be reduced due to stem local buckling. b_e = b \ (\lambda_{ey} / \lambda_e) \le b \Rightarrow b_{ew} = d_w \ (\lambda_{ey} / \lambda_e) \Rightarrow b_{ew} = 285 \ x \ (16 / 25.4) AS \ 4100 \ Cl. \ 6.2.4 b_{ew} = 179.53 \ mm A_e = \text{flanges area} + \text{effective web area} A_e = 229 \ x \ 19.6 + 179.53 \ x \ 11.9 + (7970-229 \ x \ 19.6-285 \ x \ 11.9) = 6715 \ mm^2 k_f = A_e / A_g = 6715 / 7970 = 0.843 L_{ey} = 2 \ m \lambda_n = (L_{ey} / r_y) \ \sqrt{k_f} \ \sqrt{(f_y/250)} = (2x10^3/49.7) \ x \ \sqrt{(0.843)} \ x \ \sqrt{(280/250)} = 39.1 \alpha_b = 1.0 \ (\text{other sections not listed}) AS \ 4100 \ Table \ 6.3.3(2) \alpha_c = 0.824 AS \ 4100 \ Table \ 6.3.3(3) \phi N_{cy} = \phi \alpha_c N_s = \phi \alpha_c \ k_f A_n f_y = 0.9 \ x \ 0.824 \ x \ 0.843 \ x \ 7970 \ x \ 280 \ x \ 10^{-3} = 1395 \ kN
```

## **Example 6.4.5 Checking Two Angles Connected at Intervals**

The two 150x90x8 UA angle sections in Grade 300 steel are connected together at 1.25m intervals by welding the toes of their short legs together as shown in the figure below. The compound member is braced at 5m intervals in its stiffer principle plane, and at 2.5m intervals in its weak principle plane. Determine the maximum design compression load  $N^*$  of the compound member.

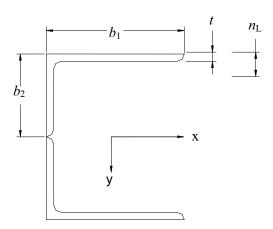


Figure 6.12 Two Angles Connected at Intervals

#### Solution

$$\begin{split} A_{\rm gc} &= 2 \times 1820 = 3640 \text{ mm}^2 \\ I_{\rm xc} &= 2 \left[ I_{\rm ps} + A_{\rm gs} \, d_y^2 \right] = 2 \times \left[ 1.18 \times 10^6 + 1820 \times (90 - 19.6)^2 \right] = 20.4 \times 10^6 \text{ mm}^4 \\ r_{\rm xc} &= \sqrt{\left[ I_{\rm xc} \, / \, A_{\rm gc} \right]} = \sqrt{\left[ (20.4 \times 10^6) \, / \, 3640 \right]} = 74.9 \text{ mm} \\ \lambda_{\rm e} &= \left[ (b_1 - t)/t \right] \times \sqrt{\left( f_y / 250 \right)} \\ \lambda_{\rm e} &= \left( (150 - 7.8) \, / \, 7.8 \right) \times \sqrt{\left( 320 \, / \, 250 \right)} = 20.63 > \lambda_{\rm ey} = 16 \text{ (HR)} \\ b_{\rm e} &= b \left( \lambda_{\rm ey} \, / \, \lambda_{\rm e} \right) = (150 - 7.8) \times (16 \, / \, 20.63) = 110.29 \text{ mm} \end{split}$$

```
k_{\rm f} = A_{\rm ec} \ / \ A_{\rm gc} = 3142.2 \ / \ 3640 = 0.863 L_{\rm ex} = 5000 \ {\rm mm}, \ L_{\rm ey} = 2500 \ {\rm mm} \lambda_{\rm nx} = (5000 \ / \ 74.9) \ {\rm x} \ \sqrt{0.863} \ {\rm x} \ \sqrt{(320 \ / \ 250)} = 70.2 \lambda_{\rm ny} = (2500 \ / \ 48.4) \ {\rm x} \ \sqrt{0.863} \ {\rm x} \ \sqrt{(320 \ / \ 250)} = 54.3 < \lambda_{\rm nx} = 70.2 \ {\rm and \ therefore} \ \lambda_{\rm n} = \lambda_{\rm nx} = 70.2 \alpha_{\rm b} = 1 \ ({\rm other \ sections}) \qquad \qquad AS4100 \ Table \ 6.3.3(2) \alpha_{\rm c} = 0.608 \qquad \qquad AS4100 \ Table \ 6.3.3(3) \phi N_{\rm cc} = 0.9 \ {\rm x} \ 0.608 \ {\rm x} \ 0.863 \ {\rm x} \ 3640 \ {\rm x} \ 320 = 549.72 \ {\rm kN} \Rightarrow N^* \le 549.72 \ {\rm kN} Find the maximum load at which buckling of a single angle about its minor principle axis will occur. L_{\rm em} = 1250 \ {\rm mm}, \ r_{\rm ys} = 19.7 \ {\rm mm} \lambda_{\rm ns} = (1250/19.7) \ {\rm x} \ \sqrt{0.863} \ {\rm x} \ \sqrt{(320/250)} = 66.7
```

 $\begin{aligned} & \lambda_{\rm ns} = (1250/19.7) \text{ x } \sqrt{0.863} \text{ x } \sqrt{(320/250)} = 66.7 \\ & \alpha_{\rm b} = 1 \\ & \alpha_{\rm c} = 0.631 \\ & \phi N_{\rm cs} = 0.9 \text{ x } 0.631 \text{ x } 0.863 \text{ x } 1820 \text{ x } 320 = 285.27 \text{ kN} \\ & N^* / 2 \leq 285.27 \text{ kN} \\ & N^* \leq 570.6 \text{ kN} > 549.72 \text{ kN} \end{aligned}$ 

*Buckling of the compound section about the* x-axis governs and  $N^* \le 549.72 \text{ kN}$ 

### **Example 6.4.6 Checking Two Angles Connected Back To Back**

 $A_{\rm ec} = 3640 - 2 \times (150 - 7.8 - 110.29) \times 7.8 = 3142.2 \text{ mm}^2$ 

Member AB in the truss shown below is made up of two 125x125x10 EA discontinuously separated back to back by a distance of 20 mm (Gusset plate- thickness), under gravity loading member AB will be under compression. The length between consecutive points where tie plates are attached is equal to 1.20m. The member is braced at A&B so that the unbraced length (i.e. system length) for minor axis buckling  $L_y = 4.8$  m. Determine the maximum design load  $N^*$  of the compound member.

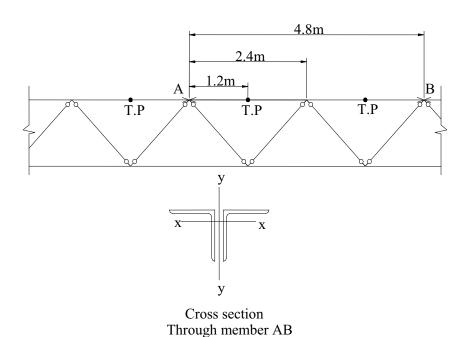


Figure 6.13 Two Angles Connected Back To Back in a Truss

#### Solution

$$A_{\rm gc} = 2 \times 2300 = 4600 \, {\rm mm}^2$$
  
 $I_{\rm yc} = 2 \times [3.42 \times 10^6 + 2300 \times (34.4 + 10)^2] = 15.91 \times 10^6 \, {\rm mm}^4$   
 $r_{\rm yc} = \sqrt{(I_{\rm yc}/A_{\rm gc})} = \sqrt{[(15.91 \times 10^6)/460]} = 58.81 \, {\rm mm}$   
 $r_{\rm xc} = r_{\rm p} = r_{\rm n} = 38.6 \, {\rm mm}$   
 $Note$ : the x-axis of the compound member is the minor axis.  
 $\lambda_{\rm e} = \{(125 - 9.5)/9.5\} \times \sqrt{(320/250)} = 13.76 < \lambda_{\rm ey} = 16$   
 $k_{\rm f} = 1$ 

### In-plane buckling (about the x-axis) / Flexural Buckling

$$\begin{aligned} L_{\text{ex}} &= k_{\text{e}} L_{\text{x}} = 1 \text{ x } 2400 = 2400 \text{ mm} \\ \lambda_{\text{nx}} &= (2400 / 38.6) \text{ x } \sqrt{1} \text{ x } \sqrt{(320 / 250)} = 70.34 \\ \alpha_{\text{b}} &= 0.5 \text{ (other sections)} \end{aligned} \qquad & \textit{AS } 4100 \textit{ Table } 6.3.3(2) \\ \alpha_{\text{c}} &= 0.678 & \textit{AS } 4100 \textit{ Table } 6.3.3(3) \\ \phi N_{\text{cx}} &= 0.9 \text{ x } 0.678 \text{ x } 1 \text{ x } 4600 \text{ x } 320 \text{ x } 10^{-3} = 897.87 \text{ kN} \end{aligned}$$

### Out of plane buckling (about the y-axis) / Flexural Torsional Buckling

$$L_{\rm ey} = k_{\rm e}L_{\rm y} = 1 \text{ x } 4800 = 4800 \text{ mm}$$

The slenderness of the compound compression member about the axis parallel to the connected surfaces shall be calculated from the following expression,

$$(L_{\rm ey}/r_{\rm y})_{\rm p} = \sqrt{[(L_{\rm e}/r)^2_{\rm m} + (L_{\rm e}/r)^2_{\rm c}} \qquad AS4100 \ Cl. \ 6.5.1.4.$$

$$(L_{\rm e}/r)_{\rm m} = 4800 / 58.81 = 81.62$$

$$(L_{\rm e}/r)_{\rm c} = 1200 / 24.7 = 48.6$$

$$(L_{\rm ey}/r_{\rm y})_{\rm p} = \sqrt{[(81.62)^2 + (48.6)^2]} = 95$$

$$(L_{\rm e}/r)_{\rm c} \text{ shall not exceed the lesser of 50 and } 0.6 \times L_{\rm ey}/r_{\rm y} = 0.6 \times 95 = 57$$

$$(L_{\rm e}/r)_{\rm c} = 48.6 < 50 \qquad OK$$

$$\lambda_{\rm ny} = 95 \times \sqrt{1} \times \sqrt{(320 / 250)} = 107.5$$

$$\alpha_{\rm b} = 0.5$$

$$\alpha_{\rm c} = 0.444$$

$$AS4100 \ Table \ 6.3.3(2)$$

$$AS4100 \ Table \ 6.3.3(3)$$

$$\phi N_{\text{cy}} = 0.9 \times 0.444 \times 1 \times 4600 \times 320 \times 10^{-3} = 588.21 \text{ kN}$$

Since  $\phi N_{\rm cy} < \phi N_{\rm cx}$  the capacity will be controlled by buckling about the y-axis of the compound section.

$$\therefore \phi N_{c} = \phi N_{cy} = 588.21 \text{ kN}$$

$$N^{*} \leq \phi N_{c}$$

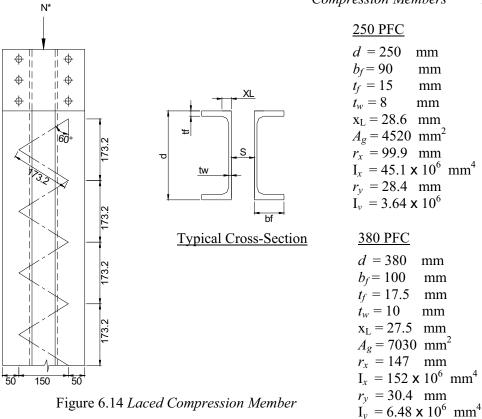
$$N^{*} \leq 588.21 \text{ kN} \quad Answer$$

### **Example 6.4.7 Laced Compression Member**

Design a built up section composed of two parallel flange channels, toes out and separated, for a design axial compression force of 2600 kN. The effective lengths for major and minor axis buckling are  $L_{\rm ex} = 7$  m and  $L_{\rm ey} = 3.5$  m.

#### Solution

Select a PFC section that is satisfactory for x-axis failure then determine the minimum spacing (s) so that the section is satisfactory for y-axis failure.



## Try two 250 PFC in Grade 300 steel

```
\overline{A_g} = 2 \times 4520 = 9040 \text{ mm}^2
I_x = 2 \times 45.1 \times 10^6 = 90.2 \times 10^6 \text{ mm}^4
r_x = 99.9 \text{ mm}
k_f = 1
\Delta ISC \ Tables
\alpha_b = 0.5
\Delta_{nx} = (7000 / 99.9) \times \sqrt{1} \times \sqrt{(300 / 250)} = 76.76
\alpha_{cx} = 0.634
\Phi N_{cx} = 0.9 \times 0.634 \times 1 \times 9040 \times 300 \times 10^{-3} = 1548 \text{ kN} < N^* = 2600 \text{ kN} \quad NG
```

## Try two 380 PFC in Grade 300 steel

 $(3500 / r_v) \times \sqrt{1} \times \sqrt{(280 / 250)} \le 61.88$ 

```
A_g = 2 \times 7030 = 14060 \text{ mm}^2
I_x = 2 \times 152 \times 10^6 = 304 \times 10^6 \text{ mm}^4
r_{\rm x} = 147 \; {\rm mm}
                                                                                                    AISC Tables
k_{\rm f} = 1
                                                                                                    AS 4100 Table 6.3.3(1)
\alpha_b = 0.5
\lambda_{\rm nx} = (7000 / 147) \times \sqrt{1} \times \sqrt{(280 / 250)} = 50.4
\alpha_{\rm cx} = 0.806
                                                                                                    AS 4100 Table 6.3.3(3)
\phi N_{\rm cx} = 0.9 \text{ x } 0.806 \text{ x } 1 \text{ x } 14060 \text{ x } 280 \text{ x } 10^{-3} = 2855.8 \text{ kN} > N^* = 2600 \text{ kN} OK
Hence Adopt two 380 PFC in Grade 300 steel
\phi N_{cv} = \phi \alpha_{cv} N_{s} \ge N^{*}
                                                                                                    AS 4100 Cl. 6.3.3
\alpha_{\rm cv} \ge 2600 \times 10^3 / (0.9 \times 1 \times 14060 \times 280) = 0.734
                                                                                                    AS 4100 Cl. 6.2.1
                                                                                                    AS 4100 Table 6.3.3(1)
\lambda_{nv} \le 65 - (65-60) \times (0.734-0.714) / (0.746-0.714) = 61.88
```

$$r_y \ge 59.9 \text{ mm}$$
  
 $I_y = 2 \times [6.48 \times 10^6 + 2 \times 7030 \times (27.5 + S/2)^2] \text{ mm}^4$   
 $I_y = 12.96 \times 10^6 + 14060 \times (27.5 + S/2)^2 \text{ mm}^4$   
 $I_y = r_y^2 A_g \ge 59.9^2 \times 14060 = 50.45 \times 10^6 \text{ mm}^4$   
 $12.96 \times 10^6 + 14060 \times (27.5 + S/2)^2 \ge 50.45 \times 10^6$   
 $(27.5 + S/2)^2 \ge 2666.43$   
 $27.5 + S/2 \ge 51.64$ 

 $S \ge 48.3 \text{ mm}$ 

The two channels need to be separated using lacing by at least 48.3mm.

Hence separate the two 380 PFC by 50 mm using single lacing made up of either flat bars or single angles.

### Lacing Design

In this design example single lacing made up of flat bars will be used.

Lacing angle 
$$\theta = 60^{\circ}$$
 AS 4100 Cl.6.4.2.3  $V^* = \pi (N_s / N_c - 1) N^* / \lambda_n \ge 0.01 N^*$  AS 4100 Cl.6.4.1  $V^* = \pi [1x14060x280/(0.734x1x14060x280) - 1]x2600 / 61.88 = 48 kN > 0.01x 2600 = 26 kN$ 

The lacing shall be proportioned to resist an axial compressive force  $N_{l}^{*}$ 

$$N_{l}^{*} = V^{*}/\sin \theta = 48 / \sin 60 = 55.43 \text{ kN}$$
 Effective length for lacing element  $(L_{\rm e})_{l} = 173.2 \text{ mm}$   $AS 4100 Cl.6.4.2.4$   $Try 40 \times 10 \text{ flat bar in Grade } 300 \text{ steel}$  
$$f_{\rm y} = 320 \text{ MPa}$$
  $AS 4100 Table 2.1$  
$$A_{\rm g} = 40 \times 10 = 400 \text{ mm}^{2}$$
 
$$I_{\rm y} = 40 \times 10^{3}/12 = 3.33 \times 10^{3} \text{ mm}^{4}$$
 
$$r_{\rm y} = \sqrt{(I_{\rm y}/A_{\rm g})} = \sqrt{(3.33 \times 10^{3} / 400)} = 2.9 \text{ mm}$$
 
$$\lambda_{\rm ny} = (173.2 / 2.9) \times \sqrt{(320 / 250)} = 67.57$$
 
$$(\lambda_{\rm ny})_{\rm max} = 140 > \lambda_{\rm ny} = 67.57 \text{ } OK$$
  $AS 4100 \text{ } Cl.6.4.2.5$  
$$\alpha_{\rm b} = 0.5$$
 
$$\alpha_{\rm cy} = 0.697$$
 
$$AS 4100 \text{ } Table \text{ } 6.3.3(1)$$
 
$$\alpha_{\rm cy} = 0.697$$
 
$$AS 4100 \text{ } Table \text{ } 6.3.3(3)$$
 
$$\phi N_{\rm cv} = \phi \alpha_{\rm cv} N_{\rm s} = 0.9 \times 0.697 \times 400 \times 320 \times 10^{-3} = 80.3 \text{ kN} > N_{l}^{*} = 55.43 \text{ kN} \text{ } OK$$

Hence Adopt 40 x 10 flat bar in Grade 300 steel

#### Comment

The design of the end tie plate is omitted from this design example.

## 6.5 REFERENCES

- 1. Standards Australia (1998). AS 4100 Steel Structures.
- 2. Chen W.F. (Ed.), (1995). *Handbook* of *Structural Engineering*.CRC Press, Boca Raton, Fla.
- 3. *SpaceGass*. www.spacegass.com.
- 4. Australian Institute of Steel Construction, (1994) *Design Capacity Tables for Structural Steel* (DCT) 2<sup>nd</sup>edition.

# 7 DESIGN OF FLEXURAL MEMBERS

### 7.1 INTRODUCTION

AS 4100[1] Section 5, "Members subject to bending" covers the design of structural members that support transverse loads or moments, which cause uniaxial bending moment and shear force. It does not deal with members that also carry axial compression or tension, nor with members subjected to biaxial bending. These are covered under Section 8, "Members subject to combined actions." Members subjected to moment plus axial compression force are called beam-columns. This chapter corresponds to AS 4100 Section 5 and deals only with uniaxial bending and shear.

Transverse loads on beams generally cause more deflection and higher stresses than do similar sized axial loads on columns. Most beams have small shear deflections, which are usually ignored, but the bending deflections of a beam are frequently a main design consideration for serviceability. Therefore deflection must be checked as well as strength.

## 7.1.1 Beam terminology

For convenience in structural design calculations, beams are categorized as follows:

- Girders: Beams spaced at the largest interval in a floor or roof system. The primary loads on girders are the reactions of other beams, or possibly some columns.
- Floor Beams or bearers: Beams that support floor joists (Fig. 3.6).
- Joists: The most closely spaced beams in a floor system. Joists support the floor deck (Fig.3.6).
- Roof Beams or rafters: Beams that support purlins (Fig.3.5).
- Purlins: The beams which directly support roof sheeting typically spaced at 0.9-1.5 m (Fig.3.5).
- Battens: Similar to purlins but more closely spaced (typically 0.33 m) to support roof tiles. These are usually timber but may be light, cold-formed steel sections.
- Girts: Exterior wall beams attached to the exterior column. Girts support the exterior wall sheeting and provide lateral resistant to the outside flange of the column (Fig.3.5).
- Spandrel Beams: Support the outside edges of a floor deck.
- Lintels: Beams that span over window and door openings in a wall. Lintels support the wall portion above a window or a door opening.
- Stringers: Beams that are parallel to the traffic direction in a bridge floor system.
- Diaphragms: Span between girders in a bridge floor system and provide some load distribution.

### 7.1.2 Compact, non-compact, and slender-element sections

Steel sections (usually I sections) used for flexural members are classified in AS 4100 as one of three types according to the width-thickness ratios of the component elements: compact,

non-compact, and slender. The effective section modulus  $Z_e$  is calculated in different ways for the three types of section and this affects the section moment capacity  $M_s$  used in design.

- 1. Compact sections are sections that can develop the full cross-section plastic moment  $M_p$  under flexure and sustain that moment through a large hinge rotation without buckling.
- 2. Non-compact sections are sections that either cannot develop the cross-section full plastic strength or cannot sustain a large hinge rotation at  $M_p$ , due to local buckling of the flanges or web. Fig.7.1(a) illustrates local buckling. Non-compact sections can reach the first yield moment  $M_y$  at the extreme fibres.
- 3. Slender-element sections are sections that fail by local buckling of component elements before  $M_y$  is reached.

To assess whether a given section is compact, non-compact or slender, the following guidance is provided. A section is classed as compact if all its component elements have width-thickness ratios less than the plasticity limit for plate element slenderness  $\lambda_{ep}$ , implying that flanges and web will yield fully without buckling. A section is considered to be slender if one or more of its component elements have width-thickness ratios that exceed yield limit *for* plate element slenderness  $\lambda_{ep}$ , implying that buckling will occur before yield. A section is considered non-compact if one or more of its component elements have width-thickness ratios that fall between  $\lambda_{ep}$  and  $\lambda_{ey}$ . The values of the plate element slenderness limits  $\lambda_{ep}$  and  $\lambda_{ey}$  are given in table 5.2 of AS 4100.

## 7.1.3 Lateral torsional buckling

Besides yield and local buckling, another important limit state that must be considered in designing beams is lateral torsional buckling, which is illustrated in Figure 7.1(b). Like buckling of a compression member, lateral torsional buckling can occur suddenly and the movement is at right angles to the direction of the applied load.

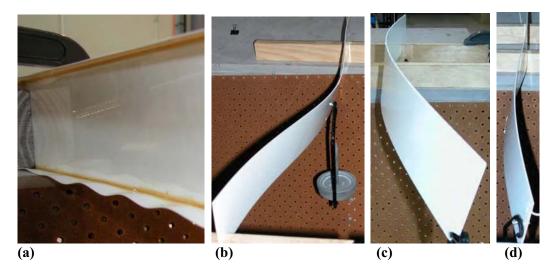


Figure 7.1 (a) Local buckling of Bottom Flange in Compression, (b) Lateral Torsional Buckling of Simply Supported Beam, (c) Lateral Torsional Buckling of Cantilever Unrestrained at End, (d) Lateral Torsional Buckling of Cantilever Restrained at End

Depending on the laterally unsupported length of the beam, lateral torsional buckling may or may not be accompanied by yielding. Thus, lateral torsional buckling can be inelastic or elastic. If the lateral unsupported length is small, the limit state is inelastic lateral torsional buckling. If the lateral unsupported length is large, the limit state is elastic lateral torsional buckling. For compact-section beams which are fully restrained against lateral torsional buckling, the limit state is plastic hinge formation. For non- compact-section beams which are fully restrained against lateral torsional buckling, the limit state is flange or web local buckling.

### 7.2 DESIGN OF FLEXURAL MEMBERS TO AS 4100

### 7.2.1 Design for bending moment

A member bent about the section major principal x-axis which is analyzed by the elastic method (see AS 4100 Clause 4.4) shall satisfy

 $M_{x}^{*} \leq \phi M_{sx}$  and

 $M_{x}^{*} \leq \phi M_{bx}$ 

where

 $M_{\rm x}^*$  = the design bending moment about the x-axis determined in accordance with clause 4.4. of AS 4100.

 $\phi$  = the capacity factor (see table 3.4 of AS 4100)

 $M_{\rm sx} = f_y Z_{\rm ex}$  is the nominal section moment capacity, as specified in clause 5.2 of AS 4100, for bending about the x-axis

 $M_{\rm bx} = \alpha_{\rm m} \ \alpha_{\rm s} \ M_{\rm s} \le M_{\rm s}$  is the nominal member moment capacity, as specified in clause 5.3 or 5.6 of AS 4100, for bending about the x-axis.

A member bent about the section minor principle y-axis, which is analyzed by the elastic method (see AS 4100 Clause 4.4) shall satisfy

 $M^*_{y} \le \phi M_{sy}$ 

where

 $M_y^*$  = the design bending moment about the y-axis determined in accordance with clause 4.4. of AS 4100.

 $M_{\rm sy} = f_y \, Z_{\rm ey}$  is the nominal section moment capacity, as specified in clause 5.2 of AS 4100, for bending about the y-axis.

Note: the design provisions for member analyzed by the plastic method are outlined in in Clause 5.1 of AS 4100.

#### 7.2.1.1 Lateral buckling behaviour of unbraced beams

When an I-beam is loaded in its major principle plane (i.e. major axis bending), one flange goes into compression, which means it is trying to get shorter. This flange will therefore tend to buckle out sideways. Effectively the flange is behaving like a column, and like a column, the longer it is the more easily it will buckle. However the tension flange does not tend to buckle. So the beam must twist as the compression flange buckles (see Figure 7.1(b)).

This buckling action is called flexural-torsional buckling since it involves both flexure (bending) and torsion (twisting). It is also referred to as lateral-torsional buckling LTB since it involves lateral movement, (i.e. movement at right angles to the direction of the load). LTB only happens when the section has a major and a minor principal axis and the applied moment is about the beam's major axis (i.e. LTB does not occur in CHS and in SHS). It is important to note that LTB can occur in steel members without flanges as in the case of a solid rectangular section or a RHS, provided that bending is about the major principal axis.

The greater the length of unrestrained compression flange, the lower the bending moment at which flexural-torsional buckling will occur. To reduce the tendency to buckle, restraints can be provided at intervals along the beam, in the same way that restraints can reduce the buckling length of a compression member. If the restraints are close enough, buckling can be prevented completely so that a compact section beam will be able to develop its full plastic moment capacity. Purlins may serve as buckling restraints on a rafter, although their primary purpose is to support the roof sheeting. Likewise the sheeting itself may serve as a buckling restraint on the top flange of the purlins. Fig.7.2 shows a rafter with its top flange restrained at close intervals by purlins, while fly braces provide restraint to its bottom flange at wider intervals. The top flanges of the purlins themselves are restrained by the fixings into the roof sheeting. The same principles can operate with floors: joists can restrain the top edge of a bearer from buckling, and the floor sheeting can restrain the top edge of the joists.



Figure 7.2

A Rafter with its Top Flange Restrained by Purlins, while Fly Braces Provide Restraint to its Bottom Flange

### 7.2.1.2 Critical flange

The critical flange (CF) as defined in clause 5.5.1 of AS 4100 is the flange which, in the absence of any restraint, would deflect the farther during buckling. For beams supported at each end, the CF is the compression flange – the top flange of a simply supported beam with a central concentrated downward load as shown in Fig.7.1(b), in which a thin strip of plastic has been used rather than an I section as it illustrates lateral torsional buckling more clearly. But for cantilevers the CF can be either the tension flange or the compression flange, depending on what kind of restraint acts at the cantilever tip, as shown in Fig.7.1(c) and (d). The CF will be the tension flange if its unsupported end is not restrained against lateral movement. But if the tension flange or the whole cross section is prevented from moving sideways, the compression flange will be the CF.

#### 7.2.1.3 Restraints at a cross section

Deep, narrow I sections are the most common steel flexural members because they are an economical way to provide a large moment capacity against uniaxial bending. However they

have little torsional stiffness and will tend to buckle as shown in Fig. 7.1(b)-(d) unless restraints are provided at intervals to prevent lateral movement and twisting.

Depending on the restraint that acts at a cross section, restrained steel cross sections are classified in AS4100 as fully, partially or laterally restrained. Examples of these restraint types are shown schematically in AS4100 Figs.5.4.2.1, 5.4.2.2 and 5.4.2.3. Trahair et al (1993), reproduced in Appendix E (check), gives a large number of examples of each restraint type.

### 7.2.1.3.1 Fully restrained cross-section

A cross section of a member may be considered to be fully restrained if either-

- (a) the restraint or support effectively prevents lateral deflection of the critical flange (CF) and effectively or partially prevents twist rotation of the section.
- (b) the restraint or support effectively prevents lateral deflection of some other point in the cross section and effectively prevents twist rotation of the section.

Fully restrained cross sections are shown schematically in AS4100 Fig.5.4.2.1. Typical practical examples include:

- (i) A rafter with its top flange in compression, laterally and partially twist restrained by a purlin (Figs.7.2, 7.3).
- (ii) A rafter with its bottom flange in compression, laterally and fully twist restrained by a lapped purlin and fly braces (Figs. 7.2, 7.4).
- (iii) A rafter with its bottom flange in compression, laterally and fully twist restrained by a purlin and a pair of web stiffeners (Fig. 7.5).
- (iv) A beam where its critical flange is attached to a column (Fig. 7.6).

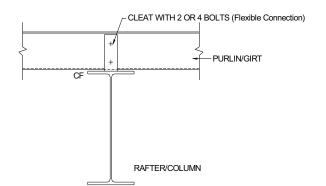


Figure 7.3 Critical Flange Restraint, Partial Twist Restraint

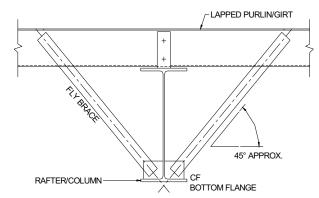


Figure 7.4 Non-Critical Flange Restraint
Full Twist Restraint

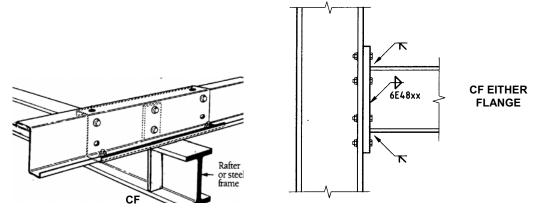


Figure 7.5 Non-Critical Flange Restraint Full Twist Restraint

Figure 7.5 Critical Flange Restraint Full Twist Restraint

## 7.2.1.3.2 Partially restrained cross-section

A cross section of a member may be considered to be partially restrained if the restraint or support effectively prevents lateral deflection of some other point in the cross section other than the critical flange and partially prevents twist rotation of the section. Generally a cross will have partial twist restraint if there was only one flexible element (brace, connection, web) between the lateral restraint and the critical flange.

Some partially restrained cross sections are shown schematically in AS 4100 Fig.5.4.2.2. Typical practical examples include

- (i) A beam with its top flange in compression (i.e. critical), the flexible web between the lateral restraint (Flexible end plate / column) and the CF will provide partial restraint against twist rotation (Fig. 7.6.).
- (ii) A main beam with its bottom flange in compression (i.e. critical), the flexible web between the lateral restraint (secondary beam) and the CF will provide partial restraint against twist rotation (Fig.7.7.).
- (iii) A secondary beam with its top flange in compression (i.e. critical), the flexible web between the lateral restraint (main beam) and the CF will provide partial restraint against twist rotation (Fig. 7.7.).

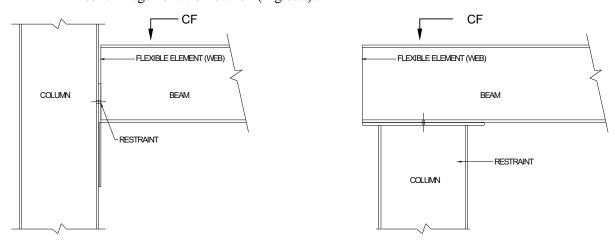


Figure 7.6 Non-Critical Flange Restraint Partial Twist Restraint

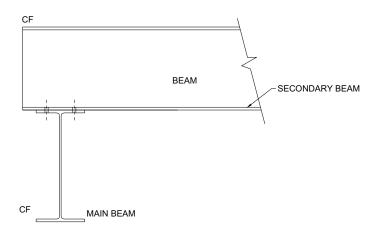


Figure 7.7 Non-Critical Flange Restraint
Partial Twist Restraint

### 7.2.1.3.2 Laterally restrained cross-section

A cross section of a member whose ends are fully or partially restrained may be considered to be laterally restrained when the restraint or support effectively prevents lateral deflection of the critical flange but is ineffective in preventing twist rotation of the section.

Laterally restrained cross sections are shown schematically in AS4100 Fig.5.4.2.4. A typical practical example, shown in Fig.7.8, is a rafter or column restrained at its critical flange by a purlin or girt. If the flange remote from the purlin or girt is critical then no restraint can be assumed unless the purlin girt is lapped (i.e stiff) and the number of bolts in the cleat connecting the lapped purlin/girt to the rafter/column is 2 or more, and the bolts were properly tensioned (i.e moment connection), in such case the section is classified as partially restrained

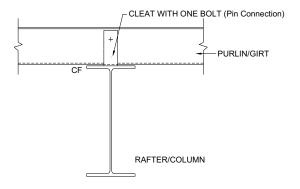


Figure 7.8 Critical Flange Restraint No twist restraint

### 7.2.1.4 Segments, Sub-Segments and Effective length

The points of restraint effectively divide a beam into segments or sub-segments which can be treated as separate beams for design purposes. This is useful because different segments or sub-segments are subjected to different bending moments and may have different member moment capacities. Each segment or sub-segment is classified according to its end restraint conditions, which in turn affect its effective buckling length and hence its moment capacity.

Thus a segment is classified as FF if both ends are fully restrained against both lateral movement and twisting. A section is fully restrained if the critical flange is laterally restrained. If one end of a segment is fully restrained and the other partially restrained, it is classified as FP, and if both ends are partially restrained it is PP. A sub-segment has one end only laterally restrained, so sub-segments may be FL, PL or LL.

The effective length  $(L_e)$  of a segment or sub-segment shall be determined as follows:

$$L_e = k_t k_l k_r L$$
 AS4100 Cl.5.6.3(1),(2),(3)

where

 $k_t$  = twist restraint factor given in table 5.6.3(1) of AS 4100. This factor accounts for the reduction in the member moment capacity resulting from thin web distortion, by increasing the effective length of a segment or sub-segment with a one or both ends partially restrained.

 $k_l$  = load height factor given in table 5.6.3(2) of AS 4100. For example a monorail or a crane runway beam or any other beam which has a gravity load on its top flange without any lateral restraint is more likely to buckle than the same beam with the same load applied to its bottom flange. This is because any twisting will cause the load on the top flange to move sideways so as to increase the twisting effect. In most case  $k_l$  = 1, but if  $k_l$  > 1, we take this effect into account by increasing the effective length.

 $k_r$  = lateral rotation restraint factor given in table 5.6.3(3) of AS 4100. This is equal to 1 unless one or both ends of the segment are restrained from rotating about the vertical or y-axis, assuming the load is in the y- direction, i.e. moment about z- axis – see figure 7.2. For example even the I section column at the left would have little torsional stiffness and could not provide much lateral rotational restraint. If it were a box section it would provide some restraint and  $k_r$  would be <1. But for most case  $k_r$  = 1.

The length L in the effective length equation shall be taken as either

- (a) the segment length, for segments without intermediate restraints, or for segments unrestrained at one end, with or without intermediate lateral restraints; or
- (b) the sub-segment length, for segments formed by intermediate lateral restraints, in a segment, which is fully or partially restrained at both ends.

## 7.2.1.5 Member moment capacity of a segment

The nominal member moment capacity of a beam, a segment of a beam, or a sub-segment of a beam is given by the following equation:

$$M_{\rm b} = \alpha_{\rm m} \alpha_{\rm s} M_{\rm s}$$
 AS4100 Cl.5.6.1.1(1)

The design capacity is given by:

$$\phi M_b = \phi \alpha_m \alpha_s M_s$$
 AS4100 Cl.5.6.1.1(1)

The greater the effective length  $L_e$  of a segment, the more easily it will buckle and the smaller is its moment capacity. This effect is taken into account by the slenderness reduction factor  $\alpha_s$ , which gets smaller as  $L_e$  gets bigger. This factor can be calculated using Equations 5.6.1.1(2) and 5.6.1.1(3) of AS 4100[1]. However it is much easier to use tables or charts prepared by the Australian Institute of Steel construction [2] These give  $\phi M_b$  for the case where the moment modification factor  $\alpha_m = 1$  (i.e.  $\phi M_b = \phi \alpha_s M_s$ ). Using these tables will always give a safe design because  $\alpha_m \ge 1$  for doubly symmetrical sections. However, they will often give a very conservative design if you do not take  $\alpha_m$  into account.

### Moment shape factor α<sub>m</sub>

As explained in 7.2.1.1, the flange of a beam in compression will tend to buckle like a column, and like a column, it may need to be restrained at intervals to prevent buckling. If a beam or segment is subjected to a uniform bending moment along its whole length, then the compressive stress in the compression flange will be uniform along its length, as it is in a centrically loaded column. But if the bending moment changes along the length of the segment (as it usually does), then some of the compression flange will be less prone to buckle. And if the bending moment reverses over the length of the segment, only part of the length of each flange will be in compression. This means there is a shorter length over which any one flange tends to buckle and the tendency to buckle is less. AS 4100 allows for this effect by introducing the moment modification factor  $\alpha_m$ , which may offset the slenderness reduction factor.

In Fig.7.9, the top flange is the critical flange over the whole length of the beam. If there are no lateral restraints to the top flange, it will be able to buckle over its full length. But if there are lateral restraints to the top flange at the two load points (typically provided by purlins or crossing beams), they effectively divide the beam into 3 sub-segments, 1-2, 2-3 and 3-4, each of which must be assessed independently for moment capacity, although in this case 3-4 is simply a mirror image of 1-2, so only 2 sub-segments need be assessed.

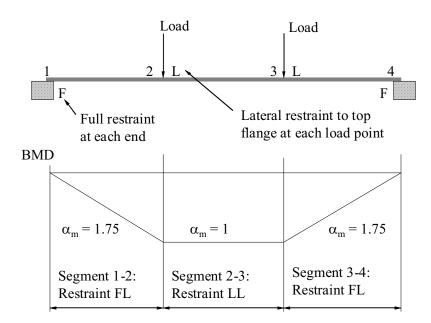


Figure 7.9 Beam under 4 point loading, with lateral restraints at load points, showing dependence of  $\alpha_m$  on moment shape

There is a uniform bending moment on sub-segment 2-3, so the maximum compressive stress in the top flange will act over its full length and it will have maximum tendency to buckle over its full length. Thus from AS 4100 Table 5.6.1,  $3^{rd}$  to bottom row,  $\alpha_m = 1$ , i.e. no increase in moment capacity. Sub-segments 1-2 and 3-4, in contrast, each have the same maximum bending moment as sub-segment 2-3 at one end, but it decreases to zero at the other end. So they are less likely to buckle and from AS 4100 Table 5.6.1,  $2^{nd}$  row from bottom,  $\alpha_m = 1.75$ , i.e. 75% increase in moment capacity before buckling will occur – as long as yielding does not occur first.

Now if the loading is changed so one load acts upwards, as shown in Fig.7.10 below, the bottom flange is now critical between sections 1 and 3, and if only lateral restraint is provided to the top flange at sections 2 and 3, without rotational restraint, this will now provide no effective restraint at section 2, so it can buckle over the full length from section 1 to section 3. The beam is now divided into only 2 sub-segments.  $\alpha_m$  for the longer sub-segment on the left is found from AS4100 Table 5.6.1,  $4^{th}$  row down, to be 1.2.

However if restraint were provided to the bottom flange at section 2, we would again have 3 sub-segments and  $\alpha_m$  for the central sub-segment is found from AS4100 Table 5.6.1, top row, to be 2.5.

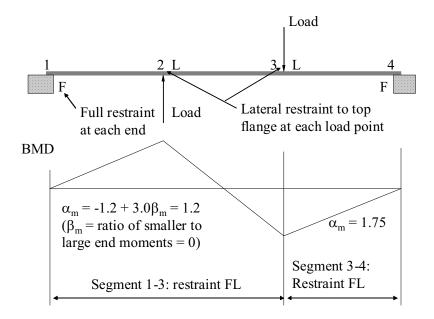
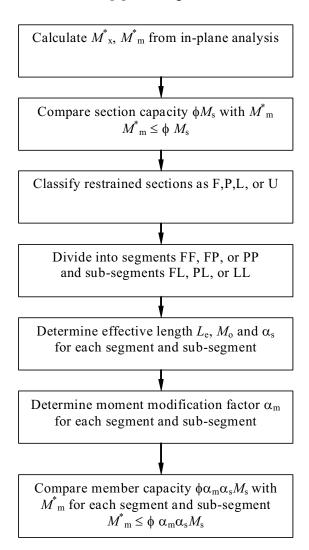


Figure 7.10 Same beam as in Figure 7.9, but one load reversed, showing change in subsegments and  $\alpha_m$ 

### 7.2.1.6 Lateral Torsional Buckling Design Methodology

The AS 4100[1] methodology for designing against lateral buckling is summarised in the Design of Unbraced Beams [3] in the figure below.



## 7.2.2 Design for shear force

A web subject to a design shear force  $V^*$  shall satisfy –

$$V^* \leq \phi V_{\rm v}$$

where

 $\phi$  = the capacity factor (see Table 3.4 of AS 4100)

 $V_v$  = the nominal shear capacity of a web determined from either clause 5.11.2 or clause 5.11.3 of AS 4100.

Note: After designing for bending and shear the designer must consider the interaction between them in accordance with clause 5.12 of AS 4100.

### 7.3 WORKED EXAMPLES

## **Example 7.3.1 Moment Capacity of Steel Beam Supporting Concrete Slab**

A simply supported non-composite beam is to span 10 m and support a concrete slab. Friction between slab and top flange will provide continuous lateral support to the whole of the top (compression) flange. The steel beam is to be designed for a factored design load 1.2G + 1.5Q = 40 kN/m including its own self-weight. Select a suitable UB section in Grade 300 steel.

- (a) from tables of section modulus.
- (b) from AISC moment capacity tables.

Assume failure will occur by bending, not by shear.

#### Solution

$$M^* = wL^2/8 = 40 \times 10^2/8 = 500 \text{ kNm} \le \phi M_{\text{bx}}$$
 AS 4100 Cl.5.1

Given that the compression flange is fully restrained, there is no possibility of lateral buckling, so

The effective length  $L_e$  over which the compression flange can buckle = 0. Thus bending failure can only occur by yielding so  $M_b = M_s$ .

$$\phi = 0.9$$

$$\phi M_b = 0.9 \times M_b = 0.9 \times M_s$$

$$M_s = f_y Z_e$$

$$M^* = 500 \le 0.9 M_s = 0.9 f_y Z_e = 0.9 \times 300 \times Z_e$$

$$Z_c \ge 500 \times 10^6 / 270 = 1852 \times 10^3 \text{ mm}^3$$
AS 4100 Table 3.4

AS 4100 Table 3.4

AS 4100 Table 3.4

- (a) Assuming a compact section,  $Z_e$  = lesser of S (plastic section modulus) and 1.5 x Z (elastic section modulus). Looking at dimensions and properties of UB sections, this will always mean S for major axis bending. So, from tables of section modulus [1], select 530UB82 with  $S = 2060 \text{ mm}^3$  (next size down, i.e. 460UB82.1, has  $S_x = 1830 \text{ x} \cdot 10^3 \text{ mm}^3$  so not quite enough).
- (b) From AISC Table 5-49[2], a 460UB82.1 has a moment capacity just below 500 kNm, so select next size up, i.e. 530UB82 (which weighs the same so costs very nearly the same the only disadvantage is it takes up a bit more headroom), i.e. same result as part (a).

### Example 7.3.2 Moment Capacity of Simply Supported Rafter Under Uplift Load

The rafter shown below is of Grade 300 steel, 250UB25.7 section. It spans 9m and is simply supported at each end by connections to tilt-up concrete panels. Tilt-up concrete panels in the end bays provide stability in the direction parallel to the rafter span. The rafter is bolted to a web side plate, which is welded to another plate embedded in the concrete panel. Two purlins cross the rafter at 3m spacing. These are attached to the top flange by two bolts each. Calculate the maximum moment capacity  $\phi M_b$  of the rafter when it is subjected to two equal upward point loads at the two purlins.

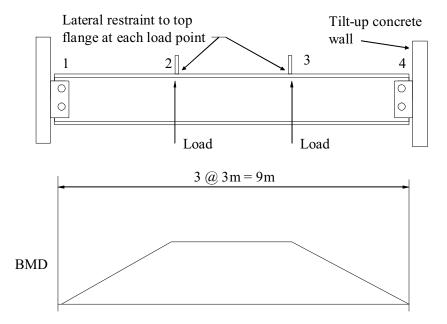


Figure 7.11 Simply Supported Beam with Upward Load from Purlins

#### Solution

The first step is to classify the restrained sections into F, P, L and U

At the purlins the section is classified as unrestrained (U) since the purlins are connected at the tension flange level and they can't be relied upon to provide partial twist restraint to the cross section because standard oversized 18 mm holes are generally used in purlins with only M12 bolts. For the restraint at the rafter-wall connection, the section is classified as fully restrained (F).

Next divide into segments FF, FP, or PP and sub-segments FL, PL, or LL We have only one segment for bending in this case with a restraint arrangement FF

Segment length L = 9m

$$L_e = k_t k_l k_r L$$
 AS4100 Cl.5.6.3(1),(2),(3)  $k_t = 1$   $k_l = 1$  (conservative)  $k_t = 1 \times 1 \times 1 \times 9 = 9 \text{ m}$ 

Now check moment capacity  $\phi M_b$  for  $\alpha_m = 1$  from AISC Tables [1]. For  $L_e = 9$ m,  $\phi M_b = 15.5$  kNm. This would be the correct value if BM was uniform over the whole segment. But it is not. It tapers down to zero at each end, which reduces the tendency for the critical flange to buckle. So  $\alpha_m > 1$ .

$$\alpha_{\rm m} = 1.0 + 0.35(1-2a/L)^2 = 1 + 0.35(2/3)^2 = 1.155.$$
 AS 4100 Table 5.6.1

Alternatively, from Section 5.6.1.1(a)(iii) of AS 4100,

$$\alpha_{m} = \frac{1.7M *_{m}}{\sqrt{\left[\left(M^{*}_{2}\right)^{2} + \left(M^{*}_{3}\right)^{2} + \left(M^{*}_{4}\right)^{2}\right]}} \le 2.5 = \frac{1.7}{\sqrt{\left(\frac{3}{4}\right)^{2} + 1^{2} + \left(\frac{3}{4}\right)^{2}}} = 1.603$$

In this case the first estimate is probably more accurate as it is for the exact bending moment shape we have. The second relies on three values at the quarter points. So we will use 1.155. This means that the actual moment capacity will be 15.5% higher than the given by the AISC Tables [2] for uniform moment over the entire segment.

$$\phi M_b = \alpha_m \mathbf{x} \ \alpha_s \phi M_s = 1.155 \ \mathbf{x} \ 15.5$$
  
$$\phi M_b = 17.9 \ kNm$$

## **Example 7.3.3 Moment Capacity of Simply Supported Rafter under Downward Load**

For the same rafter in Example 7.3.2, calculate the maximum moment capacity  $\phi M_b$  when the load is downward.

#### Solution

For the case of downward loading the critical flange (CF) is the top flange, the purlins restrain the (CF) from moving sidesway but they don't offer any restraint against twist rotation, therefore the rafter section at each purlin location is classified as laterally restrained (L) For the restraint at rafter wall connection, the section is classified as fully restrained (F).

Next divide into segments FF, FP, or PP and sub-segments FL, PL, or LL

### Sub-Segment (A-B)& (D-E)

Sub-Segment length L = 3 mRestraint arrangement FL

$$L_e = k_t k_l k_r L$$
 AS4100 Cl. 5.6.3(1),(2),(3)  $k_t = 1$   $k_l = 1$   $k_r = 1$   $L_e = 1 \times 1 \times 1 \times 3 = 3 \text{ m}$ 

Now check moment capacity  $\phi M_b$  for  $\alpha_m = 1$ , from AISC Tables [2]. For  $L_e = 3$ m,  $\phi M_b = 52$  kNm. This would be the correct value if the moment was uniform over the whole segment. But it is not. It tapers down to zero at each end, which reduces the tendency for the critical flange to buckle. So  $\alpha_m > 1$ .

From Table 5.6.1 of AS 4100,  $2^{\text{nd}}$  case from bottom,  $\alpha_m = 1.75$ .

Alternatively, from Section 5.6.1.1(a)(iii),

$$\alpha_{m} = \frac{1.7M *_{m}}{\sqrt{\left[\left(M *_{2}\right)^{2} + \left(M *_{3}\right)^{2} + \left(M *_{4}\right)^{2}\right]}} \leq 2.5 = \frac{1.7}{\sqrt{\left(\frac{1}{4}\right)^{2} + \left(\frac{1}{2}\right)^{2} + \left(\frac{3}{4}\right)^{2}}} = 1.817$$

Again the first estimate is probably more accurate as it is for the exact bending moment shape we have. So we will use 1.75. This means that the actual moment capacity before buckling will be 75% higher than the given by the AISC chart for uniform moment over the entire subsegment.i.e.  $\phi M_b = 1.75 \times 52 = 91$  kNm. But we must also check that the plastic moment capacity  $\phi M_s$  is not exceeded. This is the value of  $\phi M_b$  at  $L_e = 0$ , where no buckling can take place. From the chart, this value is about 92 kNm, so buckling just governs the design in this case and  $\phi M_b = 91$  kNm.

### Sub-Segment (B-C)

Sub-Segment length L = 3mRestraint arrangement LL

$$L_e = k_t k_l k_r L$$
 AS4100 Cl. 5.6.3(1),(2),(3)  
 $k_t = 1$   
 $k_l = 1$   
 $k_r = 1$   
 $L_e = 1 \times 1 \times 1 \times 3 = 3 \text{ m}$ 

 $\phi M_b = 52$  kNm for  $\alpha_m = 1$ . This is the correct value for sub-segment B-C since the moment is uniform over the whole sub-segment.

## Example 7.3.4 Checking a Rigidly Connected Rafter under Uplift

The rafter shown below is of Grade 300 steel, 250UB25.7 section. It spans 9m and is rigidly connected to the columns at each end. The rafter is loaded through purlins by 4 equal 10 kN point loads at 1.8m centres. Check the moment capacity of the rafter (a) with Fly Bracing at sections 3 and 4, and (b) without fly bracing.

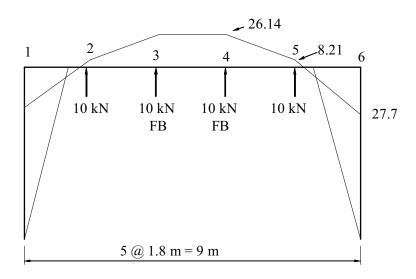


Figure 7.12 Rafter Spanning 9 m with 4 Purlins at 1.8 m Centres

#### Solution

## (a) With Fly Bracing at sections 3 and 4

The structure has been analysed using Spacegass [4]. The bending moment is shown in Fig.7.12. Nodes have been inserted at each purlin for convenience in the analysis so Spacegass gives us a value for bending moment at each purlin. We can identify the sections at each end and each purlin by a number, from 1 at the left hand end to 6 at the right (alternatively we could have just used the member numbers assigned by Spacegass [4]). If we put fly braces at sections 3 and 4, these sections will be fully restrained and sections 2 and 5 will have no restraint unless

the purlins have a wider cleat with four high strength bolts properly tensioned, and a web stiffener on one or both sides to prevent distortion.

We thus have 3 segments: 1-3, 3-4 and 4-6. Since 4-6 is a mirror image of 1-3, it will have the same moment capacity, so we need only check 1-3 and 3-4.

#### Segment (1-3)

Effective length  $L_e$ 

(i) The end restraints are FF so  $k_t = 1$ 

- *AS* 4100 *Table* 5.6.3(1)
- (ii) Although the load is applied through the top flange it is not a gravity load. Therefore

$$k_l = 1$$
 AS 4100 Table 5.6.3(2)

(iii) The end supports are assumed to provide no restraint against rotation about a vertical axis, i.e. the compression flange is acting a bit like a pin-ended column.

$$k_r = 1$$
 AS 4100 Table 5.6.3(3)

$$L_e = k_L k_L k_T L = 1 \times 1 \times 1 \times 3.6 = 3.6 \text{m}$$
 AS4100 Cl. 5.6.3(1),(2),(3)

Now check moment capacity  $\phi M_b$  for  $\alpha_m = 1$  from AISC Table 5-49 [2]. For  $L_e = 3.6$ m,  $\phi M_b = 43$  kNm. This would be the correct value if the moment was uniform over the whole segment. But it is not. It changes sign near the middle, which reduces the tendency for the critical flange to buckle. So  $\alpha_m > 1$ .

From Table 5.6.1, of AS 4100, top case, the ratio of end moments  $\beta_m = 26.14/27.7 = 1$ approx.

$$\therefore \alpha_{\rm m} = 2.5$$

Alternatively, from Section 5.6.1.1(a)(iii),

$$\alpha_m = \frac{1.7M *_m}{\sqrt{\left[ \left( M *_2 \right)^2 + \left( M *_3 \right)^2 + \left( M *_4 \right)^2 \right]}} \le 2.5 = \frac{1.7x27.7}{\sqrt{(9.745)^2 + (8.21)^2 + (17.17)^2}} = 2.20$$

This time the first estimate, although acceptable, is probably less accurate as it is for a straight line bending moment diagram, which is not the exact bending moment shape we have. So we will use 2.2. This means that the actual moment capacity before buckling will be 2.2 times higher than the given by the AISC chart for uniform moment over the entire segment. i.e.  $\phi M_b = 2.2 \times 43 = 94.7 \text{ kNm}$ . However, the plastic moment capacity  $\phi M_s$  is about 92 kNm, so yielding just governs the design in this case.

$$\therefore \phi M_{\rm b} = \phi M_{\rm s} = 92 \text{ kNm}$$

### Segment (3-4)

$$L_e = k_t k_l k_r L = 1 \times 1 \times 1 \times 1.8 = 1.8 \text{m}$$
 AS4100 Cl. 5.6.3(1),(2),(3)

 $\phi M_b = 70$  kNm approx for  $\alpha_m = 1$ . This is the correct value for segment 3-4, since  $M^*$  is uniform over the whole segment.

Note that in this case the shortest segment has the smaller moment capacity due to the effect of the moment modification factor.

#### (b) Without fly bracing

We have only one segment for bending in this case with a restraint arrangement FF Segment length L = 9m

$$L_e = k_t k_l k_r L$$
 AS4100 Cl.5.6.3(1),(2),(3)  
 $L_e = 1 \times 1 \times 1 \times 9 = 9 \text{ m}$ 

The bending moment diagram looks a bit like the 4<sup>th</sup> from the bottom of Table 5.6.1 of AS4100, where the height of the BMD =  $wL^2/8 = 27.7 + 26.14 = 53.85$  in our case, so  $wL^2/12 = 2/3 \times 53.84 = 35.9$  kNm.  $\beta_m wL^2/12 = 27.7$  so  $\beta_m = 27.7/35.9 = 0.772$ .  $\alpha_m = -2.38 + 4.8 \times 0.772 = 1.32$ 

 $\therefore \phi M_b = 1.32 \text{ x}15.6 = 20.6 \text{ kNm}$  which is not enough.

So the best design would probably be with a lighter section and two fly braces.

## **Example 7.3.5 Designing a Rigidly Connected Rafter under Uplift**

Select a suitable beam section for the loading in Example 7.3.4, with two fly braces at sections 3 and 4.

#### Solution

From Example 7.3.4, the bending moment is nearly equal at the ends and at midspan. Segment 3-4 is the critical one, and has  $\alpha_{\rm m}=1$ . So select a section with  $\phi M_{\rm b} \geq 26.14$  kNm at  $L_e=1.8$  m. From AISC Table [2], a 180UB16.1 would just do it. Next, put this section into the Spacegass model and check that the BMD is still nearly the same. Assuming the columns can also be reduced to 180UB16.1, the BMD at midspan is now 25.9 kNm, i.e. almost the same.

Check also that Segment 1-3 is *OK*: As before,  $L_e = 3.6$ m,  $\alpha_m = 2.2$  and

 $\phi M_b = 15 \text{ x } 2.2 = 33 > 27.7 \text{ kNm}$  which is the bending moment at eaves.

Therefore, the design is ok – for bending moment at least. However, we still need to check it for shear.

Sections 1 and 6 carry both the greatest moment and the greatest shear, so we will check section 1. The design shear force  $V^* = 20 \text{ kN}$ .

## Check section 1 for capacity to resist shear

From Clause 5.11 of AS 4100, the web is required to satisfy

$$V^* \leq \phi V_{v}$$

Normally the shear stress distribution in the web can be assumed approximately uniform, so Clause 5.11.2 of AS 4100, applies.

 $d_{\rm p}/t_{\rm w} = 159/4.5 = 35.3 < 82/\sqrt{(f_{\rm y}/250)} = 74.8$ , so no reduction in effective web area for shear, so  $V_{\rm u} = V_{\rm w}$ 

From Clause 5.11.4 of AS 4100

$$V_{\rm w} = 0.6 f_{\rm vw} A_{\rm w} = 0.6 \times 320 \times 4.5 \times 173 = 149.5 \text{ kN}$$

*Note*: for hot rolled sections  $A_w = d x t_w$ , for welded sections  $A_w = d_1 x t_w$ . The depth of the web panel  $d_p$  for calculating plate element slenderness is taken as  $d_1$  in both hot rolled sections and welded sections.

Thus  $V^* = 20 \text{ kN} < \phi V_w = 0.9 \text{ x } 149.5 = 134.6 \text{ kN}$ , so the section is adequate to withstand the shear on its own. But it is still necessary to check if it can take combined shear and bending.

We will again check section 1, where shear force  $V^*$  is maximum, this time for its capacity to resist combined moment and shear. We have a choice of two methods:

The proportioning method (Clause 5.12.2 of AS 4100) in which the moment is assumed to be taken by the flanges alone and the shear by the web alone, and the "Shear and bending interaction method" of Clause 5.12.3, in which the moment is assumed to be resisted by the whole section. Either is acceptable and quite straightforward if you read the code carefully.

We will use the "Shear and bending interaction method" in this example.

First we check if the design moment on the section we are checking is < 75% of the section moment capacity. If so, we can assume the shear capacity is equal to the full shear capacity of the web. If not, we must reduce the shear capacity to allow for combined stresses.

```
M^* \le 0.75 \, \phi M_s?

27.7 \le 0.75 \, \mathbf{x} \, 39.8 ? Yes.

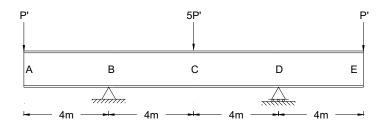
\therefore V_{\text{vm}} = V_{\text{v}} = 149.5 \, \text{kN}

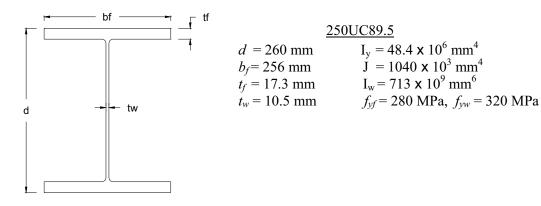
V^* = 20 \, \text{kN} \le \phi V_{\text{vm}} = 0.9 \, \mathbf{x} \, 149.5 = 134.6 \, \text{kN} OK
```

So the design is adequate. But if the design shear force on section 1 had been, say, 150 kN, it would have been necessary to increase the section to take combined shear and bending even though it was adequate for moment alone (this happens mainly with short, heavily loaded beams, not with long, lightly loaded ones as in this example).

## Example 7.3.6 Checking a Simply Supported Beam with Overhang

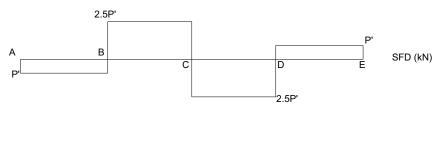
The 250UC89.5 beam in Grade 300 steel shown below is continuous over the supports at B and D and is free at A and E. The beam section is restrained against lateral deflection at B and D, fully restrained against twist rotation at B and D, and is unrestrained at A and E. A downward concentrated load of  $5P^*$  acts at C and a downward concentrated load of  $P^*$  acts at A and E. These loads act at the top flange, and are free to deflect laterally with the beam. Determine the maximum design value of  $P^*$ .

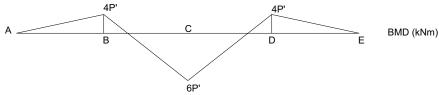




#### Solution

The bending moment diagram is determined using statics and is shown below.





## Segment (A-B) & Segment (D-E)

Segment length L = 4m

Restraint arrangement FU

Maximum moment in the segment  $M_{\rm m}^* = 4P^*$ 

$$L_e = k_t k_l k_r L$$
 AS4100 Cl. 5.6.3(1),(2),(3)

 $k_t = 1$ 

 $k_l = 2$ 

 $k_r = 1$ 

 $L_e = 1x2x1x4 = 8 \text{ m}$ 

 $I_v = 48.4 \times 10^6 \text{ mm}^4$ 

 $J = 1040 \times 10^3 \, \text{mm}^4$ 

 $I_{\rm w} = 713 \text{x} 10^9 \text{mm}^6$ 

 $Z_{ex} = 1230 \times 10^3 \text{ mm}^3$ 

 $f_y = 280 \text{ MPa}$  (the lesser of  $f_{yf}$  and  $f_{yw}$ )

$$\alpha_{\rm m} = 1.25$$

Hence using a spreadsheet program,

 $M_0 = 396.3 \text{ kNm}$ 

 $M_{\rm sx} = 344.4 \; \rm kNm$ 

 $\alpha_s = 0.64128$ 

$$\phi M_{\rm bx} = \phi \ \alpha_m \alpha_s \ M_{\rm sx} \le \phi M_{\rm sx}$$

AS4100 Cl.5.6.1.1(1)

AS4100 Table 5.6.2

 $\phi M_{\rm bx} = 0.9 \text{ x } 1.25 \text{ x } 0.64128 \text{ x } 344.4 = 248.46 \text{ kNm} < \phi M_{\rm sx} = 309.96 \text{ kNm}$ 

 $M_{\rm m}^* = 4P^* \le \phi M_{\rm bx} = 248.46 \text{ kNm}$ 

 $P^* \le 62 \text{ kN}$ 

## Segment (B-D)

Segment length L = 8m

Restraint arrangement FF

Maximum moment in the segment  $M_{\rm m}^* = 6P_{\rm m}^*$ 

$$L_e = k_t k_l k_r L$$
 AS4100Cl. 5.6.3(1),(2),(3)

 $k_t = 1$ 

 $k_1 = 1.4$ 

 $k_r = 1$ 

 $L_e = 1 \text{ x} 1.4 \text{ x} 1 \text{ x} 8 = 11.2 \text{ m}$ 

 $\beta_m$  x  $5P^*$ x 8 / 8 =  $4P^*$   $\Rightarrow$   $\beta_m$  = 0.8

 $\alpha_m = 1.35 + 0.36 \times 0.8 = 1.64$ 

Hence using a spread sheet program,

 $M_0 = 268.2 \text{ kNm}$ 

 $M_{\rm sx} = 344.4 \; \rm kNm$ 

 $\alpha_{\rm s} = 0.5232$ 

 $\phi M_{\text{bx}} = 0.9 \text{ x} 1.64 \text{x} \ 0.5232 \text{x} \ 344.4 = 265.96 \text{ kNm} < \phi M_{\text{sx}} = 309.96 \text{ kNm}$ 

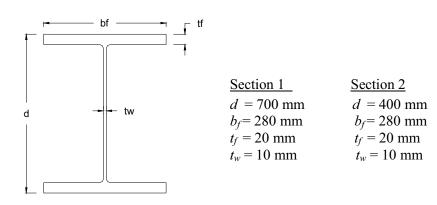
 $M_{\rm m}^* = 6 \text{ P}^* \le \phi M_{\rm bx} = 265.96 \text{ kNm}$ 

 $P^* \le 44.33 \text{ kN} < 62 \text{ kN for segment (A-B)}$ 

 $\therefore$  Segment (B-D) is the critical segment and the maximum design value of  $P^*$  under bending alone is  $P^* = 44.33 \text{ kN}$ .

## Example 7.3.7 Checking a Tapered Web Beam

A tapered web beam welded from Grade 300 steel whose end cross-sections are shown below is loaded by end moments of  $M_1^* = 800$  kNm and  $M_2^* = 500$  kNm which cause single curvature bending. The beam is 5m long with both ends fully restrained against lateral translation and twist rotation. Check if the beam is safe to carry the load.



## Solution

At Section 1,  $t_f = 20$  mm,  $t_v = 300$  MPa

AS4100 Table 2.1

AS4100 Table 5.6.1

Flange slenderness:

 $\lambda_{ef1} = ((b_f t_w) / 2t_f) \sqrt{(f_v / 250)} = [(280-10)/(2x20)] \times \sqrt{(300/250)} = 7.4$ 

From table 5.2 of AS 4100 for a flat unstiffened element (flange) under uniform compression, which is heavily welded (HW) longitudinally.

$$\lambda_{\rm ep} = 8 > \lambda_{\rm efl} = 7.4$$
 :: Flange is compact.

Web slenderness:

$$\lambda_{\text{ew}1} = (d_1/t_w)\sqrt{(f_v/250)} = ((700-40)/10) \times \sqrt{(300/250)} = 72.3$$

From table 5.2 of AS4100 for flat stiffened element (web) with compression at one edge, tension at the other.

$$\lambda_{\rm ep} = 82 > \lambda_{ewI} = 72$$
 : Web is compact.

$$S_{x1} = 2 \times 280 \times 20 \times (700-20) / 2 + 10 \times (700-2 \times 20)^2 / 4 = 4897 \times 10^3 \text{ mm}^3$$
  
 $M_{s1} = S_{x1} f_y = 4897 \times 10^3 \times 300 \times 10^{-6} = 1469 \text{ kNm}$   
 $M_1^* / M_{s1} = 800 / 1469 = 0.54$ 

At Section 2,  $t_f = 20 \text{ mm}$ ,  $f_v = 300 \text{ MPa}$ 

AS4100 Table 2.1

From the calculations done for section 1 section 2 is compact for bending about the x-axis.

$$S_{x2} = 2 \times 280 \times 20 \times (400-20) / 2 + 10 \times (400 - 2 \times 20)^2 / 4 = 2452 \times 10^3 \text{ mm}^3$$
  
 $M_{s2} = 735.6 \text{ kNm}$ 

 ${M_2}^*$  /  ${M_{\rm s2}} = 500$  / 735.6 = 0.68 >  ${M_1}^*$  /  ${M_{\rm s1}} = 0.54$  and so section (2) is the critical section in the segment.

#### For Section (2)

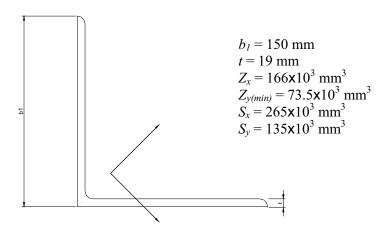
$$\begin{split} &I_y = 2 \times 20 \times 280^3 / 12 + (400 - 2 \times 20) \times 10^3 / 12 = 73.2 \times 10^6 \text{ mm}^4 \\ &I_w = I_y \, d_f^2 / 4 = 73.2 \times 10^6 \times (400 - 20)^2 / 4 = 2642.52 \times 10^9 \text{ mm}^6 \\ &J \approx &\Sigma \, (\text{bt}^3 / 3) = 2 \times 280 \times 20^3 / 3 + (400 - 2 \times 20) \times 10^3 / 3 = 1613.33 \times 10^3 \text{ mm}^4 \end{split}$$

Hence using a spreadsheet program,

```
M_0 = 1397 \text{ kNm}
r_{\rm r} = 0.5 for tapered beam
                                                                                              AS4100 Cl. 5.6.1.1 (b) (ii)
r_s = [(2x 280x20)/(2x280x 20)] x [0.6 + 0.4x 400/400] = 1
                                                                                              AS4100 Cl. 5.6.1.1 (b) (ii)
\alpha_{st} = 1 - [1.2 \text{ r}_r (1-r_s)] \Rightarrow \alpha_{st} = 1 - [1.2 \text{ x } 0.5 (1-1) = 1]
                                                                                              AS4100 Cl. 5.6.1.1 (b) (ii)
M_{\text{oa}} = \alpha_{st} M_o = 1 \times 1397 = 1397 \text{ kNm}
                                                                                              AS4100 Cl. 5.6.1.1 (b) (ii)
\alpha_s = 0.6 \text{ x} ([\sqrt{(735.6 / 1397)^2 + 3})] - (735.6 / 1397)) = 0.77
                                                                                              AS4100 Eq.5.6.1.1.(2)
\beta_{\rm m} = -500 / 800 = -0.625
                                                                                              AS4100 Table 5.6.1.
\alpha_{\rm m} = 1.75 + 1.05 \text{ x } (-0.625) + 0.3 \text{ x } (-0.625)^2 = 1.21
                                                                                              AS4100 Table 5.6.1.
\phi M_b = 0.9 \text{ x } 1.21 \text{ x } 0.77 \text{ x } 735.6 = 612 \text{ kNm} < \phi M_{sx} = 0.9 \text{ x } 735.6 = 662 \text{ kNm}
\phi M_{\rm b} = 612 \text{ kNm} > {M_2}^* = 500 \text{ kNm}
```

## **Example 7.3.8 Bending in a Non-Principal Plane**

A 150x150x19 EA in Grade 300 steel is used as a simply supported beam over a span of 1.15m with one leg vertical and one leg horizontal. A concentrated gravity design load  $P^*$  kN is acting at mid span through the shear centre. Determine the maximum design load if the load point is not restrained laterally.



#### Solution

Since the load point is not restrained laterally, the bending moment that acts in the vertical plane can be resolved into components in the inclined principal axis planes.

$$M^* = P^*L / 4 = P^* \times 1.5 / 4 = 0.2875 P^*$$
  
 $M_x^* = M_y^* = 0.2875 P^* \times \cos 45^\circ = 0.203 P^*$   
Check if the beam is fully restrained.  
 $L/t = 1150 / 19 = 60.53$   
 $\beta_m = -0.8$   
 $(210 + 175 \beta_m) \times [\sqrt{(b_2/b_1)}] \times (250/f_y) = 210 + 175 \times (-0.8) \times \sqrt{(150 / 50)} \times 250 / 280 = 62.5$   
Since  $L/t = 60.53 < 62.5$  the beam has full lateral restraint.

Since 
$$L/t = 60.33 < 62.3$$
 the beam has full lateral restraint.  
 $\lambda_e = [(150 - 19) / 19] \times \sqrt{(280 / 250)} = 7.3 < \lambda_p = 9$ 

$$1.5Z_x = 1.5 \times 166 \times 10^3 = 249 \times 10^3 \text{ mm}^3$$

$$S_{\rm x} = 265 \text{ x } 10^3 \text{ mm}^3$$

$$Z_{\rm ex} = 1.5 Z_{\rm x} = 249 \text{ x } 10^3 \text{ mm}^3$$

$$\phi M_{\text{sx}} = 0.9 \ Z_{\text{ex}} \ f_{\text{y}} = 0.9 \ \text{x} \ 249 \ \text{x} \ 10^{3} \ \text{x} \ 280 \ \text{x} \ 10^{-6} = 62.75 \ \text{kNm}$$
  
 $1.5 \ Z_{\text{y(min)}} = 1.5 \ \text{x} \ 73.5 \ \text{x} \ 10^{3} = 110 \ \text{x} \ 10^{3} \ \text{mm}^{3}$   
 $S_{\text{y}} = 135 \ \text{x} \ 10^{3} \ \text{mm}^{3}$ 

$$1.5 Z_{y(min)} = 1.5 \times 73.5 \times 10^3 = 110 \times 10^3 \text{ mm}^3$$

$$S_{\rm v} = 135 \times 10^3 \, {\rm mm}^3$$

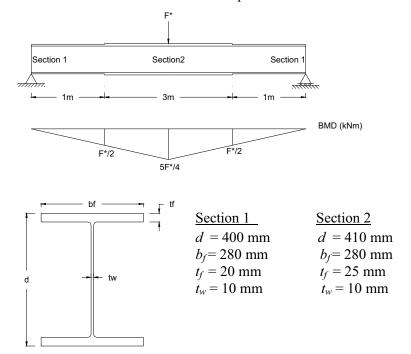
$$\phi M_{\text{sy}} = 0.9 Z_{\text{ey}} f_{\text{y}} = 0.9 \text{ x } 110 \text{ x } 10^3 \text{ x } 280 \text{ x } 10^{-6} = 27.72 \text{ kNm}$$

$$M_x^*/\phi M_{sx} + M_y^*/\phi M_{sy} \le 1$$
  
0.2875 P\* / 62.75 + 0.2875P\* / 27.72 \le 1  
P\* \le 66.88 kN

AS4100 Cl.8.3.4

## **Example 7.3.9 Checking a Flange Stepped Beam**

Determine the maximum design force F of a flange stepped beam welded from Grade 300 steel, the beam is 5m long with both ends fully restrained against lateral translation and twist rotation. The concentrated load F\* acts at mid span.



#### Solution

```
At section 1, t_f = 20 mm, f_y = 300 MPa
                                                                                                                 AS4100 Table 2.1
\lambda_{\rm efl} = (280 - 10) / (2 \times 20) \times \sqrt{(300 / 250)} = 7.4 < \lambda_{\rm ep} = 8 and so the flange is compact.
\lambda_{\text{ew}1} = [(400 - 2 \times 20) / 10] \times \sqrt{(300 / 250)} = 39.44 < \lambda_{\text{ep}} = 82 \text{ and so the web is compact.}
S_{x1} = 2 \times 280 \times 20 \times (400 - 20) / 2 + 10 \times (400 - 2 \times 20)^2 / 4
S_{\rm x1} = 2452 \text{ x } 10^3 \text{ mm}^3
M_{\rm sx1} = 2452 \times 10^3 \times 300 = 735.6 \text{ kNm}
The maximum moment acting on section 1 is M_1^* = 0.5 \text{ F}^*
M_1^* / M_{\text{sx1}} = 0.5 \text{ F}^* / 735.6 = \text{F}^* / 1471.2
At section 2, t_f = 25 \text{ mm}, f_v = 300 \text{ MPa}
                                                                                                                 AS4100 Table 2.1
By inspection section (2) is compact.
S_{x2} = 2 \times 280 \times 25 \times (410 - 25) / 2 + 10 \times (410 - 2 \times 25)^2 / 4
S_{\rm x2} = 3019 \times 10^3 \, \rm mm^3
M_{\rm sx2} = 3019 \times 10^3 \times 280 = 845.32 \text{ kNm}
M_2^* / M_{sx2} = [(5/4)F^* / 845.32] = F^* / 676.26 > F^* / 1471.2 and so section 2 is critical.
For section (2)
I_v = 2 \times 25 \times 280^3 / 12 + 360 \times 10^3 / 12 = 91.5 \times 10^6 \text{ mm}^4
I_{\rm w} = I_{\rm v} d_{\rm f}^2 / 4 = 91.5 \times 10^6 \times (410 - 25)^2 / 4 = 3391 \times 10^9 \text{ mm}^6
J = 2 \times 280 \times 25^3 / 3 + (410 - 2 \times 25) \times 10^3 / 3 = 3036.7 \times 10^3 \text{ mm}^4
L_e = 1 \times 1 \times 1 \times 5000 = 5000 \text{ mm}
Hence using a spread sheet program,
M_0 = 1920.8 \text{ kNm}
r_{\rm r} = L_{\rm r} / L = 2 / 5 = 0.4
                                                                                                    AS4100 Cl. 5.6.1.1. (b) (ii)
r_s = (280 \times 20)/(280 \times 25) [0.6 + ((0.4 \times 400)/410)] = 0.79
                                                                                                   AS4100 Cl. 5.6.1.1. (b) (ii)
\alpha_{st} = 1 - [1.2 \times 0.4 \times (1 - 0.79)] = 0.9
                                                                                                   AS4100 Cl. 5.6.1.1. (b) (ii)
M_{\text{oa}} = \alpha_{\text{st}} M_{\text{o}} = 0.9 \text{ x } 1920.8 = 1728.72 \text{ kNm}
\alpha_s = 0.6 \times [\sqrt{(845.32 / 1728.72)^2 + 3 - (845.32 / 1728.72)}] = 0.786
\beta_{\rm m} = 0
\alpha_{\rm m} = 1.35
                                                                                                    AS4100 Table 5.6
\phi M_{\rm bx} = 0.9 \text{ x } 1.35 \text{ x } 0.786 \text{ x } 845.32 = 807.3 > \phi M_{\rm sx} = 0.9 \text{ x } 845.32 = 760.8 \text{ kNm}
\phi M_{\rm bx} = \phi M_{\rm sx} = 706.8 \text{ kNm}
M_2^* = 5F^* / 4 \le \phi M_{\rm bx} = 760.8 \text{ kNm}
\Rightarrow F^* \le 608.64 \text{ kN}
```

### **Example 7.3.10 Checking a Tee Section**

A 230 BT 373 beam in Grade 300 steel is 4m long with both ends fully restrained against lateral translation and twist rotation. Determine the maximum design bending moment for uniform bending for the following arrangements:

- (1) The stem of the T-section is down and the flange in compression,
- (2) The stem is up and the flange in tension.

#### Solution

## (1) Stem is down – flange in compression

Bending about the x-axis puts the flange in almost uniform compression.  $\lambda_{\text{ef}} = [(190 - 9.1) / (2 \times 14.5)] \times \sqrt{(300 / 250)} = 6.84 < \lambda_{\text{ep}} = 9$  and so flange is compact.

Bending about the x-axis places the supported edge of the stem in compression and the unsupported edge in tension, which indicates that stem local buckling cannot occur.

$$Z_{\text{ex}}$$
 is the lesser of  $1.5Z_{\text{x}}$  and  $S_{\text{x}}$   
 $1.5Z_{\text{x}} = 1.5 \times 128 \times 10^3 = 192 \times 10^3 \text{ mm}^3$  (govern)  
 $S_{\text{x}} = 226 \times 10^3 \text{ mm}^3$   
 $Z_{\text{ex}} = 192 \times 10^3 \text{ mm}^3$   
 $M_{\text{sx}} = 192 \times 10^3 \times 300 \times 10^{-6} = 57.6 \text{ kNm}$   
 $\phi M_{\text{sx}} = 0.9 \times 57.6 = 51.8 \text{ kNm}$   
 $L_{\text{e}} = k_t k_l k_r L = 1 \times 1 \times 1 \times 4000 = 4000 \text{mm}$ 

$$L_e = k_t \ k_l \ k_r L = 1 \ \text{X} \ 1 \ \text{X} \ 1 \ \text{X} \ 4000 = 4000 \text{mm}$$
  
 $I_y = 8.3 \ \text{X} \ 10^6 \ \text{mm}^4, I_{cy} = 14.5 \ \text{X} \ 190^3 \ / \ 12 = 8.29 \ \text{X} \ 10^6 \ \text{mm}^4$   
 $I_w = 0$   
 $J = 267 \ \text{X} \ 10^3 \ \text{mm}^4$ 

$$\beta = 267 \text{ x} \text{ 10 mm}$$
  
 $\beta_{\text{x}} \approx 0.8 d_{\text{f}} [(2I_{\text{cy}} / I_{\text{y}}) - 1]$ 

Note: 
$$d_f = d$$
, for a T-section  
 $\beta_x \approx 0.8 \times 228 [(2 \times 8.29 \times 10^6) / (8.3 \times 10^6) - 1] \approx 182 \text{ mm}$ 

Hence using a spread sheet program,

$$M_{\rm o} = 267.9 \text{ kNm}$$
  
 $\alpha_{\rm s} = 0.6 \text{ x} \left[ \sqrt{(57.6 / 267.9)^2 + 3) - (57.6 / 267.9)} \right] = 0.918$   
 $\phi M_{\rm b} = 0.9 \text{ x } 1 \text{ x } 0.918 \text{ x } 57.6 = 47.6 \text{ kNm}$   
 $M^* \le 47.6 \text{ kNm}$ 

## (2) Stem is up - flange in tension

Bending about the x-axis places the stems unsupported edge under compression and the supported edge under tension.

$$\lambda_{\text{ew}}$$
 = (213 / 9.1) x √(300 / 250) = 25.63 >  $\lambda_{\text{ey}}$  = 25 and so web is slender   
∴ Slender Section  $\lambda_{\text{s}} = \lambda_{\text{ew}} = 25.63$ ,  $\lambda_{\text{ey}} = 25$   $Z_{\text{ex}} = Z (\lambda_{\text{sv}} / \lambda_{\text{s}})^2$ 

$$Z$$
 is the least elastic section modulus

$$Z = I_x / y_{max} = 22.2 \times 10^6 / 174.2 = 128 \times 10^3 \text{ mm}^3$$

$$Z_{\text{ex}} = 127.44 \times 10^3 \times (25.63 / 25)^2$$

$$Z_{\rm ex} = 121 \times 10^3 \, \rm mm^3$$

$$M_{\rm sx} = 121 \times 10^3 \times 300 = 36.3 \text{ kNm}$$

$$\phi M_{\rm sx} = 0.9 \text{ x } 36.3 = 32.7 \text{ kNm}$$

$$I_v = 8.3 \times 10^6 \text{ mm}^4$$

$$I_{yc} = 0$$
 (The flange is under Tension.)

$$I_{\rm w} = 0$$

$$J = 267 \times 10^3 \text{ mm}^4$$

$$\beta_{\rm x} \approx 0.8 d_{\rm f} \left[ (2I_{\rm cy}/I_{\rm y}) - 1 \right] = 0.8 \text{ x } 228 \text{ x } (0\text{-}1)$$

$$\beta_{\rm x} \approx -182.4 \, \rm mm$$

Hence using a spread sheet program,

$$M_0 = 81.52 \text{ kNm}$$

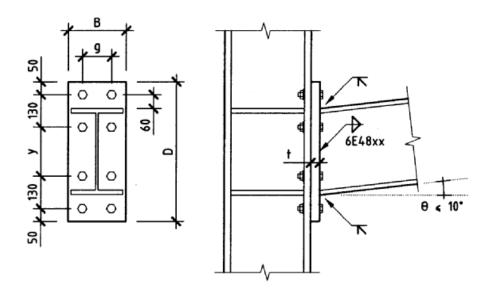
$$\alpha_s = 0.6 \text{ x} \left[ \sqrt{(36.3 / 81.52)^2 + 3) - (36.3 / 81.52)} \right] = 0.806$$

$$\phi M_b = 0.9 \times 1 \times 0.806 \times 36.3 = 26.33 \text{ kNm}$$

$$M^* \le 26.33 \text{ kNm}$$

# **Example 7.3.11 Steel Beam Complete Design Check**

A Grade 300 steel beam of 360UB50.7 section is to span 24 m. It is connected to the columns by a plate welded to each end of the beam, which is bolted to the column as shown below.



Rigid Beam-Column Connection. Note no holes in beam flanges which might reduce moment capacity

It will support purlins at 1000mm centres, which will provide lateral restraint to the top flange. Fly braces as shown in the diagram on the next page will provide effective twist restraint to the cross section which in combination with the lateral restraint offered by purlins to the top flange will provide lateral restraint to the bottom flange. We will assume that loads have been estimated and the frame has been analysed. For one particular load case the shear force and bending moment are as shown in the diagrams over the page (in a real design there would be several load cases). We will just check the capacity of this beam for this particular load case.

We will assume that the pitch on the roof is small enough to ignore for the purpose of analysis. The bending moment diagram is symmetrical, so only one half need be checked. Only the section at the end, which carries both the greatest shear force and a large bending moment, will be checked for combined shear and moment capacity.

(Strictly speaking, the rafter will carry axial tension in addition to bending moment and shear, and therefore the out of plane member moment capacity will be increased due to axial tension and the section capacity will be decreased. However we have assumed the pitch on the roof is small enough to ignore, so it is justifiable to ignore axial force. If the axial force is significant, the rafter capacity should be assessed using the provisions of Section 8 of AS4100.)

#### Solution

Load estimation and structural analysis have already been done for this example. The bending moment and the shear force diagrams are shown on the next page.

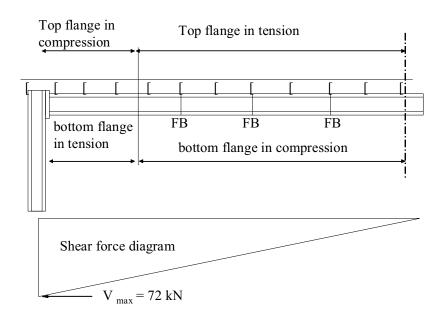
#### Section capacity

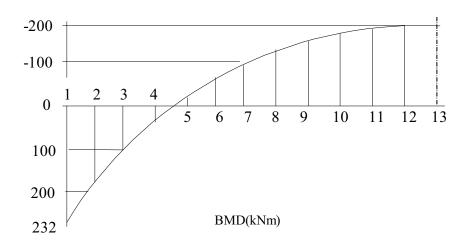
Check the 360UB50.7 section used in the computer analysis.

$$M_{\rm SX} = Z_{\rm ex} f_{\rm V}$$
 AS4100 Cl.5.2.1

$$M_{\rm sx} = 897 \times 10^3 \times 300 \times 10^{-6} = 269.1 \text{ kNm}$$
  
 $\phi M_{\rm sx} = 0.9 \times 269.1 = 242.2 \text{ kNm}$   
 $\phi M_{\rm sx}$  from AISC Tables [2] = 242 kNm

The maximum moment the rafter section have to withstand is  $M_x^* = 232 \text{ kNm}$  $\phi M_{sx} = 242 \text{ kNm} > M_x^* = 232 \text{ kNm}$  OK





# Member capacity

Classify the restrained sections into F, P, L and U.

At purlin 1,2, and 4 the section is classified as laterally restrained since the purlins can only prevent the critical flange from moving sidesway without providing any twist restraint to the cross section.

At purlin 5,6, 8,10 and 12 the section is classified as unrestrained since the purlins are connected at the tension flange level and they can't be relied upon in providing partial twist restraint to the cross section because standard oversized 18 mm holes are generally used in purlins with only M12 bolts.

At purlin 3,7,9,11 and 13 fly bracing is added and the section is classified as fully restrained.

For the restraint at the knee joint, the section is classified as fully restrained.

Divide the beam into segments and sub-segments.

Sub-Segment 1-2 contains the greatest moment so it must be checked. Sub-Segment 2-3 and 3-4 will clearly be less critical than segment 1-2. Although sub-segment 4-7 is unlikely, to be critical since it is bent in double curvature and it has relatively low moment, it is the longest out of all the segments and sub-segments and therefore it will be checked. Segments 7-9 and 9-11 need not be checked because they will be less critical than segment 11-13 which has a high moment over its entire length, which is the worst bending moment shape.

#### Sub-Segment (1-2)

```
Sub-Segment length L = 1 \text{m}
```

Restraint arrangement (LL)

Maximum moment in the sub-segment  $M_{\rm m}^* = 232 \text{ kNm}$ 

```
\begin{array}{lll} L_{e} = k_{t} \, k_{l} \, k_{r} \, L & AS4100 \, Cl. \, 5.6.3(1), (2), (3) \\ k_{t} = \, 1 & & \\ k_{l} = \, 1 & & \\ k_{r} = \, 1 & & \\ L_{e} = \, 1 \, \text{x} \, 1 \, \text{x} \, 1 = \, 1 \, \, \text{m} \\ \text{Ratio of segment end moments } \beta_{\text{m}} = -\, 0.69 & & \\ \text{Moment modification factor } \alpha_{\text{m}} = \, 1.16 & AS4100 \, Table \, 5.6.2 \\ \alpha_{\text{s}} \, \phi M_{\text{s}} = \, 230 \, \, \text{kNm} & AISC \, Tables \\ \phi M_{\text{b}} = \, \alpha_{\text{m}} \alpha_{\text{s}} \, \phi M_{\text{s}} \leq \, \phi M_{\text{s}} & AS4100 \, Cl. \, 5.6.1.1(1) \\ \phi M_{\text{b}} = \, 1.16 \, \, \text{x} \, \, 230 = \, 266.8 \, \, \text{kNm} \, > \! \phi M_{\text{s}} = \, 242 \, \, \text{kNm} \\ \phi M_{\text{b}} = \, 242 \, \, \text{kNm} \, > \, M_{\text{m}}^{*} = \, 232 \, \, \text{kNm} & OK & & \\ \end{array}
```

#### Sub-Segment (4-7)

Sub-Segment length L = 3m

Restraint arrangement (LF)

Maximum moment in the sub-segment  $M_{\rm m}^* = 100 \text{ kNm}$ 

```
L_e = k_t k_l k_r L
k_t = 1
k_l = 1
k_r = 1
L_e = 1 \times 1 \times 1 \times 3 = 3 \text{ m}
Ratio of segment end moments \beta_m = 0.4
Moment modification factor \alpha_m = 2.47
\alpha_s \phi M_s = 173 \text{ kNm}
\phi M_b = 2.47 \times 173 = 427.31 \text{ kNm} > \phi M_s = 242 \text{ kNm}
\phi M_b = 242 \text{ kNm} > M_m^* = 100 \text{ kNm} \qquad OK
AS4100 \ Table 5.6.2
AISC \ Tables
```

#### Segment (11-13)

```
Segment length L = 2m
```

Restraint arrangement (FF)

Maximum moment in the sub-segment  $M_{\rm m}^* = 200 \text{ kNm}$ 

If you do not have access to AISC Tables [2] the following method can be used to determine the member moment capacity  $\phi M_{b.}$ 

Use clause 5.3.2.4 of AS 4100 to check if the segment is fully restrained. If it is, there is no reduction due to lateral buckling and we can take  $\phi M_b = \phi M_s$ , if the segment is not fully restrained calculate  $\alpha_s$  using equations 5.6.1.1(2), and 5.6.1.1(3) of AS 4100. Calculate  $M_o$  first and then  $\alpha_s$ .

Note: fully restrained segment means that  $\alpha_s = 1$  approx and/or  $\alpha_m$  is big enough to offset the reduction due to  $\alpha_s$  like in the case of segment 4-5.

In this example, sub-segment (1-2) has  $L/r_y = 1000/38.5 = 25.97$ .  $80 + 50\beta_{\rm m}\sqrt{(250/f_y)} = (80 + 50(-0.69))\sqrt{(0.833)} = 41.53$  so sub-segment (1-2) is fully restrained. So for this segment  $\phi M_b = \phi M_s$ 

Sub-segment (4-7) has  $L/r_y = 3000/39.0 = 77.92$ 80 +  $50\beta_m\sqrt{(250/f_y)} = (80 + 50(0.4))\sqrt{(0.833)} = 91.269$  so sub-segment (4-7) is fully restrained. So for this sub-segment  $\phi M_b = \phi M_s$ 

Segment (11-13) has  $L/r_y = 2000/39.0 = 51.28$ . 80 +  $50\beta_m\sqrt{(250/f_y)} = (80 + 50(-0.95)) \sqrt{(0.833)} = 29.67$  so segment (11-13) is not fully restrained. So for this segment calculate  $\alpha_s$ .

Next,  $\alpha_s$  for segment (11-13) can be calculated from equations 5.6.1.1(2), and 5.6.1.1(3) of AS 4100. Calculate  $M_0$  first and then  $\alpha_s$ .

#### Check Web shear capacity

We will check section 1, where shear force  $V^*$  is maximum, for its capacity to resist shear. From Clause 5.11 of AS 4100, the web is required to satisfy,

$$V^* \leq \phi V_{\rm v}$$

In this case the shear stress distribution can be assumed to be approximately uniform, so Clause 5.11.2 of AS 4100 applies.

 $d_{\rm p}/t_{\rm w}$  = 45.6 < 82 /  $\sqrt{(f_{\rm y}/250)}$  = 74.8, so no reduction in effective web area for shear, so  $V_{\rm u} = V_{\rm w}$ 

From Clause 5.11.4 of AS 4100.  $V_{\rm w} = 0.6 f_{\rm yw} A_{\rm w} = 0.6 \times 320 \times 7.3 \times 356 = 498.97 \text{ kN}$  $\phi V_{\rm w} = 0.9 \times 498.97 = 449 \text{ kN} > V^* = 72 \text{ kN} \quad OK$ 

The section is adequate to withstand the shear on its own. However, it is still necessary to check if it can take combined shear and bending.

## Check capacity to resist combined moment and shear

We will again check section 1, where shear force  $V^*$  is maximum, this time for its capacity to resist combined moment and shear. We have a choice of two methods:

The proportioning method (Clause 5.12.2 of AS 4100) in which the moment is assumed to be taken by the flanges alone and the shear by the web alone, and the "Shear and bending interaction method" of Clause 5.12.3 of AS 4100. Either is acceptable and quite straightforward if you read the code carefully. The shear and bending interaction method is based on a more accurate representation of what actually happens, since the web does in fact carry a significant amount of moment. It will therefore give a less conservative design, and should be used where the design moment is close to the member capacity.

# Proportioning method

$$M_{\rm f} = A_{\rm fm} d_{\rm f} f_{\rm vf}$$

 $A_{\rm fm}$  is the lesser of the flange effective areas, determined using clause 6.2.2 of AS 4100 for the compression flange and the lesser of  $A_{\rm fg}$  and 0.85  $A_{\rm fn}f_{\rm u}/f_{\rm yf}$ 

#### For the compression flange

$$\begin{split} A_{\rm fm} &= 2 \times b_{\rm ef} \times t_{\rm f} = (b_{\rm f} - t_{\rm w}) \times (\lambda_{\rm ey} / \lambda_{\rm e}) \times t_{\rm f} \leq A_{\rm fg} \\ \lambda_{\rm e} &= (b_{\rm f} - t_{\rm w}) / 2t_{\rm f} = 7.12 \\ \lambda_{\rm ey} &= 16 \\ AS4100 \ Table \ 6.2.4 \\ A_{\rm fm} &= (171 - 7.3) \times (16 / 7.12) \times 11.5 = 4230.45 \text{mm}^2 \\ A_{\rm fm} &= A_{\rm fg} = 1966.5 \ \text{mm}^2 \end{split}$$

# For the tension flange

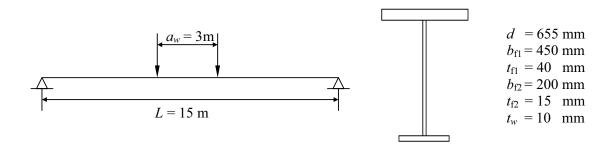
$$A_{\rm fm}$$
 is the lesser of  $A_{\rm fg}$  and 0.85  $A_{\rm fn}$   $f_{\rm u}/f_{\rm yf}$   $A_{\rm fg}$  = 1966.5 mm<sup>2</sup> 0.85  $A_{\rm fn}$   $f_{\rm u}/f_{\rm yf}$  = 0.85 x 1966.5 x 440 / 300 = 2451.6 mm<sup>2</sup>  $A_{\rm fm}$  =  $A_{\rm fg}$  = 1966.5 mm<sup>2</sup>  $A_{\rm fm}$  = 1966.5 mm<sup>2</sup>  $M_{\rm f}$  =  $A_{\rm fm}$   $d_{\rm f}$   $f_{\rm yf}$  =  $A_{\rm fm}$  x ( $d_{\rm l}$  +  $t_{\rm f}$ ) x  $f_{\rm yf}$   $M_{\rm f}$  = 1966.5 x (333+11.5) x 300  $M_{\rm f}$  = 203.24 kNm  $\phi M_{\rm f}$  = 0.9 x 203.24 = 183 kNm <  $M_{\rm f}$  = 232 kNm  $M_{\rm f}$ 

# Shear and bending interaction method

$$M^* \le 0.75 \, \phi M_{\rm s}$$
?  
 $M^* = 232 > 0.75 \, \text{x} \, 242 = 181.5 \, \text{kNm}.$   
 $V_{\rm vm} = V_{\rm v} \, [2.2 - (1.6 M^*/\phi M_{\rm s})] = 498.97 \, [2.2 - 1.6 \, \text{x} \, 232 \, / \, 242]$   
 $V_{\rm vm} = 332.4 \, \text{kN}$   
 $\phi V_{\rm vm} = 0.9 \, \text{x} \, 332.4 = 299.1 \, \text{kN} > V^* = 72 \, \text{kN}$   $OK$ 

#### Example 7.3.12 Checking an I-Section with Unequal Flanges

The 15m long simply supported beam shown below is fabricated from Grade 300 steel and is loaded by two equal symmetrically- placed concentrated loads 3m apart. The beam ends are fully restrained against lateral translation and twist rotation but unrestrained against lateral rotation. Determine the maximum design bending the beam can withstand when the two point loads act (a) at the shear centre (b) at the top flange.



#### Discussion

The example illustrates the uncertainties with the Clause 5.6.1.1 approach in AS4100[1] and presents a more fundamental and conservative approach for the design of simply supported monosymmetric beams under the action of either a central concentrated load or a two equal symmetrically placed concentrated loads.

#### Solution

Elastic Properties 
$$\begin{aligned} y_e &= \Sigma Ay \ / \ \Sigma A = \left[ 450 \ \text{x} \ 40 \ \text{x} \ 20 + 200 \ \text{x} \ 15 \ \text{x} \ (655\text{-}7.5) + 600 \ \text{x} \ 10 \ \text{x} \ (300 + 40) \right] \ / \ 27000 \\ y_e &= 160.83 \ \text{mm} \\ I_x &= 450 \ \text{x} \ 40^3 / 12 + 450 \ \text{x} \ 40 \ \text{x} \ (160.83\text{-}20)^2 + 200 \ \text{x} \ 15^3 / 12 + 200 \ \text{x} \ 15 \ \text{x} \ (494.17\text{-}7.5)^2 + \\ 10 \ \text{x} \ 600^3 / 12 + 600 \ \text{x} \ 10 \ \text{x} \ (340\text{-}160.83)^2 \\ I_x &= 1442.61 \text{x} 10^6 \ \text{mm}^4 \\ I_y &= 40 \ \text{x} \ 450^3 / 12 + 15 \ \text{x} \ 200^3 / 12 + 600 \ \text{x} \ 10^3 / 12 = 313.80 \text{x} 10^6 \ \text{mm}^4 \\ I_{cy} &= 40 \ \text{x} \ 450^3 / 12 = 303.75 \text{x} 10^6 \ \text{mm}^4 \\ I_{y} &= 15 \ \text{x} \ 200^3 / 12 = 10 \text{x} 10^6 \ \text{mm}^4 \\ I_{y} &= 15 \ \text{x} \ 200^3 / 12 = 10 \text{x} 10^6 \ \text{mm}^4 \\ Z_x &= I_x \ / \ y_{\text{max}} = 1442.61 \text{x} 10^6 \ / \ (655\text{-}160.83) = 2919.27 \ \text{x} 10^3 \ \text{mm}^3 \\ y_o &= (e_1 \ I_{yf1} - e_2 \ I_{yf2}) \ / \ (I_{yf1} + I_{yf2}) \\ e_1 &= y_e - t_{f1} / \ 2 = 160.83 - 40 / 2 = 140.83 \ \text{mm} \\ e_2 &= d - y_e - t_{f2} / \ 2 = 655 - 160.83 - 15 / 2 = 468.67 \ \text{mm} \\ y_o &= (140.83 \text{x} 303.75 \text{x} 10^6 - 486.67 \text{x} 10 \text{x} 10^6) \ / \ (313.75 \text{x} 10^6) \\ y_o &= 120.83 \ \text{mm} \\ J &= \Sigma b t^3 / 3 = 450 \ \text{x} \ 40^3 \ / \ 3 + 200 \ \text{x} \ 15^3 \ / \ 3 + 600 \ \text{x} \ 10^3 \ / \ 3 = 10025 \ \text{x} \ 10^3 \ \text{mm}^4 \\ I_w &= I_{cy} \ d_f^2 (1 - I_{cy} / I_y) = 40 \ \text{x} \ 450^3 / 12 \ \text{x} \ (655\text{-}20\text{-}7.5)^2 \ \text{x} \ (1\text{-}(40 \ \text{x} \ 450^3 / 12) / \ 313.80 \text{x} 10^6) \\ I_w &= 3830.51 \ \text{x} \ 10^9 \ \text{mm}^6 \end{aligned}$$

#### **Plastic Properties**

Assume firstly that the plastic centroid  $y_{p1}$  lies in the web. Hence,  $(y_{p1}$ - 40) x 10 + 450 x 40 = 27,000

 $A_g = 450 \times 40 + 200 \times 15 + (655-40-15) \times 10 = 27,000 \text{ mm}^2$ 

$$y_{n1} = 940 \text{mm} > d = 655 \text{ mm}$$
 NG

Therefore, the assumption of the plastic centroid location is incorrect. Try the plastic centroid  $y_{p2}$  located within the top flange.

$$y_{p2} \times 450 = \frac{27,000}{2}$$
  
 $y_{p2} = 30 \text{ mm} < t_{f1} = 40 \text{ mm}$  *OK*  
 $S_x = 30 \times 450 \times 30/2 + 10 \times 450 \times 10/2 + 10 \times 600 \times (300+10) + 15 \times 200 \times (600+7.5+10)$   
 $S_x = 3937.5 \times 10^3 \text{ mm}^3$   
 $t_{f1} = 40 \text{ mm}, f_{yf1} = 280 \text{ MPa}$  *AS4100 Table 2.1*  
 $t_{f2} = 15 \text{ mm}, f_{yy} = 310 \text{ MPa}$  *AS4100 Table 2.1*  
 $t_w = 10 \text{ mm}, f_{yw} = 310 \text{ MPa}$  *AS4100 Table 2.1*  
 $\therefore f_y = 280 \text{ MPa}$ 

# Compression Flange Slenderness

$$\lambda_{ef} = [(450 - 10) / (2 \times 40)] \times \sqrt{(280 / 250)} = 5.82$$
  
 $\lambda_{ep} = 8$ ,  $\lambda_{ey} = 14$   $\lambda_{ef} / \lambda_{ey} = 5.82/14 = 0.42$  AS4100 Table 5.2

# Web Slenderness

$$\lambda_{\text{ew}} = [600 / 10] \times \sqrt{(280 / 250)} = 63.50$$
  
 $\lambda_{\text{ep}} = 82$ ,  $\lambda_{\text{ev}} = 115$   $\lambda_{\text{ew}} / \lambda_{\text{ev}} = 63.50 / 115 = 0.55$  AS4100 Table 5.2

Hence, the web is the critical element in the section and the section slenderness and slenderness limits are the web values.

```
\lambda_{\rm s} = 63.50, \lambda_{\rm sp} = 82, \lambda_{\rm sy} = 115

\lambda_{\rm s} = 63.50 < \lambda_{\rm sp} = 82 and so the section is compact.

Z_{\rm ex} is the lesser of 1.5Z_{\rm x} and S_{\rm x}

1.5Z_{\rm x} = 1.5 \times 2919.27 \times 10^3 = 4378.91 \times 10^3 \, {\rm mm}^3

S_{\rm x} = 3937.5 \times 10^3 \, {\rm mm}^3

Z_{\rm ex} = 3937.5 \times 10^3 \, {\rm mm}^3

M_{\rm sx} = Z_{\rm ex} f_{\rm y}

M_{\rm sx} = 3937.5 \times 10^3 \times 280 \times 10^{-6} = 1102.5 \, {\rm kNm}

\phi M_{\rm sx} = 0.9 \times 1102.5 = 992.25 \, {\rm kNm}
```

# (a) Shear centre loading

#### <u>Proposed Monosymmetric Beam Design Rules</u>

It is important to note that the equation  $\phi M_b = \phi \alpha_m \alpha_s M_s$  in Clause 5.6.1.1 of AS4100 only applies to doubly symmetric sections, i.e. sections with equal flanges. It can be unsafe if used to design monosymmetric beams. According to Woolcock et al [5], "the AS 4100[1] approach for monosymmetric beams is rather unsatisfactory and can be unconservative." They propose a more rigorous set of design rules in which the two modification factors  $\alpha_m$  and  $\alpha_s$  are replaced by a single beam slenderness reduction factor  $\alpha_{sb}$ . The relevant formulae are reproduced below, followed by two illustrative examples which demonstrate the danger of using the basis AS 4100 design method for monosymmetric beams.

Unless there is a good reason for using monosymmetric beams it seems advisable to avoid them. Probably the most common situation in which they are used is for crane runway beams, in which a channel section is commonly welded to the top flange to boost the minor axis bending capacity of the top flange to resist lateral loads. However a UC or WC section will often prove more economical and is certainly easier to design.

If it is necessary to use a monosymmetric section, the following procedure is recommended (after Woolcock et al [5]):

The member bending capacity,  $\phi M_b$  is given by

$$\phi M_b = \phi \alpha_{sb} M_s \le \phi M_s \qquad AS4100 Cl. 5.6.2(ii)$$

in which  $\alpha_{sb}$  is the beam slenderness reduction factor.

The expression for the elastic buckling moment  $M_{ob}$  for one particular loading case that of a simply supported beam with two symmetrically placed point loads is given below.

$$M_{0b} = \overline{m} \sqrt{\left(\frac{\pi^2 E I_y G J}{L^2}\right)} \left\{ \sqrt{1 + 4\rho (1 - \rho)} \overline{K}^2 + \left(f_1 \overline{K}\right)^2 \left(\varsigma + f_2 \frac{\beta_x}{d_f}\right)^2 + f_1 \overline{K} \left(\varsigma + f_2 \frac{\beta_x}{d_f}\right) \right\}$$

where

$$\overline{K} = \sqrt{\frac{\pi^2 E I_y d^2_f}{4G J L^2}}$$

in which  $\overline{K}$  is the beam parameter,  $EI_y$  is the minor axis flexural rigidity, GJ is the torsional rigidity and L is the length of the beam.

$$\beta_{x} \cong 0.9d_{f} \times (2\rho - 1) \times (1 - (I_{y}/I_{x})^{2})$$

in which  $\beta_x$  is the monosymmetry section constant,  $\rho$  is the degree of beam monosymmetry given by

$$\rho = I_{yc} / I_y$$

where  $I_{yc}$  is the second moment of area of the compression flange about the section minor principal axis. Factors  $\overline{m}$ ,  $f_1$  and  $f_2$  are given in terms of the location  $\alpha$  of the point loads where  $\alpha L = (L-a_w)/2$ 

$$\overline{m} \cong 1 - 0.4\alpha (1 - 5.5 \alpha)$$

$$f_1 = \text{m sin}^2 (\pi \alpha) / \alpha \pi^2$$

$$f_2 = 0.5 \times [\alpha (1 - \alpha) \pi^2 / \sin^2 (\pi \alpha) - 1]$$

 $\xi$  is the load height parameter given by

$$\xi = 2 \overline{a} / d_f$$

where a is the height of the load below the shear centre taken as positive if the load acts below the shear centre and negative if it acts above.

Hence using a spread sheet program,

$$\rho = 0.97$$

$$\beta_{\rm x}/d_{\rm f} = 0.802$$

$$\alpha = 0.4$$

$$\overline{m} = 1.19$$

$$f_1 = 0.546$$

$$f_2 = 0.809$$

$$a = 0.0$$

$$\xi = 0.0$$

$$K = 0.58$$

$$M_{ob} = 2209.71 \text{ kNm}$$

$$\alpha_{sb} = 0.7821$$
  
 $\phi M_b = 0.7821 \times 992.25 = 776.1 \text{ kNm}$ 

# Member Capacity to AS4100

```
Segment length L = 15 \text{ m}
Restraint arrangement (FF)
L_e = k_t \, k_l \, k_r \, L
                                                                                                    AS4100 Cl. 5.6.3(1),(2),(3)
L_e = 1 \times 1 \times 1 \times 15 = 15 \text{ m}
Moment modification factor \alpha_{\rm m} = 1 + 0.35(1-3/15)^2 = 1.224
                                                                                                              AS4100 Table 5.6.2
\beta_{\rm x} \approx 0.8 d_{\rm f} \left[ (2I_{\rm cv}/I_{\rm v}) - 1 \right] = 0.8 \times 627.5 \times \left[ (2 \times 303.75 \times 10^6/(313.80 \times 10^6)) - 1 \right] = 469.85 \,{\rm mm}
\pi^2 EI_y / L_e^2 = \pi^2 x \ 200000 \ x \ 313.80 x 10^6 / 15000^2 = 2752.96 x 10^3 \ N
\pi^2 EI_w / L_e^2 = \pi^2 x \ 200000 \ x \ 3830.51 \ x \ 10^9 / 15000^2 = 33.61 x \ 10^9 \ Nmm^2
GJ = 8 \times 10^4 \times 10025 \times 10^3 = 802.0 \times 10^9 \text{ Nmm}^2
M_0 = \sqrt{(2752.96 \times 10^3)} \times [\sqrt{(802.0 \times 10^9 + 33.61 \times 10^9 + 469.85^2 \times 2752.96 \times 10^3/4)} +
                                     (469.85/2)x\sqrt{2752.96x10^3}
M_0 = 2295.57 \text{ kNm}
\alpha_s = 0.6 \text{ x} \left[ \sqrt{(1102.5 / 2295.57)^2 + 3 - (1102.5 / 2295.57)} \right] = 0.7903
\phi M_b = \alpha_m \alpha_s \phi M_s \le \phi M_s
                                                                                                          AS4100 Cl.5.6.1.1(1)
\phi M_b = 1.224 \text{ x } 0.7903 \text{ x } 992.25 = 959.8 \text{ kNm} < \phi M_s = 992.25 \text{ kNm}
\phi M_{\rm b} = 959.8 \text{ kNm}
```

The design member capacity  $\phi M_b = 959.8$  kNm exceeds the more accurate value of 776.1 kNm by 23.67% thus it can be seen that AS 4100[1] approach in designing monosymmetric beams can be quite unconservative. The reason for that is the erroneous calculation of the moment modification factor  $\alpha_m$ .

## (b) Top flange loading

# Member Capacity Using Buckling Analysis AS4100 Cl. 5.6.2(ii)

```
Hence using a spread sheet program, \rho = 0.97
\beta_{\rm x}/d_{\rm f} = 0.802
\alpha = 0.4
\overline{\rm m} = 1.19
f_1 = 0.546
f_2 = 0.809
\overline{\rm a} = -(y_{\rm e} - y_{\rm o}) = -(160.83 - 120.83) = -40 \text{ mm}
\xi = -0.127
\overline{\rm K} = 0.58
M_{ob} = 2125.17 \text{ kNm}
\alpha_{\rm sb} = 0.7736
\phi M_{\rm b} = 0.7736 \times 992.25 = 767.6 \text{ kNm}
```

#### Member Capacity to AS4100

Segment length L = 15 mRestraint arrangement (FF)

```
L_e = k_t \, k_l \, k_r \, L
                                                                                                       AS4100 Cl. 5.6.3(1),(2),(3)
L_e = 1 \times 1.4 \times 1 \times 15 = 21 \text{ m}
Moment modification factor \alpha_{\rm m} = 1 + 0.35(1-3/15)^2 = 1.224
                                                                                                                  AS4100 Table 5.6.2
\beta_{\rm x} \approx 0.8 d_{\rm f} \left[ (2 I_{\rm cv} / I_{\rm v}) - 1 \right] = 0.8 \text{ x } 627.5 \text{x} \left[ (2 \text{x} 303.75 \text{x} 10^6 / (313.80 \text{x} 10^6)) - 1 \right] = 469.85 \text{mm}
\pi^2 EI_y / L_e^2 = \pi^2 x \ 200000 \ x \ 313.80 x 10^6 / 21000^2 = 1404.57 x 10^3 \ N
\pi^2 E I_w / L_e^2 = \pi^2 x \ 200000 \ x \ 3830.51 \ x \ 10^9 / 21000^2 = 17.15 x \ 10^9 \ Nmm^2
GJ = 8 \times 10^4 \times 10025 \times 10^3 = 802.0 \times 10^9 \text{ Nmm}^2
M_0 = \sqrt{(1404.57 \times 10^3)} \times \left[\sqrt{(802.0 \times 10^9 + 17.15 \times 10^9 + 469.85^2 \times 1404.57 \times 10^3/4)} + \right]
                                      (469.85/2)x\sqrt{1404.57}x10^3
M_0 = 1452.21 \text{ kNm}
\alpha_s = 0.6 \times [\sqrt{(1102.5 / 1452.21)^2 + 3 - (1102.5 / 1452.21)}] = 0.6792
\phi M_b = \alpha_m \alpha_s \phi M_s \le \phi M_s
                                                                                                              AS4100 Cl.5.6.1.1(1)
\phi M_b = 1.224 \text{ x } 0.6792 \text{ x } 992.25 = 824.9 \text{ kNm} < \phi M_s = 992.25 \text{ kNm}
\phi M_{\rm b} = 824.9 \text{ kNm}
```

As expected the design member capacity  $\phi M_b = 824.9$  kNm exceeds the more accurate value of 767.6 kNm only by 7.46% compared to 23.67% for shear centre loading. The reason for that is the high value of the load height factor, which tends to offset the  $\alpha_m$  effect. Note that a  $k_l$  value of 1.4 is very conservative in this case as the shear centre, is very close to the top flange.

## 7.4 REFERENCES

- 1. Standards Australia (1998). AS 4100 Steel Structures.
- 2. Australian Institute of Steel Construction, (1994) *Design Capacity Tables for Structural Steel* (DCT) Second edition, Volume 1: Open Sections.
- 3. Hogan T.J., Syam A.A., Trahair N.S. (1993) *Design of Unbraced Beams* (Article in the AISC technical journal, "Steel Construction", Vol 27, No1).
- 4. SpaceGass. www.spacegass.com.
- 5. Bradford M.A., Kitipornchai S., Woolcock S.T., (1999) Design of Portal Frame Buildings Third edition (to AS 4100).

# 8 MEMBERS SUBJECT TO COMBINED ACTIONS

#### 8.1 INTRODUCTION

"Combined actions" include any combination of bending about one or both axes and/or axial tension or compression. The strength design of members subject to combined actions is covered by Clauses 8.1 to 8.4 of AS 4100 [1]. Clause 8.1 sets general guidelines, which direct the designer to subsequent clauses. Clause 8.2 defines the *design actions*  $(N^*, M_x^*, M_y^*)$  for checking the section capacity and the member capacity.

Clause 8.3 defines the *reduced section moment capacities*  $M_{rx}$  and  $M_{ry}$  which are reduced by the effect of axial force, in terms of  $M_{sx}$ ,  $M_{sy}$ ,  $N^*$  and  $N_s$ . Section capacities are limited by either yielding or local buckling, which will often control the design of highly restrained members.

Clause 8.4 defines the *member capacity*, which unlike the section capacity, is limited by overall buckling of the member, or of member segments, and will generally govern the design of members without full restraint.

The code deals with bending plus axial force in two ways. Where there is biaxial bending the ratios of the load effects to the corresponding capacities are summed. Thus for example the section capacity of a member subjected to biaxial bending plus axial force is given by

$$\frac{N^*}{\phi N_s} + \frac{M_x^*}{\phi M_{sx}} + \frac{M_y^*}{\phi M_{sy}} \le 1 \tag{8.1}$$

where the numerators are the load effects and the denominators are the section capacities. Trial and error is used to find the unknown. Thus for example if the design load effects  $N^*$ ,  $M_x^*$  and  $M_y^*$  are known, values of  $M_{\rm sx}$ ,  $M_{\rm sy}$  and  $N_{\rm s}$  for trial sections are substituted in Equation 8.1 to find the most economical section which satisfies the equation. To find either of the section capacities  $M_{\rm rx}$  or  $M_{\rm ry}$  for a given section and design axial load  $N^*$ ,  $M_y^*$  must be specified to find  $M_{\rm rx}$ , i.e. the maximum value of  $M_x^*$  that will satisfy Equation 8.1, or  $M_x^*$  must be specified to find  $M_{\rm ry}$ .

The code deals with combinations of uniaxial bending plus axial force by calculating the moment capacity reduced due to axial force. Thus for example the reduced moment capacity  $M_{rx}$  of a section carrying a bending moment  $M_x^*$  about its x axis plus an axial compression force  $N^*$  is given by

$$M_{rx} = M_{sx} \left( 1 - \frac{N^*}{\phi N_s} \right) \tag{8.2}$$

By rearranging equation (8.2) it will be seen that this is equivalent to equation (8.1) with  $M_y^*$  equal to zero and  $M_x^*$  replaced by  $M_{rx}$  for the limiting case, thus giving the design moment capacity directly.

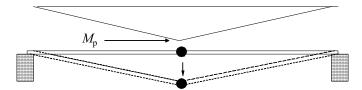
As with any code, using these clauses is just a matter of applying the formulas carefully. However the designer can save time by checking the member capacity first, rather than the section capacity, since member capacity will most often control the design.

#### 8.2 PLASTIC ANALYSIS AND PLASTIC DESIGN

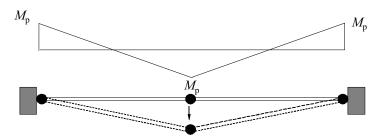
Most analysis and design is done on the basis that all members behave elastically at all times. However a structure made of ductile materials can accommodate some local yielding and will not actually collapse until enough "plastic hinges" have formes to enable the structure to behave like a mechanism (a "plastic hinge" is a cross section which has fully yielded so that it is behaving in a fully plastic manner and will continue to deform without any increase in moment until strain hardening occurs).

This fact is recognised in Clause 5.2 of AS 4100, where the effective section modulus  $Z_e$  for a compact section is the lesser of the plastic section modulus S and 1.5Z, whichever is less. For sections bent about their major axis S is usually about 1.2Z, so in effect we are using the plastic section capacity.

In plastic analysis and design we take this logic a step further and base our analysis on the fact that a structure will not actually collapse until there are enough plastic hinges to enable it to behave as a mechanism and permit collapse, as shown in Figs. 8.1 and 8.2.



(a) Simply supported beam: only one plastic hinge needed for collapse



(b) Fixed end beam: three plastic hinges needed for collapse

Figure 8.1 Plastic Collapse Mechanisms for Beams

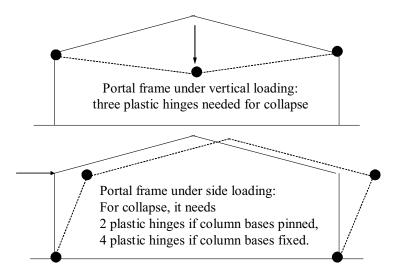


Figure 8.2 Plastic Collapse Mechanisms for Portal Frames

We then ensure that there is enough section and member capacity to carry the plastic collapse moments throughout the frame. Thus as shown in Fig.8.3, the midspan moment for a simply supported beam under UDL is  $wL^2/8$ , so the value of w to produce collapse is given by  $wL^2/8 = M_p$ .

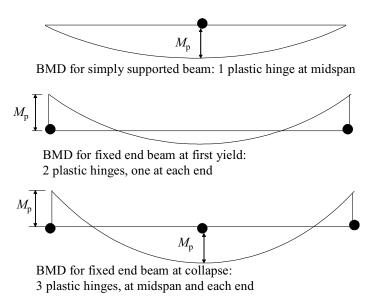


Figure 8.3 Bending Moment Diagrams for Plastic Collapse of Beams

But for a fixed end beam under UDL, plastic hinges will start to form when the end moments  $wL^2/12 = M_p$ , i.e. at 50% higher load than for a simply supported beam of the same section. But this process will redistribute moment to midspan, i.e. the beam starts to behave as if it were only partially fixed at the ends, but it does not collapse yet, and load can increase until the midspan moment and the end moments =  $M_p$ , i.e.  $wL^2/16 = M_p$ . Thus the plastic collapse load is twice the simply supported collapse load.

Plastic analysis and design is basically about moment capacity and need not always involve combined actions and so might logically have been introduced in Chapter 7, AS 4100 deals with the topic mainly in Section 8, and this format has been followed in the present work.

Plastic analysis and design is illustrated below in Example 8.3.5.

AS 4100 places the following restrictions on the use of plastic analysis and design. A member assumed to contain a plastic hinge in the plastic analysis of the frame shall satisfy the following:

- 1. The member must be hot-formed compact doubly symmetrical I-section.
- 2. The member must satisfy the in-plane slenderness limitations of clause 8.4.3 of AS 4100. A member that does not satisfy these limitations will buckle in the plane of bending under the combined actions of bending and axial compression and therefore shall not contain a plastic hinge.
- 3. The web of the compact I-section member must satisfy the web slenderness limitations outlined in clause 8.4.3.2 of AS 4100. Note that the flange of a compact section will not buckle locally under the action of bending and axial compression because its plasticity slenderness limits under bending alone will remain unchanged under combined bending and axial compression. On the other hand a compact web under bending alone may buckle locally under combined bending and axial compression because the plastic neutral axis PNA under combined bending and axial compression will not coincide with the centroid of the steel section. More than  $d_1/2$  of the web will be under compression and therefore the plastic slenderness limit of the web will be less that that under bending alone. Because of this AS 4100 requires the web to satisfy the web slenderness limitations given in Clause 8.4.3.3.
- 4. Any member segment that may contain a plastic hinge must have full lateral restraint (Clause 5.3.2.4) to prevent out of plane failure due to lateral torsional buckling. The rest of the segments may or may not have full lateral restraint. If they do not, then the limit state of lateral torsional buckling under combined bending and axial force outlined in clause 8.4.4 must be designed for. For segments that have full lateral restraint, only the requirements of clause 8.4.3 need to be satisfied (i.e. out of plane capacity (Clause 8.4.4) shall not be designed for). Note that a member analysed elastically must satisfy Clause 8.4.4 even if it is fully restrained (Clause 5.3.2.4).

A member that is not assumed to contain a plastic hinge in the plastic analysis of the frame does not need to satisfy any of the above conditions. If for such a member any of the above conditions is not satisfied, it is permissible to design it as an elastic member in a plastically analyzed frame (i.e. design the member as if an elastic analysis had been performed). However if such a member satisfies the above conditions only the requirements of clause 8.4.3.4 need to be satisfied.

#### 8.3 WORKED EXAMPLES

#### **Example 8.3.1 Biaxial Bending Section Capacity**

A cross section in a fully restrained beam has a major axis design bending moment  $M_x^* = 122$ kNm, and a minor axis design bending moment  $M_y = 27$  kNm, Select a suitable UC section in Grade 300 steel that will satisfy the strength requirement of biaxial bending.

#### Solution

Assume a section that is compact for bending about both principal axes with  $f_v = 300 \text{ MPa}$ 

$$M_{\rm x}^*/\phi M_{\rm sx} + M_{\rm y}^*/\phi M_{\rm sy} \leq 1.0$$

$$M_{\rm x}^* / (0.9 \, {\rm x} \, {\rm S}_{\rm x} \, {\rm x} \, f_{\rm y}) + M_{\rm y}^* / (0.9 \, {\rm x} \, {\rm S}_{\rm y} \, {\rm x} \, f_{\rm y}) \le 1.0$$

Time the above equation by  $(0.9 \times S_x \times f_y)$  you will get

$$M_{x}^{*} + (S_{x}/S_{y}) M_{y}^{*} \le 0.9 \times S_{x} \times f_{y}$$
  
 $S_{x} \ge [M_{x}^{*} + (S_{x}/S_{y}) M_{y}^{*}] / (0.9 \times f_{y})$ 

 $(S_x/S_y)$  is close to 2 for all UC sections therefore adopt 2 for the first trial.

Guess 
$$(S_x / S_y) = 2.0$$

$$S_x \ge [122 \times 10^6 + 2 \times 27 \times 10^6] / (0.9 \times 300)$$

 $S_x \ge 651.85 \text{ x} 10^3 \text{ mm}^3$ 

# Try 200UC59.5

$$\phi M_{\rm sx} = 177 \text{ kNm}$$
 AISC Tables  $\phi M_{\rm sy} = 80.6 \text{ kNm}$  AISC Tables

$$M_x^*/\phi M_{sx} + M_y^*/\phi M_{sy} = 122 / 177 + 27 / 80.6 = 1.024 > 1.0$$
 Just fails, so try next size up.

#### Trv 250UC72.9

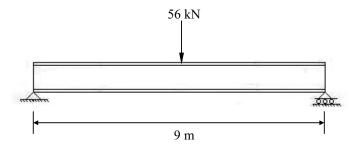
$$\phi M_{\rm sx} = 266 \ {\rm kNm}$$
 AISC Tables  $\phi M_{\rm sy} = 123 \ {\rm kNm}$  AISC Tables

$$M_x^*/\phi M_{sx} + M_y^*/\phi M_{sy} = 122/266 + 27/123 = 0.68 < 1.0$$
 OK

Hence adopt 250UB72.9 in grade 300 steel

# **Example 8.3.2 Biaxial Bending Member Capacity**

A simply supported beam with a span of 9m is loaded by a central concentrated live load Q of 56 kN. The concentrated load acts at the shear centre and is forming a 10° angle with the beam's major principal plane. The beam is fully restrained against lateral displacement and twist rotation only at the supports, and is free to rotate in plan (i.e. No restraint against lateral rotation exists at the supports). Design a suitable UB section of Grade 300 steel.



#### Solution

Factored design load 
$$P^* = 1.5Q = 1.5 \times 56 = 84 \text{ kN}$$
  
Design moment  $M^* = 84 \times 9 / 4 = 189 \text{ kNm}$   
 $M^*_x = 189 \times \cos 10^\circ = 186.13 \text{ kNm}$   
 $M^*_y = 189 \times \sin 10^\circ = 32.82 \text{ kNm}$ 

To design this beam a trial section must first be selected. To do this, choose a section that will satisfy the strength requirements for major and minor axis bending separately.

Assume a section that is compact for bending about both principal axes with

$$f_y = 300 \text{ MPa}$$
$$\alpha_m = 1.35$$

AS4100 Table 5.6.1

Guess  $\alpha_s = 0.28$ 

$$S_x \ge M_x^* / (\phi \alpha_m \alpha_s f_y)$$

$$S_x \ge 186.13 \times 10^6 / (0.9 \times 1.35 \times 0.28 \times 300) = 1823.73 \times 10^3 \text{ mm}^3$$

$$S_v \ge M_v^* / \phi f_v$$

$$S_v \ge 32.82 \text{ x} 10^6 / (0.9 \text{ x} 300) = 121.56 \text{ x} 10^3 \text{ mm}^3$$

#### Try 530 UB 92.4

Once a trial section is chosen, the self-weight bending must be added to the major axis moment caused by live loads.

$$M_{x}^{*} = 186.13 + 1.25 \text{ x} (92.4 \text{ x} 9.8 \text{x} 10^{-3} \text{ x} 9^{2}/8) = 197.59 \text{ kNm}$$

$$M_{\rm v}^* = 32.82 \, \rm kNm$$

#### STRENGTH DESIGN

Member Capacities

(i) Major axis bending capacity

$$f_{\rm vf} = 300 \, \text{MPa}$$

$$f_{vw} = 320 \text{ MPa}$$

Yield stress  $f_v = 300 \text{ MPa}$ 

Flange slenderness:

$$\lambda_{\text{ef}} = ((b_{\text{f}} - t_{\text{w}}) / 2t_{f}) \sqrt{(f_{\text{v}} / 250)} = 6.4 \text{ x} \sqrt{(300 / 250)} = 6.98$$

$$\lambda_{ep} = 9$$
  $\lambda_{ey} = 16$   $\lambda_{ef} / \lambda_{ey} = 6.98 / 16 = 0.44$ 

AS4100 Table 5.2

Web slenderness:

$$\lambda_{\text{ew}} = (d_1/t_{\text{w}})\sqrt{(f_{\text{y}}/250)} = 49.2 \sqrt{(300/250)} = 53.91$$

$$\lambda_{\rm ep} = 82$$
  $\lambda_{\rm ey} = 115$   
 $\lambda_{\rm ew} / \lambda_{\rm ey} = 53.91 / 115 = 0.47$ 

AS4100 Table 5.2

Since the web has the higher value of  $\lambda_e / \lambda_{ey}$  it is the critical element in the section.

From Clause 5.2.2 of AS 4100 the section slenderness and slenderness limits are the web values, i.e.

$$\lambda_s = 53.91$$
  $\lambda_{sp} = 82$   $\lambda_{sy} = 115$ 

Now 
$$\lambda_s = 53.91 < \lambda_{sp} = 82$$
: The section is COMPACT

 $Z_{\rm ex}$  is the lesser of  $S_{\rm x}$  and  $1.5Z_{\rm x}$ 

$$S_x = 2370 \times 10^3 \text{ mm}^3$$

$$1.5Z_x = 1.5 \times 2080 \times 10^3 = 3120 \times 10^3 \text{ mm}^3$$

$$\therefore Z_{ex} = 2370 \times 10^3 \text{ mm}^3$$

$$\phi M_{sx} = 0.9 \text{ Z}_{ex} f_v = [0.9 \text{ x } 2370 \text{ x } 10^3 \text{ x } 300] \text{ x } 10^{-6} = 639.9 \text{ kNm} > M_x^* = 197.59 \text{ kNm}$$
 OK

**Note:** A section with a shape factor  $\zeta$  (ratio of plastic moment to the moment corresponding to the onset of yielding at the extreme fiber  $\zeta = M_p/M_y$ ) greater than 1.5 may have significant inelastic deformation under service loads if it was permitted to reach  $M_p$  at factored loads. The limit of 1.5Z will control the amount of inelastic deformation for sections with shape factors greater than 1.5. If 1.5Z<sub>x</sub> is less than S<sub>x</sub> in sections with  $f_{yw} < f_{yf}$ , the flange yield stress is used in determining the section moment capacity  $M_{sx}$  not the web yield stress; because the onset of yielding at the extreme fiber will occur when the flange yield stress is reached. (i.e.  $M_{sx} = 1.5M_y = 1.5Zf_{yf}$  for a compact section with  $\zeta > 1.5$ ).

We have one segment for bending in this case with length  $L=9\mathrm{m}$  and a restraint arrangement (FF).

$$L_e = k_t k_l k_r L$$
 AS4100 Cl. 5.6.3(1),(2),(3)

$$L_e = 1 \times 1 \times 1 \times 9 = 9 \text{ m}$$

Hence using a spread sheet program

$$M_0 = 241.73 \text{ kNm}$$
 AS4100 Cl.5.6.1.1(3)  
 $\alpha_s = 0.283$  AS4100 Cl.5.6.1.1(2)

$$\phi M_{\text{bx}} = \alpha_{\text{m}} \alpha_{\text{s}} \phi M_{\text{sx}} \le \phi M_{\text{sx}}$$

$$AS4100 Cl.5.6.1.1$$

$$\phi M_{\rm bx} = 1.35 \text{ x } 0.283 \text{ x } 639.9 = 244.5 \text{ kNm} < \phi M_{\rm sx} = 639.9 \text{ kNm}$$

$$\phi M_{\rm bx} = 244.5 \text{ kNm} > M_x^* = 197.59 \text{ kNm}$$
 OK

Note: the same result may be obtained using the design capacity tables [1].

#### (ii) Minor axis bending capacity

I-section beams, which are compact for major axis bending, are always compact when bent about the minor axis because their plasticity slenderness limits are unchanged. Therefore the trial section 530UB92.4 is compact for minor axis bending.

$$\phi M_{\rm sv} = 0.9 Z_{\rm ev} f_{\rm vf}$$
 AS4100 Cl.5.2.1

 $Z_{ey}$  is the lesser of  $S_y$  and  $1.5Z_y$ 

$$S_v = 355 \times 10^3 \text{ mm}^3$$

$$1.5Z_v = 1.5 \times 228 \times 10^3 = 342 \times 10^3 \text{ mm}^3$$

$$\therefore Z_{ev} = 342 \times 10^3 \text{ mm}^3$$

$$\phi M_{\text{sy}} = 0.9 \text{ Z}_{\text{ey}} f_{\text{yf}} = [0.9 \text{ x } 342 \text{ x } 10^3 \text{ x } 300] \text{ x } 10^{-6} = 92.34 \text{ kNm} > M_{\text{y}}^* = 32.82 \text{ kNm}$$
 OK

### (iii) Biaxial bending member capacity

$$(M_x^*/\phi M_{bx})^{1.4} + (M_y^*/\phi M_{sy})^{1.4} \le 1.0$$
?
$$AS4100 Cl.8.4.5$$

$$(197.59/244.5)^{1.4} + (32.82/92.34)^{1.4} = 0.98 < 1.0 OK$$

#### Web shear capacity

The web of the UB section is required to resist the shear associated with the loads causing major axis bending. The maximum design shear force the web has to withstand is,

$$V_y^* = 1.5 \text{ x } (56 \text{ x } \cos 10^{\circ}/2) + 1.25 \text{ x } (92.4 \text{ x } 9.8 \text{x} 10^{-3} \text{ x } 9/2) = 46.5 \text{ kN}$$

$$d_{\rm p}/t_{\rm w} = d_{\rm l}/t_{\rm w} = 49.2 < 82/\sqrt{(f_{yw}/250)} = 72.48$$
, and so  $V_{\rm v} = V_{\rm u} = V_{\rm w}$ 

$$V_{\rm w} = 0.6 f_{\rm yw} A_{\rm w} = 0.6 \text{x} 320 \text{x} 10.2 \text{x} 533 = 1043.83 \text{ kN}$$

$$\phi V_{\rm w} = 0.9 \text{ x } 1043.83 = 939.4 \text{ kN} > V_{\rm v}^* = 46.5 \text{ kN}$$
 OK

*Shear and bending interaction between*  $(V_y^* \& M_x^*)$ 

$$M_{x}^{*} \leq 0.75 \ \phi M_{sx}$$
?

$$M_{\rm x}^* = 197.59 < 0.75 \text{ x } 639.9 = 479.93 \text{ kNm}$$
 and so  $V_{\rm vm} = V_{\rm v} = V_{\rm w} = 1043.83 \text{ kN}$ 

$$\phi V_{\text{vm}} = \phi V_{\text{w}} = 0.9 \text{ x } 1043.83 = 939.4 \text{ kN} > V_{\text{y}}^* = 46.5 \text{ kN}$$
 OK

*Note:*  $V_y^*$  is the design shear force coincident with the maximum bending moment  $M_x^*$ .

#### Flanges shear capacity

The two flanges of the UB section are required to resist the shear associated with the loads causing minor axis bending. The maximum design shear force the two flanges have to withstand is given by:

$$V_{\rm x}^* = 1.5 \text{ x } (56 \text{x } \sin 10^{\circ}/2) = 7.3 \text{ kN}$$

Since we have a non-uniform shear stress distribution, Clause 5.11.3 of AS 4100 applies.

$$V_{\rm f} = 2V_{\rm u} / [0.9 + (f_{\rm vm}^*/f_{\rm va}^*)] \le V_{\rm u}$$
 AS4100 Cl.5.11.3

where  $V_{\rm u}$  is the nominal shear yield capacity of the two flanges calculated assuming uniform shear stress distribution.

 $f_{\text{vm}}^*, f_{\text{va}}^*$  = the maximum and average design shear stresses in the flanges respectively.

The shear stress distribution is parabolic in this case with the maximum shear stress being 1.5 the average stress.

#### Hence

$$V_{\rm f} = 2V_{\rm u} / [0.9 + 1.5] = 0.833 V_{\rm u} = 0.833 \times 0.6 \times f_{\rm yf} \times 2A_{\rm f}$$
  
 $V_{\rm f} = f_{\rm yf} \times A_{\rm f} = 300 \times 15.6 \times 209 = 978.12 \text{ kN}$   
 $\phi V_{\rm f} = 0.9 \times 978.12 = 880.31 \text{ kN} > V_{\rm x}^* = 7.3 \text{ kN}$  OK

Clearly shear and bending interaction need not to be checked in this case because  $\phi V_{\rm f} >> V_{\rm x}^*$ 

Note: the bearing yield capacity and the bearing buckling capacity of the UB web at the location of the concentrated load and at the support are not checked in this example. A complete design must include such a check.

#### SERVICEABILITY DESIGN

After designing the beam for strength, the designer must ensure that the maximum deflection under service live load is less than the deflection limit given in table B1 of AS 4100.

Hence adopt 530UB92.4 in Grade 300 steel.

#### Comment

Deflection is checked only for service live load because the designer often chooses to eliminate the self-weight deflection by providing a steel section that is cambered.

# **Example 8.3.3 Biaxial Bending and Axial Tension**

A cross section of a 200UC59.5 of Grade 300 steel has a major axis design bending moment  $M_y^* = 100$  kNm, a minor axis design bending moment  $M_y^* = 32$  kNm, and an axial tensile load  $N_y^* = 290$  kN. Check if the section has enough strength to support the applied loading.

OK

#### Solution

#### (i) General Linear Method

| $\phi N_{\rm t} = 2060 \text{ kN}$ $\phi M_{\rm sx} = 177 \text{ kNm}$ $\phi M_{\rm sy} = 80.6 \text{ kNm}$   | AISC Tables<br>AISC Tables<br>AISC Tables |
|---|---|
| $N^*/\phi N_t + {M_x}^*/\phi M_{sx} + {M_y}^*/\phi M_{sy} \le 1.0?$<br>290/2060 + 100/177 + 32/80.6 = 1.1 > 1.0 NG  | AS4100 Cl.8.3.4                           |
| (ii) Method for Compact Doubly Symmetric I-Sections   |   |
| 200UC59.5 section is compact  | AISC Tables                               |
| $\phi M_{\text{rx}} = 1.18 \ \phi M_{\text{sx}} \ [1 - N^*/\phi N_{\text{t}}] \le \phi M_{\text{sx}}$<br>$\phi M_{\text{rx}} = 1.18 \ \text{x} \ 177 \ \text{x} \ (1 - 290/2060) = 179.5 \ \text{kNm} > \phi M_{\text{sx}} = 177 \ \text{kNm}$<br>$\phi M_{\text{rx}} = 177 \ \text{kNm}$             | AS4100 Cl.8.3.2(a)                        |
| $\phi M_{\text{ry}} = 1.19 \ \phi M_{\text{sy}} \ [1 - (N^*/\phi N_{\text{t}})^2] \le \phi M_{\text{sy}}$<br>$\phi M_{\text{ry}} = 1.19 \ \mathbf{x} \ 80.6 \ \mathbf{x} \ [1 - (290/2060)^2] = 94 \ \text{kNm} > \phi M_{\text{sy}} = 80.6 \ \text{kNm}$<br>$\phi M_{\text{ry}} = 80.6 \ \text{kNm}$ | AS4100 Cl.8.3.3(a)                        |

# Comment

y = 1.54

 $\gamma = 1.4 + (N^*/\phi N_t) \le 2.0$ 

 $\gamma = 1.4 + 290/2060 = 1.54 < 2.0$ 

In this example, the conservative general linear method gives a significantly lesser capacity than that given by the power law method.

#### Example 8.3.4 Checking the In-Plane Member Capacity of a Beam Column

 $(M_x^*/\phi M_{rx})^{\gamma} + (M_y^*/\phi M_{ry})^{\gamma} = (100/177)^{1.54} + (32/80.6)^{1.54} = 0.66 < 1.0$ 

An second order elastic analysis shows that a 6m long 310UC118 beam column in Grade 300 steel is bent in reverse curvature with end moments  $M_{\rm x1}^*=200$  kNm and  $M_{\rm x2}^*=190$  kNm. It also shows that the beam column carries a factored design axial compressive load  $N^*=1600$  kN. Check the in-plane member capacity of the 310UC118 beam column for the following arrangements (a) the beam column is a braced member for major axis column buckling with an effective length  $L_{\rm ex}=6$  m. (b) the beam column is a sway member for major axis column buckling with an effective length  $L_{\rm ex}=12$  m.

#### Solution

(a) The beam column is a braced member for major axis column buckling with an effective length  $L_{ex} = 6$  m.

#### (i) General Linear Method

$$\begin{aligned} & \phi N_{\rm cx} = \phi \alpha_{\rm cx} \ k_{\rm f} A_{\rm n} f_{\rm y} \\ & \phi N_{\rm cx} = 0.9 \ {\rm x} \ 0.876 {\rm x} \ 1 \ {\rm x} \ 15000 \ {\rm x} \ 280 \ {\rm x} \ 10^{-3} \\ & \phi N_{\rm cx} = 3311.3 \ {\rm kN} > N^* = 1600 \ {\rm kN} \\ & \phi M_{\rm sx} = 0.9 \ {\rm Z_{\rm ex}} f_{\rm y} \\ & \phi M_{\rm sx} = [0.9 \ {\rm x} \ 1960 \ {\rm x} \ 10^3 \ {\rm x} \ 280] \ {\rm x} \ 10^{-6} = 494 \ {\rm kNm} > M_{\rm x}^* = 200 \ {\rm kNm} \\ & \phi M_{\rm ix} = \phi M_{\rm sx} \left[ 1 - \frac{N*}{\phi N_{\rm cx}} \right] = 494 \ {\rm x} \ (1 - 1600/3311.3) = 255.3 \ {\rm kNm} > M_{\rm x}^* = 200 \ {\rm kNm} \\ & OK \end{aligned}$$

#### (ii) Method for compact doubly symmetric I-sections

$$\begin{split} \beta_{mx} &= 190/200 = 0.95 \\ \phi M_{rx} &= 1.18 \ \phi M_{sx} \ [1 - N^*/\phi N_s] \leq \phi M_{sx} \\ \phi M_{rx} &= 1.18 \ x \ 494 \ x \ (1 - 1600/3780) = 336.2 \ kNm < \phi M_{sx} = 494 \ kNm \\ \phi M_{rx} &= 336.2 \ kNm \\ \phi M_{ix} &= \phi M_{sx} \Big( \Big[ 1 - ((1 + \beta_m)/2)^3 \Big] \ (1 - N^*/\phi N_{cx}) + 1.18 \ x \ ((1 + \beta_m)/2)^3 \ x \ \sqrt{(1 - N^*/\phi N_{cx})} \Big) \leq \phi M_r \\ \phi M_{ix} &= 494 \Big( \Big[ 1 - ((1 + 0.95)/2)^3 \Big] \ (1 - 1600/3311.3) + 1.18 \ x \ ((1 + 0.95)/2)^3 \ x \\ \sqrt{(1 - 1600/3311.3)} \Big) &= 407 \ kNm > \phi M_{rx} = 336.2 \ kNm \\ \phi M_{ix} &= 336.2 \ kNm > M_x^* = 200 \ kNm \quad OK \end{split}$$

(b) The beam column is a sway member for major axis column buckling with an effective length  $L_{ex} = 12$  m.

Two effective lengths need to be used under Clause 8.4.2.2 of AS 4100. For combined actions the effective length for major axis buckling is the actual column length (i.e. taking  $k_{\rm ex} = 1$ ). The column also needs to be checked under axial load alone using the actual effective length (i.e.  $L_{\rm ex} = 12$  m).

# Under axial load alone

From the AISC Design Capacity Tables [2] for 
$$L_{ex} = 12$$
 m,  $\phi N_{cx} = 2200$  kN  $\phi N_{cx} = 2200$  kN  $> N^* = 1600$  kN  $OK$ 

#### Under combined bending and axial load

The solution is the same as in part (a).

#### Comment

In this example  $\phi M_{ix}$  calculated using the conservative general method is much less than  $\phi M_{ix}$  calculated by the method for compact doubly symmetric I-sections, thus it can be seen that the general method will give an uneconomical design in this case.

#### **Example 8.3.5 Checking the In-Plane Member Capacity (Plastic Analysis)**

For the haunched 460UB74/310UB46 pinned base portal frame shown in Fig.8.5 below, check the in-plane member capacity of the columns for the load combination 1.2G + LW + ISLW. The loads shown are the factored loads for strength design. Use plastic analysis to determine the design action effects based on the fact that the haunch was proportioned to remain elastic at plastic collapse.

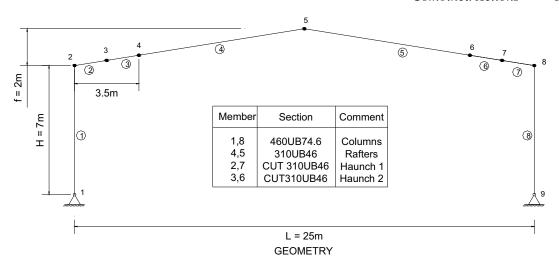


Figure 8.5 Portal Frame with 3.27m Haunches

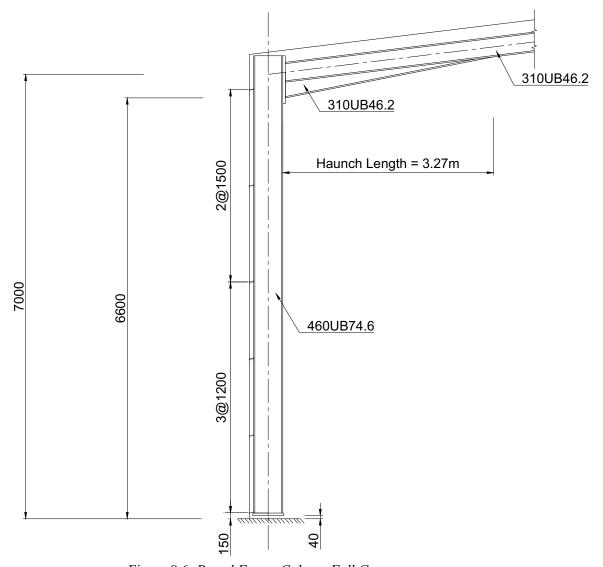
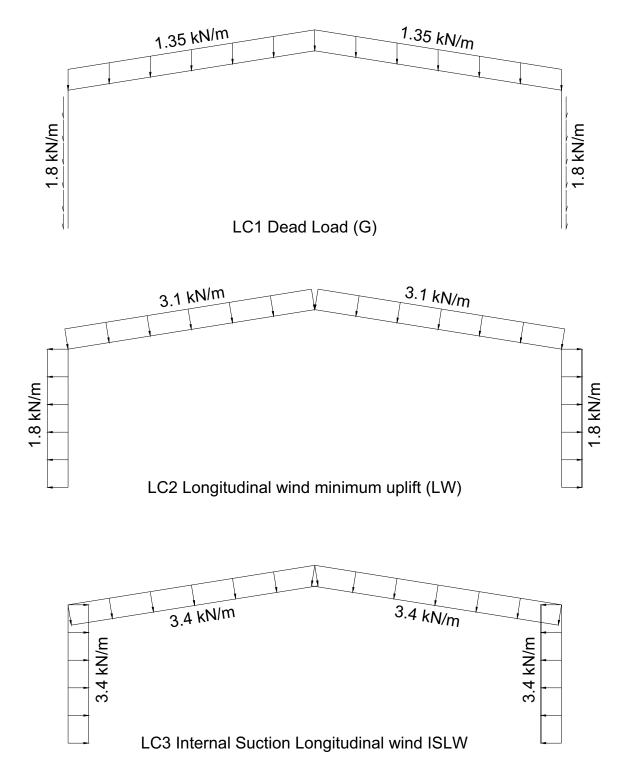
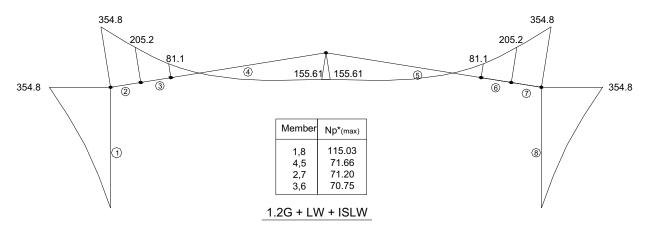
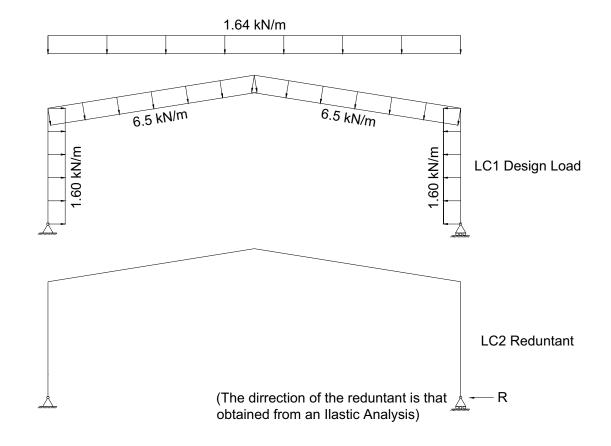


Figure 8.6 Portal Frame Column Full Geometry





First order plastic analysis BMD(kNm)



#### Solution

For the given load combination the location of the plastic hinges at collapse is known and therefore the direct mechanism method can be used.

$$w_1^* = \frac{1.2 \times 1.35}{\cos \alpha} = 1.64 \text{ kN/m (on plan projection)}$$

$$w_2^* = 3.1 + 3.4 = 6.5 \text{ kN/m}$$

$$w_3^* = 3.4 - 1.8 = 1.6 \text{ kN/m}$$

$$S_R = \frac{M_{sx(Column)}}{M_{sx(Rafter)}} = \frac{1660 \times 10^3 \times 300}{729 \times 10^3 \times 300}$$

$$S_R = 2.28$$

$$M_p^* \times \left[ S_R \left( 1 + \frac{f}{H} \right) + 1 \right] = \frac{w_1 \times L^2}{8} + \frac{w_2 \times L^2}{8} - \frac{w_2 \times f^2}{2} - \frac{w^3 \times f \times H}{2}$$

$$M_p^* \times \left[ 2.28 \times \left( 1 + \frac{2}{7} \right) + 1 \right] = \frac{1.64 \times 25^2}{8} + \frac{6.5 \times 25^2}{8} - \frac{6.5 \times 2^2}{2} - \frac{1.6 \times 2 \times 7}{2}$$

$$M_p^* = 155.61 \text{ kNm}$$

$$S_R M_p^* = 2.28 \times 155.61 = 354.8 \text{ kNm}$$

The same result can be obtained directly using the iterative mechanism method because the location of the plastic hinges is known under the action of a symmetrical loading such as that of this example. To use this method remove enough redundants from the frame to make it statically determinate. In this case there is only one redundant - the horizontal reaction at either of the two supports. Replace the hinge at the left or right support by a roller and apply a force R. Solve the equations of equilibrium for plastic hinges in the columns at the knees and in the rafter at the ridge to find the required plastic moment capacity and the value of the redundant. Combine the load case containing the redundant with that containing the design load and run a first order elastic analysis using SpaceGass [3] on the statically determinate frame to obtain the plastic bending moment and the axial forces in the columns and rafters.

The equations of equilibrium for plastic hinges at knees and ridge are  $2.28 M_p^* = 39.2 + 7 R$  $M_p^* = 561.39 - 9 R$ 

Hence

$$M_{\rm p}^*$$
 = 155.61 kNm, R = 45.086 kN  
2.28  $M_{\rm p}^*$  = 354.8 kNm

# Second Order Effects

Using the column and rafter axial forces obtained from a non-linear elastic analysis  $\lambda_c = 6.371$  from SpaceGass [3]

$$\delta_P = \frac{0.9}{1 - \frac{1}{6.371}} = 1.068$$
 AS4100 Cl. 4.5.4

$$\frac{Design \ Load \ Effects}{M^* = 1.068 \ x \ 354.8} = 378.93 \ kNm$$

$$N^* = 1.068 \ x \ 115.03 = 122.85 \ kN$$

# **Bending Capacity**

$$\overline{\phi M_{\rm sx}} = 0.9 \text{ x } 1660 \text{ x } 10^3 \text{ x } 300 = 448.2 \text{ kNm}$$

AS4100 Cl.5.2.1

## Compression Capacity

$$\phi N_s = 0.9 \times 0.948 \times 9520 \times 300$$

$$= 2436.74 \text{ kN}$$

AS4100 Cl.6.3.3

#### Check Reduced Plastic Moment Capacity

$$\phi M_{prx} = 1.18 \times \phi M_{sx} \times \left[ 1 - \frac{N^*}{\phi N_s} \right] \le \phi M_{sx}$$
 AS4100 Cl.8.4.3.4 
$$\phi M_{prx} = 1.18 \times 448.2 \times \left[ 1 - \frac{122.85}{2436.74} \right] = 502.21 \text{ kNm} > \phi M_{sx} = 448.2 \text{ kNm}$$

$$\phi M_{prx} = 448.2 \text{ kNm} > M^* = 378.93 \text{ kNm} OK$$

# Check Member Slenderness

$$\frac{N^*}{\phi N_s} = \frac{122.85}{2436.74} = 0.05 < 0.15$$
AS4100 Cl.8.4.3.2

$$N_{oL} = \frac{\pi^2 \times 2 \times 10^5 \times 335 \times 10^6}{7000^2} = 13495.2 \text{ kN}$$
 AS4100 Cl.8.4.3.4

$$\beta_m = 0$$

$$N_s = \frac{2436.74}{0.9} = 2707.5$$

$$\left[ \frac{0.6 + 0.4\beta_m}{\sqrt{\frac{N_s}{N_{ol}}}} \right]^2 = \left[ \frac{0.6}{\sqrt{\frac{2707.5}{13495.2}}} \right]^2 = 1.79 > \frac{N*}{\phi N_s} = 0.05 \quad OK \qquad AS4100 Cl.8.4.3.2$$

# Check Web Slenderness

$$\frac{d_1}{t_w} \sqrt{\frac{f_y}{250}} = \frac{457 - 2 \times 14.5}{9.1} \times \sqrt{\frac{300}{250}} = 51.5$$
AS4100 Cl.8.4.3.3

Hence

$$0.6 - \left[ \frac{d_1}{t_w} \times \frac{\sqrt{f_y/250}}{137} \right] = 0.6 - \frac{51.5}{137} = 0.22 > \frac{N^*}{\phi N_s} = 0.05 \quad OK$$

# Check Lateral Restraint Requirement

In plastic analysis the frame is only permitted to fail in the plane of bending and therefore any out of plane failure due to lateral torsional buckling must be prevented. To achieve this, any segment containing a plastic hinge at plastic collapse must have full lateral restraint. For segments not containing a plastic hinge full lateral restraint is not required and the segment can be designed as if an elastic analysis had been performed.

# Check Upper Segment

Full lateral restraint is achieved if

$$\alpha_m \alpha_s \ge 1.0$$

$$L_e = k_t k_l k_r L$$

 $k_t = 1$ 

 $k_l = 1$ 

 $k_r = 1$ 

$$L = 6600 - (150 + 3 \times 1200 + 1500) = 1350 \text{ mm}$$

$$L_e = 1350 \text{ mm}$$

Ratio of segment end moments  $\beta_m = -\frac{6600 - 1350}{6600} = -0.79$ 

$$\alpha_m = 1.75 - 1.05 \times 0.79 + 0.3 \times 0.79^2 = 1.11$$

AS4100 Table 5.6.2

AS4100 Cl. 5.6.3(1),(2),(3)

Hence using a spread sheet program,

 $\alpha_{s} = 0.968$ 

$$\alpha_m \alpha_s = 1.11 \times 0.968 = 1.07 > 1.0 \ OK$$

#### Check Lower Segment

Maximum moment in the segment  $M^* = 1.068 \times 258.75 = 276.35 \text{ kNm}$ 

Segment length  $L = 1500 + 3 \times 1200 + 150 = 5250 \text{ mm}$ 

Restraint arrangement (FF)

$$L_e = k_t k_l k_r$$
 AS4100 Cl. 5.6.3(1),(2),(3)

 $k_t = 1$ 

 $k_{l} = 1$ 

$$k_r = 0.70$$

*Note*:  $k_r$  equal to 0.7 for this segment because the upper segment is fully restrained and provides lateral rotational restraint to the lower segment, while the base plate and the holding down bolts provide lateral rotational restraint at the lower end of this segment.

$$L_e = 1 \times 1 \times 0.70 \times 5250 = 3675 \text{ mm}$$

$$\alpha_{m} = 1.75$$

AS4100 Table 5.6.2

Hence using a spread sheet program,

$$\alpha_{s} = 0.666$$

$$\phi M_{bx} = 1.75 \times 0.666 \times 448.2 = 522.4 > \phi M_{sx} = 448.2 \text{ kNm}$$

Hence lower segment is fully restrained

$$\phi M_{bx} = 448.2 \text{ kNm} > M^* = 276.35 \text{ kNm}$$
 OK

NG

Note: if the lower segment is not fully restrained the limit state of lateral torsional buckling under combined bending and axial force outlined in clause 8.4.4 of AS4100 must be designed for.

# Example 8.3.6 Checking the Out -of- Plane Capacity of a Beam Column

For the 310UC118 beam column in example 8.3.4. Check the out of plane member capacity. The out-of-plane effective lengths for column and beam buckling are both  $L_{\rm e} = 6 {\rm m}$ .

#### Solution

#### (i) General Linear Method

$$\begin{array}{lll} \beta_{\rm mx} = 190/200 = 0.95 \\ \alpha_{\rm m} = 1.75 + 1.05 \times 0.95 + 0.3 \times 0.95^2 &= 3.02 > 2.5 \\ AS4100 \ Table \ 5.6.1 \\ \alpha_{\rm m} = 2.5 \\ L_e = k_t \, k_l \, k_r \, L \\ L_e = 1 \times 1 \times 1 \times 6 = 6 {\rm m} \\ \text{Hence using a spread sheet program} \\ M_o = 1087.3 \ {\rm kNm} \\ \alpha_{\rm s} = 0.764 \\ \phi M_{\rm bx} = \alpha_{\rm m} \alpha_{\rm s} \, \phi M_{\rm sx} \leq \phi M_{\rm sx} \\ \phi M_{\rm bx} = 2.5 \times 0.764 \times 494 = 943.54 \ {\rm kNm} > \phi M_{\rm sx} = 494 \ {\rm kNm} \\ \phi M_{\rm bx} = 494 \ {\rm kNm} \\ k_f = 1.0 \\ \lambda_{\rm ny} = (6 \times 10^3 / 77.5) \times \sqrt{(1)} \times \sqrt{(280/250)} = 81.93 \\ \alpha_{\rm b} = 0 \\ \alpha_{\rm cy} = 0.681 - (81.93-80) \times (0.681-0.645) / (85-80) \\ \alpha_{\rm cy} = 0.667 \\ \phi N_{\rm cy} = \phi \, \alpha_{\rm cy} \, k_f \, A_{\rm n} f_y \\ \phi N_{\rm cy} = 0.9 \times 0.667 \times 1 \times 15000 \times 280 \times 10^{-3} \\ \phi N_{\rm cy} = 2521.3 \ {\rm kN} \\ \phi M_{\rm ox} = \phi M_{\rm bx} \left[ 1 - \frac{N^*}{\phi N_{\rm cy}} \right] = 494 \times (1 - 1600/2521.3) = 180.5 \ {\rm kNm} < M^* = 200 \ {\rm kNm} \qquad NG \\ \end{array}$$

#### (ii) Method for compact doubly symmetric I-sections

$$\begin{split} N_{\text{oz}} &= \left[ \text{GJ} + (\pi^2 \text{EI}_\text{w} \, / \, l_z^2) \right] \, / \left[ (\text{I}_\text{x} + \text{I}_\text{y}) \, / \, \text{A} \right] \\ N_{\text{oz}} &= \underline{\left[ 80,000 \text{x} 1630 \text{x} 10^3 + (\pi^2 \text{x} 200,000 \text{x} 1980 \text{x} 10^9 / 6000^2) \right]} = 9762 \text{ kN} \\ &\qquad \qquad \left[ (277 \text{x} 10^6 + 90.2 \text{x} 10^6) \, / \, 15000 \right] \\ \phi M_{\text{bxo}} &= 0.764 \text{ x} \, 494 = 377.4 \text{ kNm} \\ 1 / \alpha_{\text{bc}} &= (1 - \beta_{\text{m}}) \, / \, 2 + ((1 + \beta_{\text{m}}) \, / \, 2)^3 \, (0.4 - 0.23 \, N^* / \phi N_{\text{cy}}) \\ 1 / \alpha_{\text{bc}} &= (1 - 0.95) / 2 + ((1 + 0.95) / 2)^3 \, (0.4 - 0.23 \, \text{x} 1600 / 2521.3) = 0.2605 \\ \alpha_{\text{bc}} &= 3.8393 \\ \phi M_{\text{ox}} &= \phi \alpha_{\text{bc}} M_{\text{bxo}} \sqrt{\left[ (1 - N^* / \phi N_{\text{cy}}) (1 - N^* / \phi N_{\text{oz}}) \right]} \leq \phi M_{\text{rx}} \\ \phi M_{\text{ox}} &= 3.8393 \text{x} 377.4 \, \sqrt{\left[ (1 - 1600 / 2521.3) (1 - 1600 / (0.9 \text{x} 9762) \right]} = 792 \text{ kNm} > \phi M_{rx} = 336.2 \text{ kNm} \\ \phi M_{\text{ox}} &= 336.2 \text{ kNm} > M^* = 200 \text{ kNm} \end{split}$$

#### **Comment**

 $\phi M_{\rm ox}$  calculated using the conservative general method is much less, than  $\phi M_{\rm ox}$  calculated by the method for compact doubly symmetric I-sections because of the high value of  $\beta_{\rm mx}$ , thus it can be seen that the use of the general method may lead to a significant economic disadvantage.

#### **Example 8.3.7 Beam Column in a Multi-Story Building**

A braced column in a multi-story building has an effective length  $L_{\rm ex} = L_{\rm ey} = 4$  m for column buckling and an effective length  $L_e = 4$  m for beam buckling. A second order elastic analysis shows that it carries a factored design axial compressive load  $N^* = 2000$  kN and a design moment  $M_x^* = 96$  kNm. Select a suitable UC or WC section in Grade 300 steel.

#### Solution

- 1. Select trial selection with axial compression capacity  $\phi N_{\rm cy} = say \ 1.5N^*$  to allow for bending. From AISC table 6-29[2] try 310UC 118.
- 2. Check moment capacity  $\phi M_{\rm ox}$  for out of plane buckling from AS4100 Section 8.4.4. or AISC design capacity tables [2] Section 8.3.1.2.

$$\phi M_{ox} = \phi M_{bx} \left[ 1 - \frac{N^*}{\phi N_{cy}} \right] = 440 \text{ x} (1 - 2000/3170) = 162.4 \text{ kNm} > M^* = 96 \text{ kNm} OK$$

Strong enough but 69% over designed. Try a lighter section.

Try next size down: 310UC96.8

$$\phi M_{ox} = \phi M_{bx} \left[ 1 - \frac{N^*}{\phi N_{cy}} \right] = 368 \text{ x} (1 - 2000/2750) = 100 \text{ kNm} > M^* = 96 \text{ kNm} \quad OK$$

3. Check moment capacity  $\phi M_{ix}$  for in plane buckling from AS 4100 Section 8.4.2.2.or AISC design capacity tables [1] Section 8.3.1.2.

$$\phi M_{ix} = \phi M_{sx} \left[ 1 - \frac{N*}{\phi N_{cx}} \right] = 422 \times (1 - 2000/3100) = 149.7 \text{ kNm} > M^* = 96 \text{ kNm} OK$$

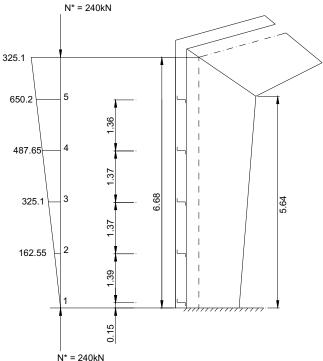
4. Check section reduced moment capacity  $\phi M_{rx}$  for axial force plus moment from AS 4100 Section 8.3.2 or AISC design capacity tables [2] Section 8.3.1.1.

$$\phi M_{rx} = \phi M_{sx} \left[ 1 - \frac{N*}{\phi N_s} \right] = 422 \text{ x} (1 - 2000/3340) = 169.3 \text{ kNm} > M^* = 96 \text{ kNm} OK$$

Hence adopt 310 UC 96.8 in Grade 300 steel

#### Example 8.3.8 Checking a Web Tapered Beam Column

The 6.68m long web-tapered member shown below is welded from Grade 300 as rolled plates and is used as a column in a pinned base portal frame building. A second order analysis shows that the column is subjected to a design axial compressive force  $N^* = 240$  kN and a design bending moment  $M_x^* = 770$  kNm at the knee joint. Check if the member is safe to carry the load.



#### Solution

Nominal Bending Capacity Mbx

Since the inside flange is in compression and there is no fly bracing, the segment length is the column length from the base plate to the underside of the haunch [4]. At the bottom of the column the base plate and the anchor bolts provide full lateral and twist restraint and hence the section is classified as fully restrained (F), the base plate and bolts also provides almost full lateral rotational restraint. At the top the wall bracing provides full lateral restraint and the rafter provides partial twist restraint, therefore the section at the top of the column is classified as partially restrained (P). No restraint against lateral rotation exists at the top of the column.

At Section 5, 
$$t_f = 12 \text{ mm}$$
,  $t_w = 10 \text{ mm}$   
 $f_{yf} = 310 \text{ MPa}$ ,  $f_{yw} = 310 \text{ MPa}$  AS4100 Table 2.1  
 $\therefore f_v = 310 \text{ MPa}$ 

Flange slenderness

$$\begin{array}{l} \lambda_{\rm ef} = ((b_{\rm f}\text{-}t_{\rm w}) \, / \, 2t_{\rm f} \, ) \sqrt{(f_{\rm y}/\, 250)} = [(200\text{-}10)/(2\text{x}12)] \, \text{x} \, \sqrt{(310/250)} = 8.82 \\ \lambda_{\rm ep} = 8 \quad , \, \lambda_{\rm ey} = 14 \\ \lambda_{\rm ef} \, / \lambda_{\rm ey} = \, 8.82 \, / \, 14 = 0.63 \\ Web \, slenderness \\ \lambda_{\rm ew} = (d_{\rm l}/t_{\rm w}) \sqrt{(f_{\rm y}/\, 250)} = ((1130\text{-}2\text{x}12)/10)\text{x} \sqrt{(310/250)} = 123.16 \\ \lambda_{\rm ep} = 82 \quad , \, \lambda_{\rm ey} = 115 \\ \lambda_{\rm ew} / \lambda_{\rm ev} = \, 123.16 \, / \, 115 = 1.07 \\ \end{array} \qquad AS4100 \, Table \, 5.2$$

Since the web has the higher value of  $\lambda_e / \lambda_{ey}$  it is the critical element in the section and the section slenderness and slenderness limits are the web values, i.e.

$$\lambda_s = 123.16, \ \lambda_{sp} = 82 \ , \ \lambda_{sy} = 115$$

Now  $\lambda_s > \lambda_{sy}$ : The section is SLENDER

$$Z_{\rm ex} = Z_{\rm x} \left( \lambda_{\rm sy} / \lambda_{\rm s} \right)$$

AS4100 Cl.5.2.5

$$Z_{x} = I_{x} / y_{\text{max}} = \frac{10 \times (1130-2 \times 12)^{3} / 12 + 2 \times [(200 \times 12^{3} / 12) + 200 \times 12 \times ((1130-12) / 2)^{2}]}{(1130 / 2)}$$

$$Z_{\rm x} = 4650.23 \text{ x } 10^3 \text{ mm}^3$$

$$Z_{\text{ex}} = 4650.23 \times 10^3 (115 / 123.16) = 4342.13 \times 10^3 \text{ mm}^3$$

$$M_{\rm sx} = Z_{\rm ex} \times f_{\rm y} = 4342.13 \times 10^3 \times 310 = 1346.1 \text{ kNm}$$

$$M^*/M_{\rm sx} = 650.2 / 1346.1 = 0.48$$

$$\phi M_{\rm sx} = 0.9 \text{ x } 1346.1 = 1211.5 \text{ kNm}$$

The same procedure is repeated for section 1, 2, 3, and 4 and the results are shown in the table below.

| Section No.        | 1     | 2      | 3     | 4      | 5      |
|--------------------|-------|--------|-------|--------|--------|
| Section depth (d)  | 320   | 523    | 725   | 928    | 1130   |
| $M_{\rm sx}$ (kNm) | 292.4 | 562.3  | 911.4 | 1184.2 | 1346.1 |
| $M^*$ (kNm)        | 0     | 162.55 | 325.1 | 487.65 | 650.2  |
| $M^*/M_{\rm sx}$   | 0     | 0.29   | 0.36  | 0.41   | 0.48   |

Hence, section (5), which is the deepest section, is the critical cross section in the segment.

#### For section (5)

$$I_y = 2 \times 12 \times 200^3 / 12 + (1130 - 2 \times 12) \times 10^3 / 12 = 16.09 \times 10^6 \text{ mm}^4$$

$$I_{\rm w} = I_{\rm y} \ d_{\rm f}^{\ 2} \ / \ 4 = 16.09 \ x \ 10^6 \ x \ (1130 - 12)^2 \ / \ 4 = 5028 \ x \ 10^9 \ mm^6$$

$$J = \Sigma (bt^3 / 3) + 2\alpha_1 D_1^4 - 0.42 t_f^4$$

where  $\alpha_1$  and  $D_1$  are given by:

$$\alpha_1 = -0.042 + 0.2204 (t_w/t_f) - 0.0725 (t_w/t_f)^2$$

$$D_1 = [t_f^2 + 0.25 t_w^2] / t_f$$

$$\alpha_1 = -0.042 + 0.2204 \text{ x } (10/12) - 0.0725 \text{ x } (10/12)^2 = 0.09132$$

$$D_1 = [12^2 + 0.25 \times 10^2] / 12 = 14.083$$

$$J = 2 \times 200 \times 12^{3} / 3 + (1130 - 2 \times 12) \times 10^{3} / 3 + 2 \times 0.09132 \times 14.083^{4} - 0.42 \times 12^{4}$$

$$J = 597.5 \times 10^3 \text{ mm}^4$$

Segment length L = 5640 mm

Restraint arrangement (FP)

$$L_e = k_t k_l k_r L$$
 AS4100 Cl. 5.6.3(1),(2),(3)

$$k_t = 1 + \left[ (d_1 / L) (t_f / 2t_w)^3 \right] / n_w = 1 + \left[ (1130 - 2 \times 12) / 5640 \right] \times (12 / (2 \times 10))^3 / 1 = 1.042$$

$$k_{l} = 1$$

$$k_r = 0.85$$

$$L_e = 1.042 \times 1 \times 0.85 \times 5640 = 4997 \text{ mm}$$

Hence using a spread sheet program,

$$M_0 = 752.6 \text{ kNm}$$

```
r_{\rm r} = 0.5 for tapered beam
                                                                                                    AS4100 Cl. 5.6.1.1 (b) (ii)
r_{\rm s} = (A_{\rm fm}/A_{\rm fc}) \times [0.6 + 0.4d_{\rm m}/d_{\rm c}]
                                                                                                    AS4100 Cl. 5.6.1.1 (b) (ii)
r_s = [(2x200x12) / (2x200x12)] \times [0.6 + 0.4 \times 320 / 1130] = 0.713
\alpha_{st} = 1 - [1.2 \text{ r}_r (1 - r_s)] = 1 - [1.2 \text{ x} 0.5 (1 - 0.713)] = 0.828
                                                                                                    AS4100 Cl. 5.6.1.1 (b) (ii)
M_{\text{oa}} = \alpha_{\text{st}} M_{\text{o}} = 0.828 \text{ x } 752.6 = 623.15 \text{ kNm}
                                                                                                    AS4100 Cl. 5.6.1.1 (b) (ii)
\alpha_s = 0.6 \times [\sqrt{(1346.1 / 623.15)^2 + 3)} - (1346.1 / 623.15)]
                                                                                                    AS4100 Eq.5.6.1.1 (2)
\alpha_{\rm s} = 0.3652
\alpha_{\rm m} = 1.75
                                                                                                    AS4100 Table 5.6.1.
\phi M_b = \alpha_m \mathbf{x} \ \alpha_s \mathbf{x} \ \phi M_{sx} = 1.75 \mathbf{x} \ 0.3652 \mathbf{x} \ 1211.5 = 774.3 \ kNm < \phi M_{sx} = 1211.5 \ kNm
\phi M_{\rm b} = 774.3 \text{ kNm}
```

# Major Axis Compression Capacity N<sub>cx</sub>

For buckling about the major axis the column is a sway member, as the relative displacement of one end to the other is not prevented. Under the combined actions rules of Clause 8.4.2.2 of AS 4100 [1] the designer need to consider two effective lengths for this sway member. For combined actions the effective length used to determine the in-plane major axis compression capacity  $N_{cx}$  is the actual column length (i.e. taking  $k_{ex} = 1$ ), because the effects of end restraints, which influence member buckling, are already taken into account by performing a second order analysis. AS 4100 Supplement [5] indicates that the previous procedure may be unsafe for some unbraced compression members, which have small bending moments. For this reason the design compression capacity determined using the actual effective length should satisfy Clause 6.1 of AS 4100. In portal frames without runway crane girders the columns are principally flexural members with low axial load and therefore the check under axial loads alone is unlikely to be critical. For this reason this check is omitted from this design example.

Since the column has varying cross section Clause 6.3.4 of AS 4100 applies. This clause states that Clause 6.3.3 of AS 4100 shall be used to determine  $N_{\rm cx}$  provided the following are satisfied-

- (i) The nominal section capacity  $N_s$  is the minimum value for all cross sections along the member.
- (ii) The modified member slenderness  $\lambda_n$  given in Clause 6.3.3 is replaced by the following:

$$\lambda_{\rm n} = 90 \sqrt{(N_{\rm s}/N_{\rm om})}$$

where  $N_{\rm om}$  is the elastic flexural buckling load of the member in axial compression determined using rational elastic buckling analysis.

Solutions for  $N_{om}$  are available in literature but are somewhat limited. For this reason  $N_{om}$  will be obtained by performing a buckling analysis using SpaceGass[3]. To perform such an analysis we first need to model the non-uniform member. This is done by dividing the member into an equal number of prismatic segments (say 2 to 4), the properties for each being the average for that segment. The effective length for combined actions is the actual column length therefore buckling analysis will be performed on an isolated member composed of two uniform segments with a pin support at one end and a roller at the other.

The elastic flexural buckling load [3] for this member is  $N_{\text{omx}} = 32343 \text{ kN}$ 

Flange slenderness

$$\begin{split} \lambda_{\rm ef} &= ((b_{\rm f}\text{-}t_{\rm w}) \ / \ 2t_{\rm f} \ ) \\ \sqrt{(f_{\rm y} \ / \ 250)} &= [(200\text{-}10) \ / (2\text{x}12)] \ \text{x} \ \sqrt{(310 \ / 250)} = 8.82 \\ \lambda_{\rm ey} &= 14 \end{split}$$
 
$$AS4100 \ Table \ 6.2.4$$
 
$$\lambda_{\rm ef} &= 8.82 \ < \lambda_{\rm ey} = \ 14 \end{split}$$

Web slenderness

$$\begin{array}{l} \lambda_{\rm ew} = (d_1/t_{\rm w})\sqrt{(f_{\rm y}\,/\,250)} = ((320\text{-}2\text{x}12)/10)\text{x}\sqrt{(310/250)} = 32.96 \\ \lambda_{\rm ey} = 35 \\ \lambda_{\rm ew} = 32.96 \,<\,\lambda_{\rm ey} = \,35 \\ k_{\rm f} = 1.0 \\ N_{\rm s} = k_{\rm f}\,A_{\rm n}\,f_{\rm y} = 1.0\,\,\text{x}\,\,7760\,\,\text{x}\,\,310 = 2405.6\,\,\text{kN} \\ \lambda_{\rm nx} = 90\,\,\text{x}\,\,\sqrt{(2405.6\,/\,32343)} = 24.55 \end{array} \qquad \begin{array}{l} AS4100\,\,Cl.6.2.2 \\ AS4100\,\,Cl.6.2.1 \\ AS4100\,\,Cl.6.3.4 \end{array}$$

Hence using a spread sheet program,

$$\alpha_{\rm c} = 0.9446$$
  
 $\phi N_{\rm cx} = 0.9$  x  $\alpha_{\rm c}$  x  $N_{\rm s} = 0.9$  x  $0.9446$  x  $2405.6 = 2045$  kN

# Minor Axis Compression Capacity New

The girts are not connected to the column centrelines and the effects of rotational restraint offered by the girts are uncertain. This uncertainty can be overlooked when designing portal frame buildings without runway crane girder and the effective length  $L_{ey}$  is then taken as the maximum girt spacing. However for the design of heavily loaded columns it would be wise and safer to disregard the restraint offered by the girts and to take the effective length for minor axis column buckling as the distance between fly braces, or the overall column length if there was no fly braces.

By inspection, the minor axis compression  $N_{\rm cy}$  is governed by the buckling strength of the column on the unbraced length between the first girt from the base plate and the second girt restraint. For this unbraced length the nominal member capacity  $N_{\rm cy}$  can be determined using one of the following approaches:

- (i) use the properties of the minimum cross section along the tapered member within the unbraced length under consideration.
- (ii) use Clause 6.3.4 of AS 4100.

In this example, the first approach will be used to determine the nominal member capacity  $N_{\text{cy.}}$ The depth of the section at the first girt from the bottom is d = 332 mm.

$$L_{\text{ey}} = 1390 \text{ mm}$$
  
 $N_{\text{s}} = k_{\text{f}} A_{\text{n}} f_{\text{y}} = 1.0 \text{ x } 7880 \text{ x } 310 = 2442.8 \text{ kN}$ 

Hence using a spread sheet program,

$$\alpha_{\rm cy}=0.8948$$
  $\phi N_{\rm cy}=0.9$  x  $\alpha_{\rm cy}$  x  $N_{\rm s}=0.9$  x 0.8948 x 2442.8 = 1967 kN

Check In- Plane Design Member Capacity  $\phi M_{ix}$ 

$$\phi M_{ix} = \phi M_{sx} \left[ 1 - \frac{N^*}{\phi N_{cx}} \right]$$

$$AS4100 Cl.8.4.2.2$$

 $\phi M_{ix} = 1211.5 \text{ x } (1 - 240 / 2045) = 1069.3 \text{ kNm} > M^* = 650.2 \text{ kNm}$  OK

Check Out-of-plane Design Member Capacity  $\phi M_{ox}$ 

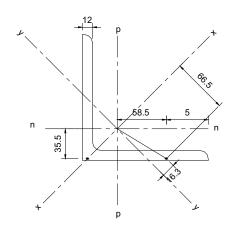
$$\phi M_{ox} = \phi M_{bx} \left[ 1 - \frac{N*}{\phi N_{cy}} \right]$$

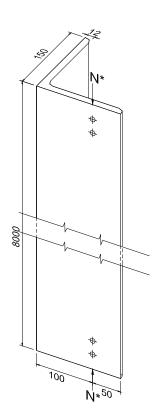
$$\phi M_{ox} = 774.3 \times (1 - 240 / 1967) = 679.8 \text{ kNm} > M^* = 650.2 \text{ kNm} \quad OK$$

Hence the member is safe.

# **Example 8.3.9 Eccentrically Loaded Single Angle in a Truss**

A 3m long pin-ended truss member in Grade 300 steel is made of 150x150x12 EA. The member is loaded through one leg by an axial compressive force  $N^* = 200$  kN. The angle is fully restrained at both ends against lateral deflection and twist rotation, but unrestrained against lateral rotation. Check if the member is safe.





# Solution

# Design Actions

 $N^* = 200 \text{ kN}$ 

 $M_{x}^{*} = 200 \times 0.0665 = 13.3 \text{ kNm}$   $M_{y}^{*} = 200 \times 0.0163 = 3.3 \text{ kNm}$ 

# Major Axis Member Bending Capacity $\phi M_{bx}$

Segment length L = 3000mm

Restraint arrangement (FF)

$$L_e = k_t \, k_l \, k_r \, L$$

 $k_t = 1$ 

 $k_l = 1$ 

 $k_r = 1$ 

AS4100 Cl. 5.6.3(1),(2),(3)

$$L_e = 1 \times 1 \times 1 \times 3000 = 3000 \text{ mm}$$

Hence using a spread sheet program

$$M_{\rm o} = 96.65 \text{ kNm}$$

$$\alpha_{\rm s} = 0.790$$

$$\phi M_{\rm bx} = \alpha_{\rm m} \alpha_{\rm s} \ \phi M_{\rm sx} \le \phi M_{\rm sx}$$
 AS4100 Cl.5.6.1.1

$$\phi M_{\rm bx} = 1.0 \text{ x } 0.790 \text{ x } 41.85 = 33.1 \text{ kNm} > \phi M_{\rm sx} = 41.85 \text{ kNm}$$

$$\phi M_{\rm bx} = 33.1 \text{ kNm}$$

# Minor Axis Bending Capacity $\phi M_{sy}$

$$\phi M_{sv} = 0.9 Z_{ev} f_{vf}$$
 AS4100 Cl.5.2.1

The minor axis moment will cause compression on the unsupported edge of the angle legs therefore the effective minor axis section modulus that correspond to load B will be used.

$$Z_{\text{ey}} = 72.3 \text{ x} 10^3 \text{ mm}^3$$
 AISC Tables

$$\phi M_{\text{sy}} = 0.9 \ Z_{\text{ey}} f_{\text{yf}} = [0.9 \ \text{x} \ 72.3 \ \text{x} 10^3 \ \text{x} \ 300] \ \text{x} \ 10^{-6} = 19.5 \ \text{kNm}$$

#### *Major Axis Compression Capacity* $\phi N_{cx}$

The member is pin connected at both ends and therefore the effective length for major and minor axis column buckling is equal to the member length.

$$k_{\rm f} = 1.0$$
 AISC Tables

$$\lambda_{\text{nx}} = (3x10^3 / 58.4) \text{ x } \sqrt{(1)} \text{ x} \sqrt{(300/250)} = 56.3$$

$$\alpha_b = 0.5$$
 AS4100 Table 6.3.3(1)

$$\alpha_{cx} = 0.778 - (56.3-55) \times (0.778-0.746) / (60-55)$$
 AS 4100 Table 6.3.3(3)

 $\alpha_{cx} = 0.770$ 

$$\phi N_{\rm cx} = \phi \alpha_{\rm cx} k_{\rm f} A_{\rm n} f_{\rm y}$$
 AS4100 Cl.6.3.3

$$\phi N_{\rm cx} = 0.9 \times 0.770 \times 1 \times 3480 \times 300 \times 10^{-3}$$

$$\phi N_{\rm cx} = 723.5 \, {\rm kN}$$

## Minor Axis Compression Capacity $\phi N_{cv}$

$$\lambda_{\text{ny}} = (3 \times 10^3 / 29.6) \times \sqrt{(1)} \times \sqrt{(300/250)} = 111$$

$$\alpha_b = 0.5$$
 AS4100 Table 6.3.3(1)

$$\alpha_{cy} = 0.431 - (111-110) \times (0.431-0.406) / (115-110)$$
 AS 4100 Table 6.3.3(3)

 $\alpha_{\rm cv} = 0.426$ 

$$\phi N_{\rm cy} = \phi \alpha_{\rm cy} k_{\rm f} A_{\rm n} f_{\rm y}$$
 AS4100 Cl.6.3.3

$$\phi N_{\rm cy} = 0.9 \times 0.426 \times 1 \times 3480 \times 300 \times 10^{-3}$$

$$\phi N_{\rm cv} = 400 \text{ kN}$$

#### Check Design Member Capacity $\phi M_{ix}$

$$\phi M_{ix} = \phi M_{sx} \left[ 1 - \frac{N*}{\phi N_{cx}} \right]$$
AS4100 Cl.8.4.2.2

$$\phi M_{ix} = 41.85 \text{ x} (1 - 200 / 723.1) = 30.3 \text{ kNm} > M_x^* = 13.3 \text{ kNm}$$
 OK

Check Design Member Capacity  $\phi M_{ox}$ 

$$\phi M_{ox} = \phi M_{bx} \left[ 1 - \frac{N^*}{\phi N_{cy}} \right]$$

$$\phi M_{ox} = 33.1 \times (1 - 200 / 400) = 16.55 \text{ kNm} > M_x^* = 13.3 \text{ kNm} \quad OK$$

Check Design Member Capacity  $\phi M_{iy}$ 

$$\phi M_{iy} = \phi M_{sy} \left[ 1 - \frac{N^*}{\phi N_{cy}} \right]$$

$$\phi M_{iy} = 19.5 \times (1 - 200 / 400) = 9.75 \text{ kNm} > M_y^* = 3.3 \text{ kNm} \quad OK$$

#### Biaxial Bending Member Capacity

$$(M_x^*/\phi M_{cx})^{1.4} + (M_y^*/\phi M_{iy})^{1.4} \le 1.0$$
?   
  $AS4100 Cl.8.4.5.1$   
 $(13.3 / 16.55)^{1.4} + (3.3 / 9.75)^{1.4} = 0.96 < 1.0$  OK

Hence the member is safe.

# 8.4 REFERENCES

- 1. Standards Australia (1998). AS 4100 Steel Structures.
- 2. Australian Institute of Steel Construction, (1994) *Design Capacity Tables for Structural Steel* (DCT) Second edition, Volume 1: Open Sections.
- 3. SpaceGass. www.spacegass.com
- 4. Bradford M.A., Kitipornchai S., Woolcock S.T., (1999) Design of Portal Frame Buildings Third edition (to AS 4100).
- 5. Standards Australia (1999). AS 4100 Steel Structures Commentary.

# 9 CONNECTIONS

#### 9.1 INTRODUCTION

Connections are covered in Section 9 of AS 4100[1]. These are the structural elements used for connecting different members of a framework; they are also the means through which forces and moments are transferred from the structural member through the connection and its components to other parts of the structure. Analysing this method of force transfer, and proportioning each of the components so that it has adequate capacity for the force that it is required to transmit, is how a connection is designed. To do this one must evaluate the design capacities of all the components of a connection; these include connectors such as bolts, rivets, welds and pins, connection components such as plates or cleats, and the affected elements of connected members such as webs and flanges.

The designer must also consider if the connection is economic to fabricate and erect. Connection detailing and the consequential cost of fabrication and erection have a profound effect on the final cost of the erected structure. Though it sometimes seems impractical to provide for repetition in the fabrication shop, the designer should nevertheless bear in mind that it can lead to substantial overall savings. A balance between function and economy must be achieved.

This chapter is concerned primarily with the design of a range of connections commonly used for industrial buildings in Australia. The design of the presented connections is in accordance with the AISC Connection Manual [2] with the exceptions of the design of extended end plates with more than eight bolts and the design of base plates subject to both axial force and bending moment, both of which follow the British practise BS 5950[3]. These have been included as the AISC Connection Manual [2] is restricted to the design of extended end plates with four bolts placed symmetrically about each of the flanges and no design model for base plates subject to an axial force and a bending moment is given. This chapter addresses these, presenting a design model for an extended end plate with more than eight bolts and also demonstrates the design of base plates subject to both axial force and bending moment using two approaches; the first is based on elastic analysis and the second based on plastic analysis.

# 9.2 DESIGN OF BOLTS

# 9.2.1 Bolts and bolting categories

The bolting categories identification system adopted in clause 9.3.1 of AS4100 is summarised in Table 9.2.1.

High strength structural bolt

Bolt Bearing type connection High strength structural bolt

Bolt Friction type connection

Bolts and Bolting CategoryMethod of tensioningMinimum tensile strength  $(f_{uf})$  MPaBolt NameSnug tight400Commercial boltSnug tight830High strength structural bolt

Table 9.2.1

Bolts and Bolting Category

830

830

## 9.2.2 Bolt strength limit states

Bolt

grade

4.6

8.8

8.8

8.8

## **9.2.2.1** *Bolt in shear*

**Bolting** 

category

4.6/S

8.8/S

8.8/TB

8.8/TF

Clause 9.3.2.1 of AS 4100 requires that for a single bolt with a single shear plane the design shear force  $V_{\rm f}^*$  shall be no greater than the design shear capacity  $\phi V_{\rm f}^-$ 

$$V_{\rm f}^* \leq \phi V_{\rm f}$$
 AS4100 Cl.9.3.2

#### where

 $\phi$  = capacity factor = 0.8

 $V_{\rm f}$  = nominal shear capacity of a bolt

The nominal shear capacity of a bolt  $V_f$  shall be calculated as follows:

 $V_{\rm f} = 0.62 f_{\rm uf} A_{\rm c}$  if the shear plane passes through the threads

**Full Tensioning** 

**Full Tensioning** 

=  $0.62 f_{\rm uf} A_{\rm o}$  if the shear plane passes through the unthreaded portion or "shank" of the bolt (see Table 9.2.2 for values of  $A_{\rm c}$  and  $A_{\rm o}$ )

 $f_{\rm uf}$  = ultimate tensile strength of the bolt, so ultimate shear strength  $\approx 0.62 f_{\rm uf}$ 

For a bolt in double or multiple shear, the above formula is simply expanded to allow for the number of shear planes  $n_x$  passing through the shank and the number  $n_n$  passing through the threads. A reduction factor  $k_r$  is included to allow for non-uniform loading of bolts in a long bolted connection.

$$V_{\rm f} = 0.62 f_{\rm uf} k_{\rm r} (n_{\rm n} A_{\rm c} + n_{\rm x} A_{\rm o})$$
 AS4100 Cl. 9.3.2.1

Connections affected by the requirement for lap splice connections and for which  $k_r$  may not be taken as 1.0 without calculation using Table 9.3.2.1 of AS4100 are:

- (1) Bracing cleat.
- (2) Bolted flange splice

The capacity of a bolt group is simply the sum of the capacities of each individual bolt

| Nom.             | Design   | $A_{\rm c}$ | $A_{\mathrm{o}}$ | $A_{\mathrm{s}}$ |
|------------------|----------|-------------|------------------|------------------|
| Dia. $d_{\rm f}$ | Notation | core        | shank            | tensile          |
| 12               | M12      | 76.2        | 113              | 84.3             |
| 16               | M16      | 144         | 201              | 157              |
| 20               | M20      | 225         | 314              | 245              |
| 24               | M24      | 324         | 452              | 353              |
| 30               | M30      | 519         | 706              | 561              |
| 36               | M36      | 759         | 1016             | 817              |

Table 9.2.2 Effective Areas of Bolts

# Table 9.2.3 Strength Limit State High Strength Structural Bolts 8.8/S,8.8/TB,8.8/TF Bolting Categories ( $f_{uf}$ = 830 MPa, $\phi$ = 0.8)

| Bolt<br>Size | Axial Tension $\phi N_{\rm tf}$ (kN) | Single Shear Threads included in shear plane $\phi V_{\rm fn}  (kN)$ | Single Shear Threads excluded from shear plane $\phi V_{\rm fx}$ (kN) |
|--------------|--------------------------------------|--|---|
| M16          | 104                                  | 59.3   | 82.7  |
| M20          | 163                                  | 92.6   | 129   |
| M24          | 234                                  | 133  | 186   |
| M30          | 373                                  | 214  | 291   |

# Table 9.2.4 Strength Limit State Commercial Bolts 4.6/S Bolting Category ( $f_{uf}$ = 400 MPa, $\phi$ = 0.8)

| Bolt<br>Size | Axial Tension $\phi N_{\rm tf}$ (kN) | Single Shear Threads included in shear plane $\phi V_{\rm fn}  (kN)$ | Single Shear Threads excluded from shear plane $\phi V_{\rm fx}$ (kN) |
|--------------|--------------------------------------|--|---|
| M12          | 27.0                                 | 15.1   | 22.4  |
| M16          | 50.2                                 | 28.6   | 39.9  |
| M20          | 78.4                                 | 44.6   | 62.3  |
| M24          | 113                                  | 64.3   | 89.7  |
| M30          | 180                                  | 103  | 140   |
| M36          | 261                                  | 151  | 202   |

## 9.2.2.2 Bolt in tension

A bolt subject to a design tension force  $N_{\rm tf}^*$  shall satisfy-

$$N_{\rm tf}^* \leq \phi N_{\rm tf}$$

where

 $\phi$  = capacity factor = 0.8

 $N_{\rm tf}$  = nominal tensile capacity of a bolt.

The nominal tension capacity  $(N_{\rm tf})$  of a bolt shall be calculated as follows:

$$N_{\rm tf} = A_{\rm s} f_{\rm uf}$$

where  $A_s$  is the tensile stress area of a bolt as specified in AS 1275.

## **9.2.2.3** *Bolt subject to combined shear and tension*

A bolt required to resist both a design shear force  $V_{\rm f}^*$  and a design tensile force  $N_{\rm tf}^*$  at the same time shall satisfy —

$$\left(\frac{V_f^*}{\phi V_f}\right)^2 + \left(\frac{N_{if}^*}{\phi N_{if}}\right)^2 \le 1.0$$

 $\phi$  = capacity factor = 0.8

 $V_{\rm f}$  = nominal shear capacity of a bolt.

 $N_{\rm tf}$  = nominal tensile capacity of a bolt.

## **9.2.2.4** *Ply in bearing*

A bolted connection in shear can also fail if the plates joined by the bolt group either fail in bearing (i.e. the high tensile bolt ploughs into the softer mild steel plate) or by tearing (i.e. shearing) at the end of the plate or between holes. A further possible failure mode is tensile failure of the plate between holes. These are shown below.

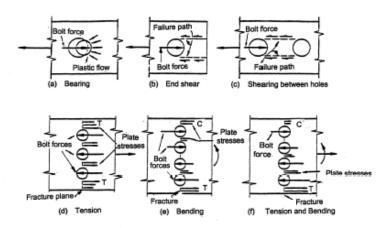


Figure 9.1 Bolted Plates in Bearing, Shear, Tension, or Bending

AS 4100 requires that

 $V_b^* \le \phi V_b$  where

 $V_b = 3.2 d_f t_p f_{up}$  for bearing, i.e. 3.2 times the bearing contact area times the uts (ultimate tensile strength) of the plate, or

 $V_{\rm b} = a_{\rm e} \, t_{\rm p} f_{\rm up}$  for tearing, i.e. half the area of the shear planes times the uts of the plate, or approx the area times the ultimate shear strength.

## **9.2.3** Bolt serviceability limit state for friction type connections

**9.2.3.1** Design for friction type connections (bolting category 8.8/TF) in which slip in the serviceability limit state is required to be limited, a bolt subjected only to a design shear force  $(V_{\rm sf}^*)$  in the plane of the interfaces shall satisfy-

$$V_{\rm sf}^* \leq \phi V_{\rm sf}$$

where

 $V_{\rm sf}$  = nominal shear capacity of a bolt, for friction type connection (slip resistance).

 $V_{\rm sf} = \mu \, n_{\rm ei} \, N_{\rm ti} \, k_{\rm h}$  AS4100 Cl. 9.3.3.1

where

 $\mu$  = slip factor.

 $n_{\rm ei}$  = number of effective interfaces.

 $N_{\rm ti}$  = minimum bolt tension at installation.

 $k_h$  = factor for different hole types, as specified in Clause 14.3.5.2 of AS4100

= 1.0 for standard holes

= 0.85 for short slotted holes

= 0.70 for long slotted holes

## **9.2.4** *Design details for bolts and pins*

To prevent tearing out of bolts and pins, minimum bolt centre to centre spacing is set at 2.5  $d_{\rm f}$  and edge distances range from 1.25 to 1.75  $d_{\rm f}$ , depending on how the edge is cut.

## 9.3 DESIGN OF WELDS

#### **9.3.1 Scope**

## **9.3.1.1** *Weld types*

AS 4100 lists the following weld types: complete penetration butt weld, incomplete penetration butt weld, fillet, slot, plug and compound weld. The most common types of welds are fillet and butt. Fillet welds are easy for the welder and therefore relatively cheap because they require no special edge preparation, but they are less strong because they act mainly in shear. Complete and incomplete penetration butt welds generally require edge preparation and are more expensive but are the strongest, complete penetration butt weld develops nearly the full strength of the plate being joined.

## 9.3.1.2 Weld quality

Welds can be specified as GP (general purpose), or SP (structural purpose) where strength is important. The capacity factor  $\phi$  is higher for SP (Table 3.4 of AS 4100), but they are more expensive because more rigorous inspection is required to ensure quality is maintained.

## 9.3.2 Complete and incomplete penetration butt welds

AS 4100 defines the complete and incomplete penetration butt welds as follows:

Complete penetration butt weld -a butt weld in which fusion exists between the weld and parent metal throughout the complete depth of the joint.

*Incomplete penetration butt weld* - a butt weld in which fusion exists over less than the complete depth of the joint.

A complete penetration SP butt weld is usually used in tension. Its cross section is the same as that of the weaker part being joined (e.g. a beam flange being joined to a column, or two plates being joined end to end). Provided the tensile strength of the weld metal (normally 410 MPa for E41XX rods or 480 MPa for E48XX rods) is no lower than that of the parts being joined, the design strength is the same as that of the weaker part being joined ( $\phi N_t = \phi A_g f_y$ ). The incomplete penetration butt weld is designed as a fillet weld with the appropriate design throat thickness, the design throat thickness is determined in accordance with Clause 9.7.2.3 of AS 4100.

#### 9.3.3 Fillet welds

## 9.3.3.1 Size of a fillet weld

A fillet weld is approximately triangular in section and its size  $t_w$  is specified by the leg length (Refer to Fig.9.7.3.1 in AS 4100), but its strength is governed by the throat thickness  $t_t$  which is ideally about  $0.7t_w$ . Clauses 9.7.3.2 and 9.7.3.3 of AS 4100 specify minimum and maximum sizes of fillet welds: there is no point in making them so small that they will break easily, nor so big that they are far stronger than the plates they join.

## 9.3.3.2 Capacity of a fillet weld

Regardless of the direction of loading of a fillet weld, its strength depends on the cross sectional area of the throat, which will generally be in shear. Thus its shear capacity (which is normally specified in N/mm or kN/mm run of weld) must satisfy

$$V_{\mathrm{w}}^* \leq \phi V_{\mathrm{w}}$$

where

 $V_{\rm w}^*$  is the design load on the weld and nominal capacity  $V_{\rm w} = 0.6 f_{\rm uw} t_{\rm t} k_{\rm r}$ 

 $f_{\rm uw}$  is the ultimate tensile strength of the weld metal, i.e. 410 MPa if using E41XX rods (equivalent to Grade 250 mild steel), or 480 MPa if using E48XX rods (equivalent to Grade 350 steel).

Thus  $0.6f_{uw}$  is approximately the ultimate shear strength of the weld.

 $k_{\rm r}$  is a reduction factor for non-uniform stress on a weld, which only applies if the weld is more than 1.7 m long.

Thus for a straight tensile or shear load, just divide the design load by the mm run of weld to get  $V_{\rm w}^*$  in kN/mm, and check that its capacity  $\phi V_{\rm w}$  in kN/mm is not less than  $V_{\rm w}^*$ .

## 9.4 WORKED EXAMPLES

## 9.4.1 Flexible Connections

## 9.4.1.1 Double Angle Cleat Connection

Design a bolted web cleat connection for a 410UB59.7 beam in grade 300 steel to carry a reaction of 190 kN (due to factored loads). The connection is to the flange of 250UC 89.5 column in grade 300 steel.

Note: this example uses the same angle component and dimensions adopted in AISC Connection Manual [2].

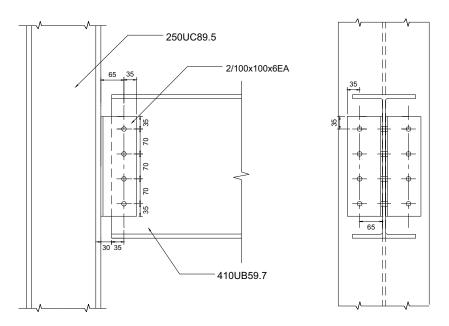


Figure 9.2 Bolted Web Cleats (Double Cleat)

#### Discussion

The connection at each end of the beam must be able to transmit the ultimate shear force of 190 kN to the column or other support. The connection forms part of the beam, (i.e. the point of support is the column to cleat interface). Web cleat connections are assumed to rely on the local distortion of the cleats to accommodate part of the end rotation of the beam. To obtain the flexibility required to meet the requirements of AS 4100 [1] for simple construction the designer should be aware of the following design consideration.

AISC Connection Manual [2] points out that the flexibility in this connection is provided by:

- (1) Using relatively thin cleats (8 and 10mm) so that local distortion allows end rotation of the supported member.
- (2) The use of snug-tightened bolts which allows the legs attached to the supporting member to slip horizontally and allows slip in the bolts attached to the supported member.

#### Solution

## (1) Connection to web of beam

Try 4 Bolts at 70 mm vertical pitch, 65mm from heel of cleats. The bolts are M20 bolts grade  $8.8 \ (f_{\rm uf} = 830 {\rm MPa})$  snug tightened. As the connection forms part of the beam the point of support is the column to cleat interface, therefore the bolt group connecting the cleats to the web of the beam is subjected to a vertical shear force plus a bending moment this bending moment will cause horizontal shear force on the bolts. The bending moment acts at the centroid of the bolt group and the horizontal loads on the bolts are proportional to their vertical distance from the centroid of the bolt group see figure below.

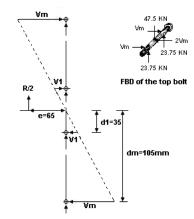


Figure 9.3 Web Bolt Group

Check Shear Capacity of the bolts connecting the cleats to the beam's web

End reaction at the column to cleat interface = 190 kN Hence reaction at each cleat = 190 / 2 = 95 kN Moment acting at the bolt group centroid at each cleat  $M_c^*$ 

$$M_{\rm c}^* = 95 \times 65 = 6175 \text{ kNmm}$$
  
 $M_{\rm c}^* = 2 V_{\rm xm}^* d_{\rm m} + 2 V_{\rm x1}^* d_{\rm l}$ 

From triangular symmetry:

$$\frac{V_{xm}^{*}}{d_{m}} = \frac{V_{x1}^{*}}{d_{1}}$$

$$M_{c}^{*} = \frac{V_{xm}^{*}}{d_{m}} \cdot \left(2d_{m}^{2} + 2d_{1}^{2}\right)$$

$$V_{xm}^{*} = \frac{M_{c}^{*}d_{m}}{\left(2d_{m}^{2} + 2d_{1}^{2}\right)}$$

$$V_{xm}^{*} = \frac{(6175 \times 105)}{2 \times \left(105^{2} + 35^{2}\right)}$$

$$V_{xm}^{*} = 26.46 \text{ kN}$$

Total Horizontal shear force on the top and bottom bolt due to moment due to eccentricity.  $2V_{ym}^{*} = 2 \times 26.46 = 52.92 \text{ kN}$ 

Note: the horizontal shear force is the force exerted by the beam's web on the bolt to balance the two horizontal forces acting on the bolt at each cleat.

Vertical shear force per bolt  $V_v^* = 190 / 4 = 47.5 \text{ kN}$ 

Resultant shear force acting on the top and bottom bolt  $V_f^*$  is given by:

$$V_{\rm f}^* = \sqrt{(52.92^2 + 47.5^2)} = 71.1 \text{ kN}$$

This Resultant shear force is acting on two shear planes. (i.e. The bolt is in double shear.) Shear capacity of bolt in double shear taking the conservative assumption that the two shear planes passes through the threads.

$$\phi V_{\text{fn}} = 2 \times 92.6 = 185.2 \text{ kN} > V_{\text{f}}^* = 71.1 \text{ kN } OK$$

Check Bearing and Tearing Capacity of the web at the bolt -holes:

(a) Bearing:

$$\phi V_b = 0.9 \ (3.2 \ d_f t_p f_{up})$$
  
 $\phi V_b = 0.9 \ x \ 3.2 \ x \ 20 \ x \ 7.8 \ x \ 440 \ x \ 10^{-3}$   
 $\phi V_b = 197.7 \ kN > V_f^* = 71.1 \ kN$  OK

(b) Tearing:

$$\phi V_{\rm b} = 0.9 \ a_{\rm e} \ t_{\rm p} f_{\rm up}$$

 $a_{\rm e}=$  minimum distance from the edge of a hole to the edge of a ply, measured in the direction of the component of a force, plus half the bolt diameter. The edge of ply shall be deemed to include the edge of an adjacent bolt-hole.

The bolt is exerting a force  $V_f^* = 71.1$  kN on the beam web at the bolt-hole, this force has two components:

(i) 
$$V_x^* = 2V_{xm}^* = 52.92 \text{ kN}$$
  
 $a_{ex} = 35 - r_{hole} + r_{bolt} = 35 - 11 + 10 = 34 \text{mm}$ 

(ii) 
$$V_y^* = 47.5 \text{ kN}$$

Tearing between bolt holes:

$$a_{\rm ey} = 70 - d_{\rm hole} + r_{\rm bolt} \Rightarrow 70 - 22 + 10 = 58 \text{ mm}$$

Tearing towards an edge:

Not relevant as web of supported beam is uncoped

As  $V_x^* > V_y^*$  and  $a_{ex}$  is less than  $a_{ey}$  we only need to check tearing capacity for  $a_{ex}$  and compare it with  $V_x^*$ .

$$\phi V_b = 0.9 \text{ x } 34 \text{ x } 7.6 \text{ x } 440 \text{ x } 10^{-3} = 102.3 \text{ kN} > V_x^* = 52.92 \text{ kN}$$
 OK

Check bearing and tearing in the angles:

Try 2 -100 x 100 x 6 angle cleats 280 mm long in grade 300 steel

(a) Bearing:

$$\phi V_{\rm b} = 0.9 \times 3.2 \times 20 \times 6 \times 440 \times 10^{-3} = 152.1 \text{ kN}$$

Vertical bolt force acting at the bolt hole in the angle leg connected to the beam's web = 47.5 / 2 = 23.75 kN

Maximum Horizontal bolt force acting at the top and bottom hole in the angle leg  $V_{vm}^* = 26.46 \text{ kN}$ 

Resultant Bolt force = 
$$\sqrt{(23.75^2 + 26.46^2)}$$
 = 35.6 kN <  $\phi V_b$  = 152.1 kN OK

## (b) Tearing:

The vertical bolt force can cause tearing towards an edge and it can also cause tearing between the bolt holes which is less critical in this case as  $a_e$  towards an edge is less than  $a_e$  between bolt holes.

$$\phi V_b = 0.9 \times 34 \times 6 \times 440 \times 10^{-3} = 80.78 \text{ kN} > \text{Vertical Bolt force} = 23.75 \text{ kN}$$
 OK

The horizontal bolt force can cause tearing only towards an edge.

$$\phi V_b = 0.9 \text{ x } 34 \text{ x } 6 \text{ x } 440 \text{ x } 10^{-3}$$
  
 $\phi V_b = 80.78 \text{ kN} > V_{vm}^* = 26.46 \text{ kN}$  OK

Use 4 x M20 bolts in 8.8/ S category to connect the cleats to the beam's web

#### (2) Connection to column flange

Design practice assumes that the column bolts support shear force only, while Graham [3] suggests that with the lower safety factors adopted in current design codes it would be prudent to allow for the effect of eccentricity. It is important to know that while the effect of eccentricity can be neglected in the case of double angle cleat it must be taken into account in cases where a single angle cleat is to be used. For simplicity the effect of eccentricity will be neglected in this book for the case of double angle cleat

Try 8 M20 grade 8.8/S bolts, 2 rows at 70 mm vertical pitch one row in each angle cleat, these two rows of bolts are 137.8 mm c/c apart. (Bolts are in single shear).

Vertical shear force per bolt  $V_f^* = 190 / 8 = 23.75 \text{ kN}$ 

Shear capacity of M20 Bolt grade 8.8 in single shear with threads included in the shear plane.  $\phi V_f = 92.6 \text{ kN} > V_f^* = 23.75 \text{ kN}$  OK

Check Bearing and tearing in the angles:

(a) Bearing:

$$\phi V_b = 152.1 \text{ kN} > V_f^* = 23.75 \text{ kN}$$
 OK

#### (b) Tearing:

The vertical bolt force can cause tearing towards an edge and it can also cause tearing between the bolt holes which is less critical in this case as  $a_{\rm ey}$  towards an edge is less than  $a_{\rm ey}$  between bolt holes.

$$a_{\text{ey}} = 34 \text{mm}$$
  
 $\phi V_b = 80.78 \text{ kN} > V_f^* = 23.75 \text{ kN}$  OK

*Check Bearing and tearing in the columns flange:* 

(b) Bearing:

$$\phi V_b = 0.9 \times 3.2 \times 20 \times 17.3 \times 440 \times 10^{-3}$$

$$\phi V_b = 438.5 \text{ kN} > V_f^* = 23.75 \text{ kN}$$
 OK

(b) Tearing:

The vertical bolt force can cause tearing between the bolt holes  $a_{\rm ey} = 70 - d_{\rm hole} + r_{\rm bolt} = 70 - 22 + 10 = 58 \text{ mm}$   $\phi V_{\rm b} = 0.9 \times 58 \times 17.3 \times 440 \times 10^{-3} = 397.4 \text{ kN} > V_{\rm f}^* = 23.75 \text{ kN}$  *OK* 

Use 8 x M20 bolts in 8.8/ S category to connect the cleats to the column's flange

Use 2 / (100 x 100 x 6 EA) angle cleats x 280 mm

## 9.4.1.2 Angle Seat Connection

Design an angle seat connection for 410UB59.7 beam in grade 300 steel, to carry a reaction of 160 kN (due to factored loads). The connection is to the flange of 250UC89.5 column in grade 300 steel.

Note: this example uses the same angle components and dimensions adopted in AISC Connection Manual [2].

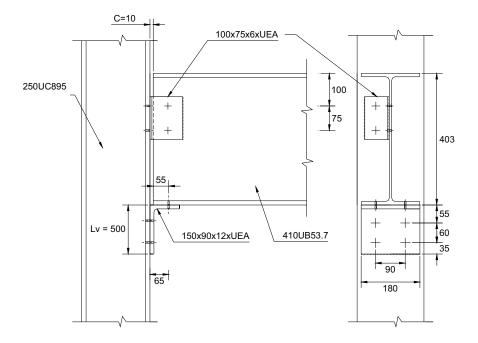


Figure 9.4 Bolted Seating Cleat

## Design Capacities of the Connection:

(a) Web crippling (yielding) capacity of the supported member

$$V_{\rm a} = \phi (1.25 \ b_{\rm bf} \ t_{\rm wb} f_{\rm yw})$$
 AS4100 Cl. 5.13.3

where 
$$b_{\rm bf} = b_{\rm s} + 2.5 \ t_{\rm fb}$$
,  $\phi = 0.9 \ V_{\rm a} = 0.9 \ x \ 1.25 \ (b_{\rm s} + 2.5 \ t_{\rm fb}) \ t_{\rm wb} f_{\rm yw} \ V_{\rm a} = k_1 \ (b_{\rm s} + k_4) \ b_s = \frac{V_a}{k_1} - k_4$ 

where:

$$k_1 = \phi(1.25 f_{\text{yw}} t_{\text{wb}}), k_4 = 2.5 t_{\text{fb}}$$

Web buckling capacity of the supported member

$$V_{\rm b} = \phi \, \alpha_{\rm c} \, (b_{\rm b} \, t_{\rm wb} \, f_{\rm vw})$$
 AS4100 Cl. 5.13.3

Bending capacity of outstanding leg of angle seat

The design plastic moment capacity of the outstanding leg of the angle seat is

$$\phi M_{\rm p} = \phi S f_{\rm ya}$$
  

$$\phi M_{\rm p} = \phi (L_{\rm a} t_{\rm a}^2 / 4) f_{\rm ya}$$
  

$$\phi M_{\rm p} = V_{\rm c} e_{\rm v}$$

where  $V_{\rm c}$  is the design end reaction at which a plastic hinge forms in the angle seat and  $e_v$  is the eccentricity of the design end reaction.

$$V_{\rm c} = \phi \left( \frac{f_{ya} L_a t_a^2}{4e_v} \right), \, \phi = 0.9$$

$$e_{\rm v} = c + b_{\rm s}/2 - (t_{\rm a} + r_{\rm a})$$

(c) Capacity of bolts in angle seat in shear

$$V_{\rm d} = n_{\rm b} \ (\phi V_{\rm df}), \ \phi = 0.8$$
  
where  $n_{\rm b} =$  number of bolts-usually 4  
 $\phi V_{\rm df} =$  design capacity of a single bolt in shear

The maximum capacity of the connection  $V_{\text{cap}}$  for any supported member is given when  $V_{\rm a} = V_{\rm c}$ 

$$b_{\rm s} = \frac{V_{cap}}{k_{\rm s}} - k_4 \dots (1)$$

$$V_{\text{cap}} = V_{\text{c}} = \phi \left( \frac{f_{ya} L_{a} t_{a}^{2}}{4 e_{v}} \right) = \frac{k_{2}}{e_{v}}$$

where 
$$k_2 = \phi \left( \frac{f_{ya} L_a t_a^2}{4} \right)$$

 $V_{\rm cap} = V_{\rm a} = k_1 (b_{\rm s} + k_4)$ 

$$e_{\rm v} = c + b_{\rm s}/2 - (t_{\rm a} + r_{\rm a})$$

Let 
$$k_3 = t_a + r_a - c$$

$$b_s = \frac{2k_2}{V_{cap}} + 2k_3 \dots (2)$$

Solving Equations (1) and (2) we get

$$V_{cap} = \frac{\left(k_5 + \sqrt{k_5^2 + 4k_6}\right)}{2} \dots (3)$$

where  $k_5 = k_1 k_4 + 2k_1 k_3$ ,  $k_6 = 2 k_1 k_2$ 

The stiff bearing length must satisfy the two constraints:

(i) 
$$c + b_s \le L_h$$
 thus if  $b_s > L_h - c$  use  $b_s = L_h - c$ 

(ii) 
$$c + b_s/2 \ge t_a + r_a$$
 thus if  $b_s > 2(t_a + r_a - c)$  use  $b_s = 2(t_a + r_a - c)$ 

*Note*: the AISC connection manual [2] points out that c is assumed to be 10mm nominal, but 14 mm is used for design purposes in order to provide for possible under-run on the beam length. AS 4100, clause 14.4.5 gives a maximum under run of 4mm for beams over 10m in length.

To check if the connection can carry the design load the following steps need to be followed:

- (1) Calculate the  $V_{\text{cap}}$  from Eq. (3)
- (2) Calculate  $b_s$  from Eq. (1) or (2)
- (3) Check if  $b_s$  satisfies the two constrains mentioned above
- (4) Calculate the web buckling capacity of the supported member  $V_b$
- (5) Check the shear capacity of bolts in angle seat
- (6) Check bearing and tearing in the angle seat
- (7) Check bearing and tearing in the column flange

#### Solution

Design Capacities of the Connection:

$$k_1 = 0.9 \times 1.25 \times 320 \times 7.8 \times 10^{-3} = 2.808 \text{ kN /mm}$$
 $k_2 = [(0.9 \times 300 \times 180 \times 12^2) / 4] \times 10^{-3} = 1749.6 \text{ kNmm}$ 
 $k_3 = 12.0 + 10 - 14 = 8 \text{ mm}$ 
 $k_4 = 2.5 \times 12.8 = 32 \text{ mm}$ 
 $k_5 = k_1 k_4 + 2k_1 k_3$ 
 $k_5 = 2.808 \times 32 + 2 \times 2.808 \times 8$ 
 $k_5 = 134.78 \text{ kN}$ 
 $k_6 = 2 k_1 k_2 = 2 \times 2.808 \times 1749.6 = 9573.81 \text{ (kN)}^2$ 

$$V_{cap} = \frac{\left(134.78 + \sqrt{134.78^2 + 4 \times 9573.81}\right)}{2} = 186.2 \text{ kN} > V^* = 160 \text{ kN}$$
 $C = \frac{186.2}{2.808} = 32 = 34.3 \text{ mm}$ 
 $C + b_s = 14 + 34.3 = 48.3 \text{ mm} < L_h = 90 \text{ mm}$ 
 $C + b_s / 2 = 14 + 34.3 / 2 = 31.2 > t_a + r_a = 12 + 10 = 22 \text{ mm}$ 
 $C = \frac{12.2 \times 10.2 \times 10.$ 

$$b_{b} = b_{bf} + b_{bw} = 66.3 + 380.4 / 2 = 256.5 \text{ mm}$$

$$A_{n} = t_{wb} b_{b} = 7.8 \times 256.5 = 2000.7 \text{ mm}^{2}$$
Slenderness Ratio =  $l_{ey} / r_{y} = 2.5 d_{1} / t_{wb} = 2.5 \times 380.4 / 7.8 = 121.92$ 

$$\lambda_{n} = \frac{l_{e}}{r} \sqrt{k_{f}} \sqrt{\frac{f_{yw}}{250}} = 121.92 \times \sqrt{1} \times \sqrt{\frac{320}{250}} = 137.94$$
for  $\lambda_{n} = 137.94$  and  $\alpha_{b} = 0.5, \alpha_{c} = 0.311$ 

$$V_{b} = \phi \alpha_{c} (b_{b} t_{wb} f_{yw}) = 0.9 \times 0.311 \times 2000.7 \times 320 = 179.2 \text{ kN} > V^{*} = 160 \text{ kN}$$

#### **Bolted angle seat:**

Try 4 x M20 bolts in 8.8/ S category in the vertical leg of angle seat. Bolts are in single shear.

OK

Shear force / Bolt (
$$V_f^*$$
) = 160 / 4 = 40 kN  
 $\phi V_f$  = 92.6 kN >  $V_f^*$  = 40 kN  $OK$   
or  $V_d$  = 4 x 92.6 = 370.4 kN>  $V_s^*$  = 160 kN  $OK$ 

Check bearing capacity of the angle seat at the bolt-holes:

$$\phi V_b = 0.9 (3.2 d_f t_p f_{up})$$
  
 $\phi V_b = 0.9 \times 3.2 \times 20 \times 12 \times 440 \times 10^{-3} = 304.13 \text{ kN}$   
 $\phi V_b = 304.13 \text{ kN} > V_f^* = 40 \text{ kN} OK$ 

Check tearing capacity of the angle seat at the bolt -holes:

$$\phi V_b = 0.9 \ a_e \ t_p f_{up}$$
  
 $a_e = 60 - d_{hole} + r_{bolt} = 60 - 22 + 10 = 48 \ \text{mm} \ (\text{tearing between bolt holes})$   
 $\phi V_b = 0.9 \ \text{x} \ 48 \ \text{x} \ 12 \ \text{x} \ 440 \ \text{x} \ 10^{-3} = 228.1 \ \text{kN} > V_f^* = 40 \ \text{kN}$  *OK*

Check bearing capacity of the column flange at the bolt-holes:

$$\phi V_b = 0.9 \times 3.2 \times 20 \times 17.3 \times 440 \times 10^{-3} = 438.45 \text{ kN} > V_f^* = 40 \text{ kN}$$
 OK

Check tearing capacity of the column flange at the bolt-holes:

$$a_e$$
 = 48 mm (tearing between bolt holes)  
 $\phi V_b$  = 0.9 x 48 x 17.3 x 440 x 10<sup>-3</sup> = 328.8 kN >  $V_f^*$  = 40 kN  $OK$ 

Use 4 x M20 bolts in 8.8/ S category in the vertical leg of angle seat.

Use 2 x M20 bolts in 8.8/ S category in the horizontal leg of angle seat.

Use 4 x M20 bolts in 8.8/ S category in the restraining cleat.

#### 9.4.1.3 Web Side Plate Connection

Design a web side plate connection for 410UB59.7 beam in grade 300, to carry a reaction of 250 kN (due to factored loads). The connection is to the flange of 250UC89.5 column in grade 300 steel.

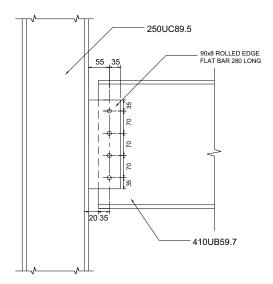


Figure 9.5 Web Side Plate

#### Discussion

The recommended design model adapted in this book follows the design model in the AISC Connection Manual [2] it treats the web side plate as an extension of the web of the supported member to which it is bolted. This is the behaviour if the connection is made to a flexible support. However if the support is stiff the web side plate cantilevers from the support and the bolt group becomes the hinge point.

The AISC Connection Manual [2] lists the design actions that the connection elements must be able to resist according to application as follows:

| SUPPORT  | WELD                      | PLATE      | BOLT GROUP                |
|----------|---------------------------|------------|---------------------------|
| Flexible | $\operatorname{V}^*$ only | $V^*,V^*e$ | $V^*,V^*e$                |
| Stiff    | $V^*,V^*e$                | $V^*,V^*e$ | $\operatorname{V}^*$ only |

As indicated in the AISC connection manual [8] in the worst case each connection element (weld, plate, bolts) must be capable of transmitting the design shear force  $(V^*)$  plus a design bending moment  $(V^*e)$  and therefore the recommended design model requires each connection element to transmit the design shear force  $(V^*)$  and a design bending moment  $(V^*e)$ .

#### Solution

## **Bolt Group**

Try 4 Bolts at 70 mm vertical pitch, 55mm from the column flange. The bolts are M20 bolts grade 8.8 ( $f_{\rm uf} = 830 {\rm MPa}$ ) snug tightened. As the connection forms part of the beam the point of support is the column to plate interface, therefore the bolt group connecting the plate to the web of the beam is subjected to a vertical shear force plus a bending moment this bending moment will cause horizontal shear force on the bolts. The bending moment acts at the centroid of the bolt group and the horizontal forces on the bolts are proportional to their vertical distance from the centroid of the bolt group.

End reaction at the weld = 250 kN Eccentricity of end reaction = 55 mm Moment acting at the bolt group centroid  $M_c^*$ 

$$M_{\rm c}^* = 250 \times 55 = 13750 \text{ kNmm}$$
  
 $M_{\rm c}^* = 2 V_{\rm xm}^* d_{\rm m} + 2 V_{\rm x1}^* d_{\rm 1}$ 

From triangular symmetry:

$$\frac{V_{xm}^{*}}{d_{m}} = \frac{V_{x1}^{*}}{d_{1}}$$

$$M_{c}^{*} = \frac{V_{xm}^{*}}{d_{m}} \cdot \left(2d_{m}^{2} + 2d_{1}^{2}\right)$$

$$V_{xm}^{*} = \frac{M_{c}^{*}d_{m}}{\left(2d_{m}^{2} + 2d_{1}^{2}\right)}$$

$$V_{xm}^{*} = \frac{\left(13750 \times 105\right)}{\left(2d_{m}^{2} + 2d_{1}^{2}\right)}$$

$$V_{xm}^* = \frac{(13750 \times 105)}{2 \times (105^2 + 35^2)}$$

$$V_{xm}^* = 58.93 \text{ kN}$$

Vertical shear force per bolt  $V_y^* = 250 / 4 = 62.5 \text{ kN}$ 

Resultant shear force acting on the top and bottom bolt  $V_f^*$  is given by:

$$V_{\rm f}^* = \sqrt{(58.93^2 + 62.5^2)} = 85.9 \text{ kN}$$
  
 $\phi V_{\rm fn} = 92.6 \text{ kN} > V_{\rm f}^* = 85.9 \text{ kN} \text{ OK}$ 

## Beam Web

Design bearing capacity:

$$\phi V_b = 0.9 (3.2 d_f t_p f_{up})$$
  
 $\phi V_b = 0.9 \times 3.2 \times 20 \times 7.8 \times 440 \times 10^{-3}$   
 $\phi V_b = 197.7 \text{ kN} > V_f^* = 85.9 \text{ kN}$  OK

Design tearing Capacity:

$$\phi V_b = 0.9 \ a_e \ t_p f_{up}$$

The bolt is exerting a force  $V_f^* = 85.9$  kN on the beam web at the bolt-hole, this force has two components:

(i) 
$$V_{\text{fx}}^* = V_{\text{xm}}^* = 58.93 \text{ kN}$$
  
 $a_{\text{ex}} = 35 - r_{\text{hole}} + r_{\text{bolt}} = 35 - 11 + 10 = 34 \text{mm}$   
 $\phi V_{\text{b}} = 0.9 \times 34 \times 7.6 \times 440 \times 10^{-3} = 102.3 \text{ kN} > V_{\text{fx}}^* = 58.93 \text{ kN}$  *OK*

(ii) 
$$V_{\rm fy}^* = 62.5 \text{ kN}$$

Tearing between bolt holes:

$$a_{\text{ey}} = 70 - d_{\text{hole}} + r_{\text{bolt}} \Rightarrow 70 - 22 + 10 = 58 \text{ mm}$$
  
 $\phi V_b = 0.9 \times 58 \times 7.6 \times 440 \times 10^{-3} = 174.6 \text{ kN} > V_{\text{fy}}^* = 62.5 \text{ kN}$  OK

Tearing towards an edge:

Not relevant as web of supported beam is uncoped

#### Web Side Plate

Try 90x 8 Rolled edge flat bar 280 mm long in grade 300

Design shear capacity:

Since we have a non-uniform shear stress distribution, Clause 5.11.3 of AS 4100 applies.

$$V_f = 2V_u / [0.9 + (f^*_{vm} / f^*_{va})] \le V_u$$
 AS4100 Cl.5.11.3

Where  $V_u$  is the nominal shear yield capacity of the web side plate assuming uniform shear stress distribution.

 $f_{vm}^*, f_{va}^*$  = the maximum and average design shear stresses in the web side plate respectively.

The shear stress distribution is parabolic in this case with the maximum shear stress being 1.5 the average stress.

Hence

$$V_i = 2V_u / [0.9 + 1.5] = 0.833 V_u = 0.833 \times 0.6 \times f_{yi} A_i$$
  
 $\phi V_i = 0.9 \times 0.833 \times 0.6 \times f_{yi} A_i$   
 $\phi V_i = 0.45 f_{yi} t_i d_i$   
 $\phi V_i = 0.45 \times 320 \times 8 \times 280 \times 10^{-3} = 322.6 \text{ kN} > V^* = 250 \text{ kN}$  OK

Design moment capacity:

The maximum bending moment acting on the web side plate is:  $M_i^* = 250 \times 55 \times 10^{-3} = 13.750 \text{ kNm}$ 

The design plastic moment capacity of the web plate is given by:

$$\phi M_{\text{pi}} = \phi \, \text{S} \, f_{\text{yi}} 
\phi M_{\text{pi}} = \phi \, (t_{\text{i}} \, d_{\text{i}}^2 / 4) \, f_{\text{yi}} 
\phi M_{\text{pi}} = 0.225 \, f_{\text{yi}} \, t_{\text{i}} \, d_{\text{i}}^2 
\phi M_{\text{pi}} = 0.225 \, \text{x} \, 320 \, \text{x} \, 8 \, \text{x} \, 280^2 \, \text{x} \, 10^{-6} = 45.2 \, \text{kNm} > M_{\text{i}}^* = 13.750 \, \text{kNm} \quad OK$$

Design bearing capacity:

$$\phi V_b = 0.9 \ (3.2 \ d_f t_p f_{up})$$
  
 $\phi V_b = 0.9 \ x \ 3.2 \ x \ 20 \ x \ 8 \ x \ 440 \ x \ 10^{-3}$   
 $\phi V_b = 202.75 \ kN > V_f^* = 85.9 \ kN$  *OK*

Design tearing capacity:

The bolt is exerting a force  $V_f^* = 85.9$  kN on the web plate at the bolt-hole, this force has two components:

(i) 
$$V_{fx}^* = 58.93 \text{ kN}$$
  
 $a_{ex} = 35 - r_{hole} + r_{bolt} = 35 - 11 + 10 = 34 \text{mm}$   
 $\phi V_b = 0.9 \times 34 \times 8 \times 440 \times 10^{-3} = 107.7 \text{ kN} > V_{fx}^* = 58.93 \text{ kN}$  *OK*

(ii) 
$$V_{\rm fy}^{*} = 62.5 \text{ kN}$$

The vertical bolt force can cause tearing towards an edge and it can also cause tearing between the bolt holes which is less critical in this case as  $a_{\rm ey}$  towards an edge is less than  $a_{\rm ey}$  between bolt holes.

$$a_{\text{ey}} = 35 - r_{\text{hole}} + r_{\text{bolt}} = 35 - 11 + 10 = 34 \text{ mm}$$
  
 $\phi V_{\text{b}} = 0.9 \times 34 \times 8 \times 440 \times 10^{-3} = 107.7 \text{ kN} > V_{\text{fy}}^{*} = 62.5 \text{ kN}$  *OK*

## Weld Group

The weld is assumed to transmit  $V^*$ ,  $M_i^*$ 

 $L_{\rm w}$  = Total run of fillet weld along one side of the plate

 $L_{\rm w} = 280 \text{ mm}$ 

Design vertical force pert unit length  $V_y^*$ 

$$V_{\rm y}^* = \frac{V^*}{2L_{\rm w}} = \frac{250}{2 \times 280} = 0.45 \text{ kN/mm}$$

Treating the weld group elements as a line elements with one unit thickness

Weld second moment

$$I_{xx} = 2 \times \frac{L_w^3}{12} = \frac{L_w^3}{6}$$

Max force per unit length at the critical points due to moment acting on the weld group:

$$V_z^* = \frac{M_w^* (L_w/2)}{\frac{L_w^3}{6}} = \frac{3M_w^*}{L_w^2}$$

$$V_z^* = \frac{3 \times 13750}{280^2} = 0.71 \text{ kN/mm}$$

$$V_{\text{Res}}^* = \sqrt{0.45^2 + 0.71^2} = 0.84 \text{ kN/mm}$$

Try 8mm E48XX SP fillet weld category

$$\phi V_{\text{w}} = \phi \times 0.6 \times f_{\text{uw}} t_{\text{t}} = 0.8 \times 0.6 \times 480 \times \frac{8}{\sqrt{2}}$$
  
 $\phi V_{\text{w}} = 1.3 \text{ kN/mm} > V_{\text{Res}}^* = 0.84 \text{ kN/mm} \quad OK$ 

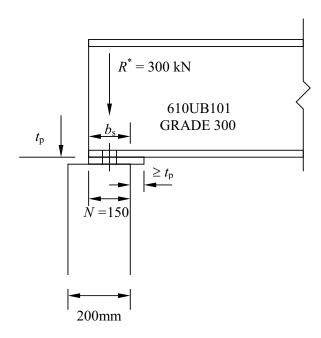
Use 8mm E48XX SP fillet weld category

Use 4 x M20 bolts in 8.8/ S category

Use 90x 8 rolled edge flat bar 280 mm long in grade 300

#### 9.4.1.4 Stiff Seat Connection

The 610UB101 beam shown below is seated on a reinforced concrete wall with  $f_c'=25$  MPa. If the design end reaction is 300 kN, design the bearing plate and if needed, load bearing stiffeners. Steel is Grade 300.



#### Solution

The beam end reaction is assumed to be uniformly distributed from the beam to the bearing plate over an area =  $b_s \times N$ , where  $b_s$  is the stiff bearing width and N is the width of the plate bearing on the support. The bearing plate is assumed to distribute the beam end reaction uniformly to the area of concrete under it. The bearing area  $A_1$  must be such that the design bearing strength of concrete  $\phi N_p$  is not exceeded.

```
\begin{split} & \phi N_{\rm p} \geq R^* \\ & \text{where} \\ & \phi = 0.6 \\ & N_{\rm p} = 0.85 \, f_{\rm c}' A_1 \\ & \phi N_{\rm p} = 0.6 \, \text{x} \, 0.85 \, f_{\rm c}' A_1 \geq R^* \\ & A_1 \geq R^* \, / \, (0.51 \, f_{\rm c}') \\ & A_1 \geq 300 \, \text{x} \, 10^3 \, / \, (0.51 \text{x} 25) = 23529 \, \text{mm}^2 \\ & A_1 = N \, \text{x} \, B \geq 23529 \, \text{mm}^2 \\ & B \geq 23529 \, / \, N = 23529 \, / \, 150 = 157 \text{mm} < b_{\rm f} = 228 \, \text{mm} \\ & \text{Use} \, B = b_{\rm f} = 228 \, \text{mm} \\ & b_{\rm s} = t_{\rm w} + 1.172 \, \text{r} + 2 \, t_{\rm f} \\ & b_{\rm s} = 10.6 + 1.172 \, \text{x} \, 14 + 2 \, \text{x} \, 14.8 = 56.6 \, \text{mm} \end{split}
```

The bearing plate is designed as an inverted cantilever loaded with the bearing pressure, The critical section for cantilever action occurs at section 1-1.

The length of the plate that can deform in flexure  $n = (B - b_s) / 2 = (228-56.6) / 2 = 85.7$  mm  $\phi M_s = 0.9 \times (N \times t^2 / 4) \times f_y \ge M^* = (R^* / A_1) \times N \times n^2 / 2$ 

 $t_p \ge \sqrt{(2.22 \ R^* \ n^2 / (A_1 \times f_y))} = \sqrt{(2.22 \times 300 \times 10^3 \times 85.7^2) / (228 \times 150 \times 280))} = 22.6 \text{ mm}$ As indicated by the AISC Connection Manual [2] add 2.5  $t_p$  to the plate width N

Hence use 228x210x24-bearing plate in Grade 300

## Design Bearing Yield Capacity

$$\phi R_{\text{by}} = 0.9 \ (1.25 \ b_{\text{bf}} t_{\text{w}} f_{\text{yw}})$$
 $AS4100 \ Cl. 5.13.3$ 
 $b_{\text{s}} = 150 \ \text{mm}$ 
 $b_{\text{bf}} = b_{\text{s}} + 2.5 \ (t_{\text{p}} + t_{\text{f}}) = 150 + 2.5 \ (24 + 14.8) = 247 \ \text{mm}$ 
For 610UB101 in Grade 300  $(f_{\text{yf}} = 300 \ \text{MPa}, f_{\text{yw}} = 320 \ \text{MPa})$ 
 $\phi R_{\text{by}} = 0.9 \ \text{x} \ 1.25 \ \text{x} \ 247 \ \text{x} \ 10.6 \ \text{x} \ 320 = 942.5 \ \text{kN}$ 

## Design Bearing Buckling Capacity

$$\phi R_{bb} = 0.9 \ (\alpha_c \ k_f A_{wb} f_{yw})$$
 AS4100 Cl. 5.13.4

where:

 $k_{\rm f}$  = 1.0 since local buckling is not a design consideration

 $A_{\rm wb} = b_{\rm b} \times t_{\rm w}$ 

 $b_b = b_{bf} + 0.5d_2$  for an End Support

 $d_2$  = twice the clear distance from the neutral axis to the compression flange =  $d_1$  for a symmetrical section

 $\alpha_c$  = the member slenderness reduction factor

The web is treated like a column of a cross section  $A_{wb}$  and a length  $d_1$ 

$$A_{\rm wb} = b_{\rm b} \times t_{\rm w}$$

$$b_b = 247 + 0.5 \times 572 = 533 \text{ mm}$$

$$A_{\rm wb} = 533 \times 10.6 = 5649.8 \text{ mm}^2$$

The slenderness ratio for this column is  $l_e/r$ , where  $l_e=k_e l$ . Since  $l_{ex}=l_{ey}$  the least radius of gyration which is  $r_y$  will give the highest slenderness ratio and therefore the least member capacity. Thus the design member buckling capacity is governed by buckling of the web about the minor-axis (i.e out of the web plane)

$$r_y = \sqrt{(I_y/A)} = \sqrt{((b_b t_w^3)/12)/(b_b t_w)}$$

$$r_v = t_w / \sqrt{12}$$

Since both edges of the web are fixed to the flanges, the effective buckling length  $l_{ey}$  between flanges is  $l_{ey} = k_e l = 0.7 d_1$ 

Slenderness Ratio = 
$$l_{\rm ey}/r_{\rm y} = 0.7d_1/(t_{\rm w}/\sqrt{12}) \approx 2.5~d_1/t_{\rm w}$$
 AS 4100 Cl. 5.13.4

$$L_{\rm ey}/r_{\rm y} = 2.5d_1/t_{\rm w} = 2.5 \times 572/10.6 = 134.91$$

$$\lambda_{\rm n} = (L_{\rm e}/{\rm r})\sqrt{k_{\rm f}}\sqrt{(f_{\rm yw}/250)}$$

$$\lambda_n = 134.91 \times 1 \times \sqrt{(320/250)} = 152.63$$

$$\alpha_b = 0.5$$
 (other sections not listed)

AS4100 Table

6.3.3(1)

$$\alpha_{\rm c}$$
 = 0.266 AS4100 Table

6.3.3(3)

$$\phi R_{bb} = 0.9 \times 0.266 \times 1 \times 5649.8 \times 320 = 432.82 \text{ kN}$$

$$\phi R_b = [\phi R_{by}, \phi R_{bb}]_{min} = 432.82 \quad kN > R^* = 300 \text{ kN} \quad OK$$

#### 9.4.1.5 Column Pinned Base Plate

Design a pinned base plate for a 460UB82.1 column subject to the following design actions:

Axial tension  $N_t^* = 120 \text{ kN}$ Shear force  $V_y^* = 90 \text{ kN}$  (acting parallel to the member y-axis)

#### Solution

## Check Standard AISC Base Plate for 460UB82.1 Column

490 x 200 x 25 mm plate in grade 250 steel 4-M24 4.6/S holding down bolts, hole dia. = 30mm Pitch p = 300 mmGauge  $s_g = 100 \text{ mm}$ 

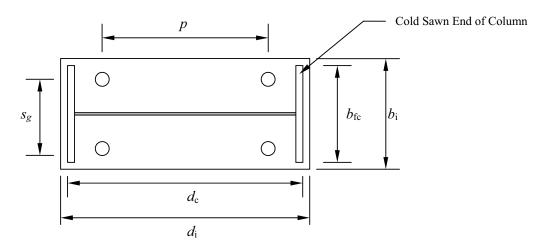


Figure 9.6 Base Plate Arrangements

#### Check Bolt Capacity **(1)**

(i) Tensile Capacity

 $N_{\rm tf}^* = 120 / 4 = 30 \text{ kN}$ 

For M24 4.6/S bolt  $\phi N_{\rm tf} = 113 > N_{\rm tf}^* = 30 \text{ kN}$ 

(ii) Shear Capacity

 $V_{yf}^* = 90/4 = 22.5 \text{ kN}$ For M24 4.6/S bolt  $\phi V_{fn} = 64.3 \text{ kN} > V_{yf}^* = 22.5 \text{ kN}$  OK

## (iii) Combined Shear and Tension

Linear interaction is adopted in this example as AISC Connection Manual [2] recommends the use of the conservative linear interaction rather than the less conservative circular interaction adopted in AS4100.

$$\frac{30}{113} + \frac{22.5}{64.3} = 0.62 < 1.0 \quad OK$$

## (2) Check Plate Bending Capacity for Tension in Column

The plate thickness must be such that,

$$N_{\rm t}^* < \phi N_{\rm s}$$

 $\phi N_s$  = design strength of steel base plate in bending

$$\phi N_{\rm s} = \frac{\phi + 4 b_{fo} f_{y_i} t_i^2}{\sqrt{2} s_g} \times \frac{n_b}{2} \text{ when } \sqrt{2} b_{fo} < d_c \qquad AISC Connection Manual Sec. 4.12.4$$

Note: for I-section members  $b_{\rm fo} = b_{\rm fc}$ 

$$\sqrt{2} \ b_{fo} = \sqrt{2} \times 191 = 270.1 \,\text{mm} < d_c = 460 \,\text{mm}$$

$$\phi N_{\rm s} = \frac{0.9 \times 4 \times 191 \times 250 \times 25^{2}}{\sqrt{2} \times 100} \times \frac{4}{2} \times 10^{-3} \text{ kN}$$

$$\phi N_s = 1519.4 \text{ kN} > N_t^* = 120 \text{ kN}$$
 OK

*Comment*: The standard AISC base plate is thicker than strictly necessary for a pin base. However a thick, robust base plate is favoured as it will be less likely to be damaged during erection and will provide some moment restraint which leads to an improvement in the frame stiffness.

## (3) Weld Design

Try 6mm E41XX fillet weld GP category all around column profile.

$$L_w = total run of the fillet weld$$

$$L_w = 2 \times 191 + 2 \times (191-9.9) + 2 \times (460-2 \times 16) = 1600 \text{ mm}$$

$$V_{\rm y}^* = \frac{V^*}{L_{\rm w}} = \frac{90}{1600} = 0.056 \text{ kN/mm}$$

$$V_z^* = \frac{N_t^*}{L_w} = \frac{120}{1600} = 0.075 \text{ kN/mm}$$

$$V_{\text{Res}}^* = \sqrt{0.056^2 + 0.075^2} = 0.094 \text{ kN/mm}$$

$$\phi V_{\rm w} = \phi \times 0.6 \times f_{\rm uw} t_{\rm t} = 0.6 \times 0.6 \times 410 \times \frac{6}{\sqrt{2}}$$

$$\phi V_{\rm w} = 0.626 \text{ kN/mm} > V_{\rm Res}^{*} = 0.094 \text{ kN/mm}$$
 OK

*Comment*: it is common to weld all around the profile, but as seen in the above example, this may lead to an over-designed and unnecessarily expensive connection.

#### 9.4.2 **Rigid Connections**

## 9.4.2.1 Fixed Base Plate

Design a base plate for 800WB122 column, subject to the following design actions:

Bending moment  $M_x^* = 590 \text{ kNm}$ Axial compression  $N_c^* = 1150 \text{ kN}$ Shear force  $V_y^* = 182 \text{ kN}$  (acting parallel to the member y-axis)

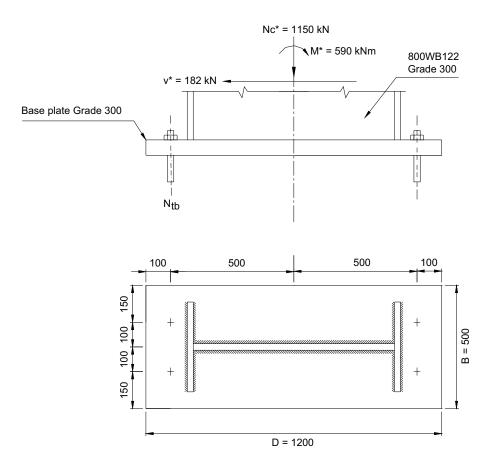


Figure 9.7 Fixed Base Plate

## Solution

The applied load and moment are equivalent to an axial load of 1150 kN acting at an eccentricity of  $e = M_x^* / N_c^* = 590 \times 10^3 / 1150 = 513 \text{ mm}$ 

Since the base plate is subjected to large bending moment so that  $e > d_i/6$  and e > half the col. depth  $d_c$  / 2 (i.e. large eccentricity) one must take into consideration the tensile force that develops in the holding down bolts.

Try 1200 x 500 base plate in grade300 steel

Offset = 500 - (792 / 2) = 104 mm (Distance from the centre line of bolts to the column face) Bolt spacing parallel to flange = 200mm

As a rule of thumb: Bolt spacing  $\leq 2$  (offset + bolt diameter)

 $200 \le 2 (104 + 24) = 256 \text{ mm}$  OK

## (1) Elastic Analysis:

This analysis follows the basic assumptions of the reinforced concrete theory, which are:

- 1. Linear elastic behaviour (i.e. stress is proportional to strain) It is assumed that the bearing pressure has a linear distribution to a maximum value of  $0.5 f_c$ , where  $f_c$  is the concrete characteristic compressive strength.
- 2. Plane sections before bending remain plane during bending (i.e. the strain varies linearly through the depth of the section).

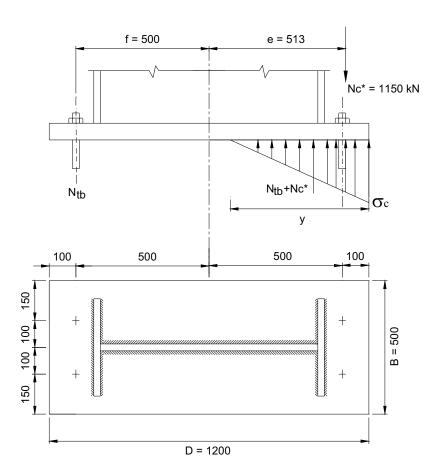


Figure 9.8 Elastic Bearing Stress Distribution for Eccentrically Loaded Column Base

With the notation of Figure 9.8 and where:

 $A_s$  = area of the holding-down bolts in tension,

 $\sigma_s$  = stress in steel bolts,

 $E_s$  = modulus of elasticity of steel bolts,

 $\sigma_c$  = bearing stress in concrete

 $E_c$  = modulus of elasticity of concrete

 $n = modular ratio = E_s/E_c$ 

There are three unknowns:  $N_{\rm tb}$ , y and  $\sigma_{\rm c}$ 

## Vertical Equilibrium gives:

$$\Sigma F_{y} = 0$$

$$0.5y \ \sigma_{c} \ b_{i} - N_{tb}^{*} - N_{c}^{*} = 0$$

$$\sigma_{c} = \frac{\left(N_{c}^{*} + N_{tb}^{*}\right)}{0.5yb_{i}}$$
(1)

## Moment Equilibrium gives:

Taking the summation of moment about the columns centroid gives

$$\sum M_{c} = 0$$

$$N_{tb} \times f + (N_{tb}^{*} + N_{c}^{*}) [(d_{i}/2) - (y/3)] - M^{*} = 0$$

$$M^{*} = N_{c}^{*} \times e$$

Substituting  $N_c^* \mathbf{x}$  e instead of  $M^*$ , the above equation becomes

$$N_{\text{tb}}^* \mathbf{x} f + (N_{\text{tb}}^* + N_{\text{c}}^*) \mathbf{x} \left[ \frac{d_i}{2} - \frac{y}{3} \right] - N_{\text{c}}^* \mathbf{x} e = 0$$
 (2)

## Plane sections condition gives:

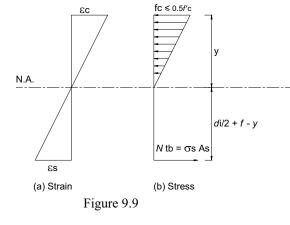
From triangular symmetry see Figure 9.9 (a)

$$\frac{\varepsilon_{s}}{\varepsilon_{c}} = \frac{d_{i}/2 + f - y}{y}$$

$$\varepsilon_{s} = \sigma_{s} / E_{s} \text{ and } \varepsilon_{c} = \sigma_{c} / E_{c}$$

$$Hence,$$

$$\frac{\sigma_{s}/E_{s}}{\sigma_{c}/E_{c}} = \frac{d_{i}/2 + f - y}{y}$$
(3)



Eliminating  $N_{tb}^*$  and  $\sigma_c$  from the above three equations, noting that  $N_{tb}^* = \sigma_s A_s$ , gives:

$$y^{3} + 3 \times \left(e - \frac{d_{i}}{2}\right)y^{2} + \left(\frac{6 \ n \ A_{s}}{b_{i}}\right)(f + e)y - \left(\frac{6 \ n \ A_{s}}{b_{i}}\right)\left(\frac{d_{i}}{2} + f\right)(f + e) = 0$$

$$(4)$$

Solve equation (4) to determine y then substitute y into equation (2) to determine  $N_{tb}^*$ , which may be written as:

$$N_{\text{tb}}^* = -N_{\text{c}}^* \times \frac{d_i/2 - y/3 - e}{d_i/2 - y/3 + f}$$

Then find  $\sigma_c$  from equation (1)

## Try 4-M24 4.6/TB holding down bolts

Tensile area of the holding down bolts in tension  $A_s = 2 \times 353 = 706 \text{ mm}^2$ 

Using a concrete mix for the foundation which has cylinder strength of  $f'_c = 25$  MPa, the design bearing strength of concrete  $\phi N_p$  is the minimum of:

$$A_1 \times \phi \times 0.85 f_c \sqrt{\frac{A_2}{A_1}}$$
 AS 3600 Cl.12.3

AS 3600 Cl.12.3  $A_1 \mathbf{x} \phi \mathbf{x} 2 f_c$ 

where

 $A_1$  = area of base plate component =  $d_i \times b_i$ 

 $A_2$  = area of supporting concrete foundation that is geometrically similar to and concentric with the base plate area.

AS 3600 Table 2.3  $\phi = 0.6$ 

Making the conservative assumption that the whole area of the concrete support is covered by the base plate, the design bearing strength of concrete  $\phi N_p$  is given by:

$$\phi N_p = 0.6 \times 0.85 f_c A_1$$

$$\phi N_{\rm p} = 0.51 \, f_{\rm c} \, A_1$$

$$\phi N_p / A_1 = 0.51 f'_c = 0.51 \times 25 = 12.75 \text{ MPa}$$

$$e = 513 \text{ mm}, f = 500 \text{ mm}, b_i = 500 \text{ mm}, d_i = 1200 \text{ mm}$$

$$n = E_s / E_c$$

 $E_s = 200000 \text{ MPa}$ 

$$E_s = 200000 \text{ MPa}$$
 $E_c = \rho^{1.5} \ 0.043 \ \sqrt{f_{cm \cdot 28}}$ 

where

 $\rho = 2400 \text{ kg/m}^3$  (normal weight concrete), and the mean compressive strength

 $f_{\text{crn.28}} = 29.5 \text{ MPa}$  (mean compressive strength at 28 days for  $f'_{\text{c}} = 25 \text{MPa}$ )

$$E_c = 27500 \text{ MPa}$$

$$n = 200000 / 27500 = 7.5$$

Substituting all the above values in equation (4) and solving to find y:

$$y^{3} + 3 \times \left(513 - \frac{1200}{2}\right)y^{2} + \left(\frac{6 \times 7.5 \times 706}{500}\right)\left(500 + 513\right)y - \left(\frac{6 \times 7.5 \times 706}{500}\right)\left(\frac{1200}{2} + 500\right)\left(500 + 513\right) = 0$$

Hence y = 458 mm

Substitute y into equation (2) to determine  $N_{tb}^*$ 

$$N_{\text{tb}}^* = -1150 \text{ x} \quad \frac{1200/2 - 458/3 - 513}{1200/2 - 458/3 + 500}$$

$$N_{\rm th} = 79.7 \, \rm kN$$

$$\phi N_{\text{tb}} = 2 \phi A_{\text{s}} f_{\text{uf}} = 2 \times 0.8 \times 353 \times 400 = 226 \text{ kN} > N_{\text{tb}}^* = 79.7 \text{ kN}$$
 OK

Use 4-M24 4.6/TB holding down bolts

$$\sigma_{c} = \frac{(1150 + 79.7) \times 10^{3}}{0.5 \times 458 \times 500} = 10.74 \text{ MPa} < \phi N_{p} / A_{I} = 12.75 \text{ MPa}$$
 OK

#### Plate thickness:

The plate thickness must be such that,

$$M_{\rm c}^* \leq \phi M_{\rm p}$$

$$\phi M_{\rm p} = \phi S f_{\rm y} = 0.9 \frac{b_i t_i^2}{4} f_{\rm y}, \phi = 0.9$$

$$M_{\rm c}^* = [M_{\rm cf}^*, M_{\rm tf}^*]_{\rm max}$$

where

 $M_{\rm cf}^*$  = Design cantilever moment at the edge of the column's compression flange

 $M_{\rm tf}^*$  = Design cantilever moment at the edge of the column's tension flange

Hence

$$t_i \ge \sqrt{\frac{4 \ M_c^*}{\phi \ b_i \ f_y}}$$

From triangular symmetry the bearing stress at the edge of the columns compression flange is

$$\frac{\sigma_{cf}}{458 - 204} = \frac{10.74}{458}$$

$$\sigma_{\rm cf} = 5.96 \, \rm MPa$$

Design cantilever moment at the edge of the columns compression flange:

$$M_{\text{cf}}^* = 5.96 \times \frac{204^2}{2} \times 500 + \frac{1}{2} \times (10.74 - 5.96) \times 204 \times 500 \times \frac{2}{3} \times 204$$
  
 $M_{\text{cf}}^* = 95.2 \text{ kNm}$ 

Design cantilever moment at the edge of the columns tension flange:

$$M_{\rm tf}^* = 79.7 \times 0.104 = 8.3 \text{ kNm}$$

$$M_c^* = [95.2, 8.3]_{\text{max}} = 95.2 \text{ kNm}$$

Hence

$$t_i \ge \sqrt{\frac{4 \times 95.2 \times 10^6}{0.9 \times 500 \times 280}} = 54.97 \text{ mm}$$

Provide 1200 x 500 base plates 55 mm thick grade 300steel

*Comment*: the elastic analysis will certainly offer an acceptable design; however, a practical alternative should to be sought which is less complex and more agreeable to design office use.

## (2) Plastic Analysis and Effective Area Concept:

If the eccentricity of the load is more than half the column depth, an effective compression area is assumed to be arranged symmetrically around the compression flange. Assuming uniform stress over this area, the line of action of the stress resultant is located at the centroid of the column flange. Taking moments about this point gives the magnitude of the bolt force directly. The required effective area of the concrete is given by  $(N_c^* + N_{tb}^*) / (\phi N_p / A_1)$ , after determining the effective area the width of the effective strip can be determined by dividing this area on the width of the base plate  $(b_i)$ .

The calculation is more difficult if the eccentricity of the load is less than half the column depth; the effective area is required to be extended along the web until its centroid coincides with the line of action of the applied load. The calculation may then proceed as before. Like before, the applied load and moment are equivalent to an axial load of 1150 kN acting at an eccentricity  $e = M_x^* / N_c^* = 590 \times 10^3 / 1150 = 513$  mm from the centreline of the column. This is outside the compression flange therefore the axial load and moment will be assumed to

be resisted by tension in the holding down bolts and an area of concrete in compression balanced about the compression flange. By taking the summation of moments about the centre of the compression flange we can obtain the uplift on the anchor bolts  $(N_{tb}^*)$ .

Taking moment about the centroid of columns compression flange

$$N_{\text{tb}}^* = \frac{590 \times 10^3 - 1150 \times \frac{1}{2} (792 - 16)}{500 + \frac{1}{2} (792 - 16)} = 161.94 \text{ kN}$$

Try 4-M24 4.6/TB holding down bolts

$$\phi N_{\text{tb}} = 226 \text{ kN} > N_{\text{tb}}^* = 161.94 \text{ kN}$$
 OK

Compressive load on concrete under compression flange = 161.94 + 1150 = 1311.94 kN

Design bearing strength of grout and concrete under base plate

$$= 0.51 f_c' = 0.51 \times 25 = 12.75 \text{ MPa}$$

Effective area of base plate required = 
$$\frac{1311.94 \times 10^3}{12.75}$$
 = 102897.3 mm<sup>2</sup>

Length of effective strip = 
$$\frac{102897.3}{500}$$
 = 205.8 mm

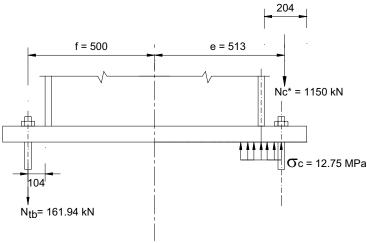


Figure 9.10 Plastic Analysis and Effective Area Concept

## Plate thickness:

$$t_i \ge \sqrt{\frac{4 \ M_c^*}{\phi \ b_i \ f_y}}$$

Design cantilever moment at the edge of the columns compression flange:

$$M_{\text{cf}}^* = 12.75 \times 500 \times \frac{\left(\frac{205.8 - 16}{2}\right)^2}{2} = 28.7 \text{ kNm}$$

Design cantilever moment at the edge of the columns tension flange:

$${M_{\rm tf}}^* = 161.94 \text{ x } (500 - 792 / 2) \text{ x } 10^{-3} = 16.84 \text{ kNm}$$

$$M_c^* = [28.8, 16.84]_{\text{max}} = 28.7 \text{ kNm}$$

Hence 
$$t_i \ge \sqrt{\frac{4 \times 28.7 \times 10^6}{0.9 \times 500 \times 280}} = 30.1 \text{ mm}$$

Provide 1200 x 500 base plate 35 mm thick grade 300steel

#### (3) Weld Design:

For the design of the flange and web welds the AISC Connection Manual [2] assumes that the proportion of the bending moment transmitted by the web is  $k_{\text{mw}}$  while the proportion of the bending moment transmitted by the flanges is  $(1-k_{\rm mw})$  provided that the applied bending moment is less than the design moment capacity for the one set of yielding at the extreme fibbers  $(\phi M_v)$ , if the applied bending moment is more than  $\phi M_v$  the proportion of the bending moment transmitted by the flanges  $M_{\rm f}^*=0.9\,f_{\rm yf}\,$  x flange area x  $(d_{\rm b}-t_{\rm fb})$  and the proportion of the bending moment transmitted by the web  $M_{\rm w}^*=M^*$ - 0.9  $f_{\rm yf}\,$  x flange area x  $(d_{\rm b}-t_{\rm fb})$ . The flanges and the web transmit a share of the design axial force  $N^*$ , the proportion taken by each being proportional to their contribution to the total cross sectional area. The web weld transmits the design shear force  $V^*$ . Because the end of the column is cold sawn it will be in tight bearing contact with the base plate and hence the flange compression force will be transmitted in direct bearing and not through the weld. The resultant compression acting on the web weld will also be transmitted in direct bearing and not through the weld.

Try 6mm E41XX fillet weld SP category all around column profile.

Column's tension flange weld:

$$N_{\text{tf}}^* = \frac{(1-k_{mw})M^*}{d_c - t_{f_c}} - \frac{(1-k_w)N_c^*}{2}$$
 (tension flange design force)

where

$$k_{mw} = I_{wx} / I_x$$

 $k_w$  = web area / total cross sectional area

$$\begin{aligned} k_{mw} &= (10 \text{ x } (792\text{-}2\text{x}16)^3/12) / (1570\text{x}10^6) = 0.23 \\ k_w &= (792\text{-}2\text{x}16)\text{x}10 / 15600 = 0.49 \\ N_{tf}^* &= \frac{(1-0.23)\times 590\times 10^3}{792-16} - \frac{(1-0.49)\times 1150}{2} = 290.5 \text{ kN} \end{aligned}$$

 $L_{\rm w}$  = total run of the fillet weld around the tension flange profile

$$L_w = 250 + (250-10) = 490 \text{ mm}$$

$$V_z^* = \frac{N_{tf}^*}{L_{co}} = \frac{290.5}{490} = 0.593 \text{ kN/mm}$$

Use 6mm E41XX fillet weld SP category for the flanges weld

$$\phi V_{\text{w}} = \phi \times 0.6 \times f_{\text{uw}} t_{\text{t}} = 0.8 \times 0.6 \times 410 \times \frac{6}{\sqrt{2}}$$
  
 $\phi V_{\text{w}} = 0.835 \text{ kN/mm} > V_{\text{z}}^* = 0.593 \text{ kN/mm} OK$ 

## Column web weld:

$$\begin{split} & \text{L}_{\text{w}} = \text{total run of the fillet weld along one side of the web} \\ & \text{L}_{\text{w}} = 792 - 2 \times 16 = 760 \text{ mm} \\ & V_{z}^{*} = \frac{3M_{w}^{-}}{L_{w}^{-2}} - \frac{N_{w}^{-}}{2L_{w}} \\ & V_{z}^{*} = \frac{3 \times 0.23 \times 590 \times 10^{3}}{760^{2}} - \frac{0.49 \times 1150}{2 \times 760} \\ & V_{z}^{*} = 0.334 \text{ kN/mm} \\ & V_{y}^{*} = \frac{V_{z}^{*}}{2L_{w}} = \frac{182}{2 \times 760} = 0.12 \text{ kN/mm} \\ & V_{\text{Res}}^{*} = \sqrt{V_{z}^{*2} + V_{y}^{*2}} = \sqrt{0.334^{2} + 0.12^{2}} \\ & V_{\text{Res}}^{*} = 0.355 \text{ kN/mm} < \phi V_{\text{w}} = 0.835 \text{ kN/mm} & OK \end{split}$$

Use 6mm E41XX fillet weld SP category for the web weld

#### (4) Base Plate with Shear:

Under normal circumstances, the frictional force developed between the plate and its support adequately resists the factored column base shear. The anchor bolts also provide additional shear capacity. For cases in which exceptionally high shear force is expected, such as in bracing connection or a connection in which uplift reduces the frictional resistance, the use of shear lugs (i.e. shear key) may be necessary. Shear lugs can be designed based on the limit states of bearing on concrete and bending of the lugs. The size of the lug should be proportioned such that the bearing stress on concrete does not exceed  $0.6 \times (0.85 \, f'_c)$ . The thickness of the lug can be determined so that the moment acting on the critical section of the lug is less than the plastic moment capacity of the lug. The critical section is taken to be at the junction of the lug and the plate.

$$V^*=182~\text{kN}$$
 By relying on friction alone to resist the shear force.  $\phi V_d = \text{design shear capacity relying on friction alone}$   $\phi V_d = \phi \ \mu \ N_c^* \ \text{where } \phi = 0.8$   $\mu = \text{coefficient of friction}$   $\mu = 0.55$  for grouted conditions with the contact plane between the grout and the as-rolled steel above the concrete surface (normal condition).

$$\phi V_d = 0.8 \times 0.55 \times 1150 = 506 \text{ kN} > V^* = 182 \text{ kN}$$
 OK

## Additional:

Assume that the shear force acting on the base plate  $V^* = 600 \text{ kN}$ By relying on friction plus a shear key to resist the shear force the design shear capacity is:

$$\phi V_{\rm dc} = \phi V_{\rm d} + \phi V_{\rm k}$$

where  $\phi V_k$  = design shear capacity of a shear key which is a minimum of:

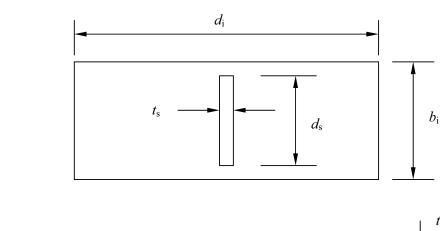
- 1)  $\phi \times 0.85 f_c \times (b_s t_g) \times d_s$  (limit state of bearing on concrete,  $\phi = 0.6$ )
- 2)  $\phi \times d_s t_s^2 \times f_{ys} / (2 b_s)$  (limit state of bending of the lugs,  $\phi = 0.9$ )

Assume that the shear key has these dimensions refer to Figure 9.11 for notation:

 $b_{\rm s} = 140 \; {\rm mm}$ 

 $d_{\rm s} = 300 \text{ mm}$ 

 $t_{\rm g}$  for grout under base plate = 40 mm



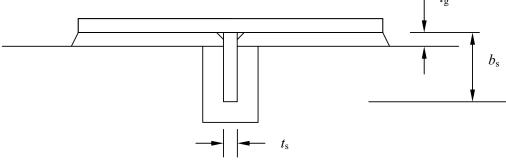


Figure 9.11 Shear Key Arrangements

Shear force acting on the shear key is given by:

$$V_{\text{key}}^* = V_{\text{total}}^* - \phi V_{\text{d}}$$
  
 $V_{\text{key}}^* = 600 - 506 = 94 \text{ kN}$ 

Bearing stress on concrete =  $V_{\text{key}}^* / (d_s \times (b_s - t_g)) = 94 \times 10^3 / (300 \times (140 - 40)) = 3.13 \text{ MPa}$ 

The concrete design bearing stress is given by:  $\phi 0.85 f'_c = 0.6 \times 0.85 \times 25 = 12.75 \text{ MPa} > 3.13 \text{ MPa}$  OK

Moment at the critical section (the plate shear key junction):

$${M_{\rm cs}}^* = {V^*}_{\rm key} \times b_{\rm s}/2$$
  
 ${M_{\rm cs}}^* = 94 \times 0.140 / 2 = 6.58 \text{ kNm}$   
 $\phi M_{\rm p \ Key} = 0.9 \ (d_{\rm s} \ t_{\rm s}^2/4) f_{\rm ys} \ge {M_{\rm cs}}^*$ 

Hence

$$t_{s} \ge \sqrt{\frac{4M_{cs}^{*}}{0.9 \times d_{s} \times f_{ys}}}$$
$$t_{s} \ge \sqrt{\frac{4 \times 6.58 \times 10^{6}}{0.9 \times 300 \times 300}}$$

 $t_{\rm s} \ge 18~{\rm mm}$ 

Use a shear key 20x140x300 mm in grade 300 steel

If the dimensions of the shear key including its thickness where all assumed the solution proceeds as follows:

Try a shear key 20x140x300 mm in grade 300 steel

 $\phi V_k$  is lesser of:

1) 
$$\phi \times 0.85 \times f_c^r \times (b_s - t_g) \times d_s = 0.6 \times 0.85 \times 25 \times (140 - 40) \times 10^{-3} \times 300 = 382.5 \text{ kN}$$
  
2)  $\phi \times d_s t_s^2 \times f_{ys} / (2 b_s) = [0.9 \times 300 \times 20^2 \times 300 / (2 \times 140)] \times 10^{-3} = 115.7 \text{ kN}$   
 $\phi V_k = 115.7 \text{ kN} > V_{key}^* = 94 \text{ kN}$   $OK$ 

Use a shear key 20x140x300 mm in grade 300 steel

## 9.4.2.2 Welded Moment Connection [Crane Girder Bracket (Corbel)]

Design a crane girder bracket (corbel) connected to the flange of 800WB122 column in grade 300 steel to support a girder reaction of 386.9 kN acting at an eccentricity of 600mm (this reaction is due to factored load), the shear force in the column above and below the bracket is 110.8 kN.

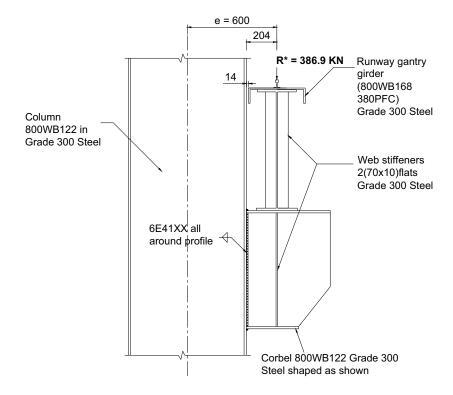


Figure 9.12 Welded Moment Connection

## Solution

## (1) Bracket (corbel):

Try using a deep section say 800WB122 in grade 300 steel to increase the lever arm(i.e. the distance between flanges centroids) to avoid the need of stiffening the column web at the compression flange of the bracket and the stiffening of the column flange at the tension flange of the bracket.

```
Max. BM in Bracket:

M_x^* = 386.9 \times 0.204 = 78.93 \text{ kNm}

\phi M_{\text{sx}} = 0.9 \text{ Z}_{\text{ex}} f_{\text{y}} = 0.9 \times 4500 \times 10^{-3} \times 300 \times 10^{-6}

\phi M_{\text{sx}} = 1215 \text{ kNm} > M_x^* = 78.93 \text{ kNm} OK
```

Shear force in bracket 
$$V^* = 386.9 \text{ kN}$$

Shear capacity  $\phi V_v$ :  $d_p / t_w = 760 / 10 = 76, 82 / \sqrt{(f_{yw} / 250)} = 82 / \sqrt{(310 / 250)} = 73.64$   $d_p / t_w = 76 > 82 / \sqrt{(f_{yw} / 250)} = 73.64$   $V_v = V_b = \alpha_v V_w \le V_w$   $\alpha_v = [82 / ((d_p / t_w) \times \sqrt{(f_{yw} / 250)})]^2 = [73.64 / 76]^2 = 0.939$   $V_v = 0.939$   $V_w$   $V_v = 0.93$   $V_w = 0.9$ 

## Check if web stiffeners are needed in the Corbel:

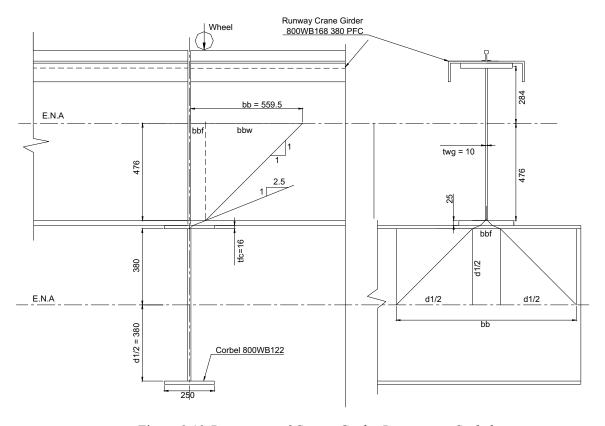


Figure 9.13 Dispersion of Gantry Girder Reaction in Corbel

## Refer to Figure 9.13.

 $b_{\rm s}$  for corbel =  $t_{\rm wg}$  + 1.17  $r_{\rm g}$  + 2  $t_{\rm fg}$   $b_{\rm s}$  = 10 + 1.17 x 0 + 2 x 25 = 60mm  $b_{\rm bf}$  for the corbel =  $b_{\rm s}$  + 5  $t_{\rm fc}$   $b_{\rm bf}$  = 60 + 5 x 16 = 140 mm  $b_{\rm b}$  for corbel =  $b_{\rm bf}$  +  $d_{\rm 1c}$  $b_{\rm b}$  = 140 + 760 = 900mm

## Check bearing yield capacity

$$R_{\text{by}} = 1.25 \ b_{\text{bf}} t_{\text{W}} f_{\text{y}}$$
  
 $R_{\text{by}} = 1.25 \times 140 \times 10 \times 310 = 542.5 \text{ kN}$   
 $\phi R_{\text{by}} = 0.9 \times 542.5 = 488.25 \text{ kN} > 386.9 \text{ kN}$  *OK*

## Check bearing buckling capacity

$$R_{\rm bb} = N_{\rm c} = \alpha_{\rm c} N_{\rm s} = \alpha_{\rm c} k_{\rm f} A_{\rm n} f_{\rm v}$$

AS 4100 Section 6

The web is treated like a column of cross section  $A_n = t_w b_b$  and a depth equals the depth between flanges  $d_1$ .

$$A_{\rm n} = t_{\rm w} b_{\rm b} = 10 \text{ x } 900 = 9000 \text{ mm}^2$$

Since both edges of the web are fixed to the flanges, the effective buckling length  $l_{ey}$  between flanges is  $l_{ey} = k_e l = 0.7 d_1$ 

$$R_{bb} = 0.152 \times 2790 = 424.1 \text{ kN}$$
  
 $\phi R_{bb} = \phi N_c = 0.9 \times 424.1 = 381.7 \text{ kN} < 386.9 \text{ kN}$  NG

Hence web stiffeners are needed to resist buckling only in this case.

## Selection of Stiffener Section:

The main cost of stiffeners is in the design time and in cutting and welding. The cost of the steel for the web stiffeners themselves will be negligible, so it is better to be on the safe side and choose a section that will be more than adequate, so choose a flat section for the stiffeners that has:

- 1. enough cross sectional area
- 2. low enough plate element slenderness, and
- 3. will not stick out beyond the column/beam flanges i.e. Outstand < ( $b_{\rm f}$   $t_{\rm w}$ ) / 2 = (250 10) / 2 = 120 mm

Try 2 / 70 x 10 mm flat for the stiffeners  $A_s = 1400 \text{ mm}^2$ 

## Check Stiffener slenderness:

$$\lambda_{\rm e} = b/t \, \sqrt{(f_{\rm y} \, / \, 250)}$$
 AS4100 Cl. 6.2.3  $\lambda_{\rm e} = (70 \, / 10) \, {\rm x} \, \sqrt{(310 \, / \, 250)} = 7.8$ 

From Table 6.2.4 of AS 4100:

 $\lambda_{\rm ey} = 15$  (For one longitudinal edge supported, lightly welded longitudinally (LW)) Since  $\lambda_{\rm e} < \lambda_{\rm ey}$  (i.e. No local buckling will occur in the outer edge of the web stiffener), thus  $k_{\rm f}$  for the cross shaped member = 1

 $L_{\rm w}$  = 17.5  $t_{\rm w}$  /  $\sqrt{(f_{\rm y}/250)}$  = 17.5 x 10 /  $\sqrt{(310/250)}$  = 157.15 mm or S/2 which is not applicable in this case.

Use  $L_w$ = 78.56mm on each side of the centre line giving a total width of 157.15 mm

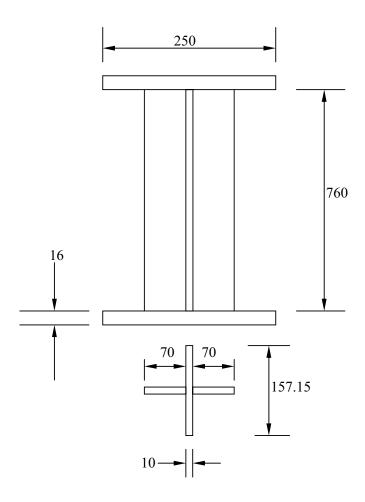


Figure 9.14 Cross Shaped Member

The web and the two stiffeners will behave as a cross-shaped column. Area of the cross-shaped member

```
A_{\rm n} = 70 \times 10 \times 2 + 157.15 \times 10

A_{\rm n} = 2971.5 \text{ mm}^2

I_{\rm y} = 10 (70+70+10)^3/12 + (157.15-10) \times 10^3/12 = 3.08 \times 10^6 \text{ mm}^4

r_{\rm y} = \sqrt{(I_{\rm y}/A)} = \sqrt{(3.08 \times 10^6/2971.5)} = 32.2 \text{ mm}

l_e/r = d_1/r = 760/32.1 = 23.6
```

The effective length for buckling is equal to  $d_1$  see Figure 9.14, since the corbel Bottom flange is not restrained against rotation in the plane of the stiffeners, thus if buckling is going to occur in the cross shaped member it will be on the whole length  $d_1$  out of the web plane.

No Flange Restraint One Flange only Restrained Both Flanges Restrained

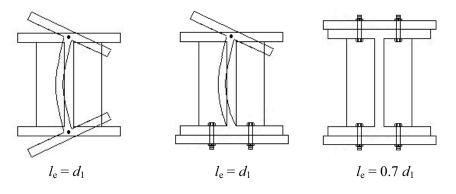


Figure 9.15 Effective Length of the Cross Shaped Member

$$\begin{split} &\lambda_{n} = (l_{e}/r) \; \sqrt{k_{f}} \; \sqrt{(f_{y}/\,250)} = 23.6 \text{x} \sqrt{1} \text{x} \sqrt{(310/250)} \\ &\lambda_{n} = 26.3 \\ &\alpha_{b} = 0.5 \\ &\alpha_{c} = 0.924 \end{split} \qquad \qquad \textit{AS4100 Table 6.3.3(3)} \\ &\phi N_{c} = 0.9 \; \alpha_{c} \; k_{f} \, A_{n} \, f_{y} \\ &\phi N_{c} = 0.9 \; \text{x} 0.924 \; \text{x} \; 1 \text{x} 2971.5 \text{x} 310 = 766.04 \; \text{kN} > \text{R}^{*} = 386.9 \; \text{kN} \end{split}$$

Adopt 2 -70x10mm flats in grade 300 steel as stiffeners

#### (2) Bracket weld:

Try 6mm fillet weld E41XX SP category all around the profile of the bracket. Design actions:

$$V^* = 386.9 \text{ kN}, M^* = 78.93 \text{ kNm}$$

The upper and lower weld at each flange is assumed to have the same line load tension or compression due to bending, to achieve that the stresses due to bending is calculated at the level of the flanges centroid.

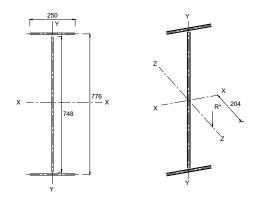


Figure 9.16 Fillet Weld Group Loaded Out of Plane

Total run of fillet weld = 4 x 250 + 2 x 748 mm = 2496 mm  $V_y^* = 386.9 / 2496 = 0.16$  kN/mm

Treating the weld group elements as a line elements with one unit thickness  $I_{xx} = 2 \times 748^3 / 12 + 4 \times 250 \times (776/2)^2$   $I_{xx} = 220.3 \times 10^6$  mm<sup>3</sup>

Max force per unit length at the critical points due to moment acting on the weld group:  $V_z^* = M^* \times y / I_{xx} = 78.93 \times 10^3 \times 388 / 220.3 \times 10^6$   $V_z^* = 0.139$  kN/mm

$$V_{\text{Res}}^* = \sqrt{(0.16^2 + 0.139^2)} = 0.211 \text{ kN/mm}$$
  
 $\phi V_{\text{w}} = 0.835 \text{ kN/mm} > V_{\text{Res}}^* = 0.211 \text{ kN/mm}$  OK

Use 6mm fillet weld E41XX SP category all around the profile of the corbel

#### *Additional*:

Assume using full penetration butt weld E48XX SP category to connect the flanges of the corbel to the column flange and fillet weld E41XX SP category to connect the web of the corbel to the column flange.

For the design of the flange and web welds the AISC Connection Manual [2] assumes that the proportion of the bending moment transmitted by the web is  $k_{\rm mw}$  while the proportion of the bending moment transmitted by the flanges is (1-  $k_{\rm mw}$ ) provided that the applied bending moment is equal to or less than the design moment capacity for the one set of yielding at the extreme fibbers ( $\phi M_y$ ), if the applied bending moment is more than  $\phi M_y$  the proportion of the bending moment transmitted by the flanges  $M_{\rm f}^* = 0.9 \ f_{\rm yf} \ x$  flange area x ( $d_{\rm b}$ - $t_{\rm fb}$ ) and the proportion of the bending moment transmitted by the web  $M_{\rm w}^* = M^*$ - 0.9  $f_{\rm yf}$  x flange area x ( $d_{\rm b}$ - $t_{\rm fb}$ ). The flanges and the web transmit a share of the design axial force  $N^*$ , the proportion taken by each being proportional to their contribution to the total cross sectional area. The web weld transmits the design shear force  $V^*$ .

### Flange welds:

$$\begin{split} &M_{\rm x}^{~*} = 78.93~{\rm kNm} < \phi M_{\rm y} = 0.9~f_{\rm yf}~Z_{\rm x} = 0.9~{\rm x}~3970~{\rm x}~10^{3}~{\rm x}~300 = 1071.9~{\rm kNm}\\ &k_{\rm mw} = I_{\rm wx}/I_{\rm x}\\ &k_{\rm mw} = (10~{\rm x}~(792\text{-}2\text{x}16)^{3}/12)~/~(1570\text{x}10^{6}) = 0.23\\ &M_{\rm f}^{~*} = (1\text{-}~k_{\rm mw})~{\rm x}~M_{\rm x}^{~*}\\ &M_{\rm f}^{~*} = 0.77~{\rm x}~78.93 = 60.78~{\rm kNm}\\ &N_{\rm ft}^{~*} = N_{\rm fc}^{~*} = M_{\rm f}^{~*}/~(d\text{-}~t_{\rm f}) = 60.78~{\rm x}~10^{3}~/~(792\text{-}16) = 78.32~{\rm kN}\\ &\phi N_{\rm w} = {\rm design~capacity~of~the~butt~weld} = \phi~f_{\rm yf}~b_{\rm f}~t_{\rm f} = (0.9~{\rm x}~300~{\rm x}~250~{\rm x}~16)~{\rm x}~10^{-3}\\ &\phi N_{\rm w} = 1080~{\rm kN} > N_{\rm ft}^{~*} = N_{\rm fc}^{~*} = 78.32~{\rm kN} \end{split}$$

Use full penetration butt weld E48XX SP category

# Web weld:

$$M_{\rm w}^{\ *} = {\rm k_{mw}} \times M_{\rm x}^{\ *} = 0.23 \times 78.93 = 18.15 \text{ kNm}$$
  
 $V^{*} = 386.9 \text{ kN}$ 

 $L_{\rm w}$  = total run of the fillet weld along one side of the web  $L_{\rm w}$  = 792 - 2 x 16 = 760 mm

$$V_z^* = \frac{M_w^* \times (L_w/2)}{I_w}$$

$$I_w = \frac{L_w^3}{6}$$

$$V_z^* = \frac{3M_w^*}{L_w^2} = \frac{3 \times 18.15 \times 10^3}{760^2}$$

$$V_z^* = 0.094 \text{ kN/mm}$$

$$V_y^* = V^* / (2L_w) = 386.9/(2 \times 760) = 0.25 \text{ kN/mm}$$

$$V_{Res}^* = \sqrt{(0.25^2 + 0.094^2)} = 0.27 \text{ kN/mm}$$

$$\phi V_w = 0.835 \text{ kN/mm} > V_{Res}^* = 0.27 \text{ kN/mm}$$

$$\phi V_w = 0.835 \text{ kN/mm} > V_{Res}^* = 0.27 \text{ kN/mm}$$

Use 6mm fillet weld E41XX SP category

Comment: the procedure mentioned above is not only limited for cases involving the use of full penetration butt weld for the flange weld, it's a general method that can be used to asses the forces acting on the flange and web welds even if the use of fillet welding for the flange weld was adopted in lieu of the butt weld.

# 9.4.2.3 Bolted Moment Connection [Crane Girder Bracket (Lapped)]

Design a crane girder bracket connected to the flanges of 310UC96.8 column in grade 300 steel to support a girder reaction of 465 kN due to factored loads, this reaction is acting at an eccentricity of 600 mm.

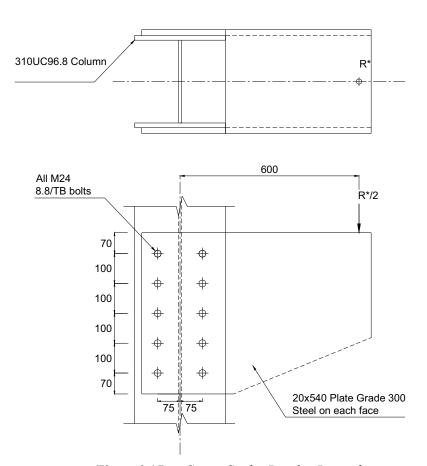


Figure 9.17 Crane Girder Bracket Lapped

#### Solution

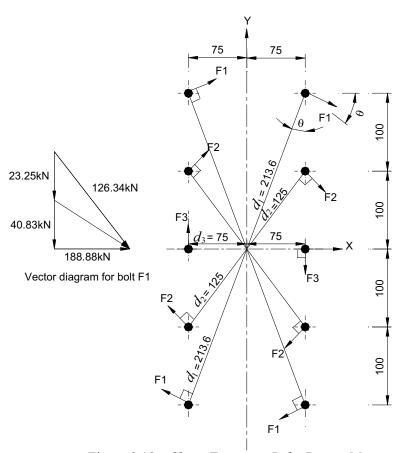
# Column Bolts:

Each bracket supports half the reaction  $R^*/2 = 465/2 = 232.5$  kN

Shear force per bolt due to vertical load = 232.5 / 10 = 23.25 kN

The moment acting at bolt group centroid  $M^* = 232.5 \times 0.6 = 139.5 \text{ kNm}$ 

This moment will cause additional shear forces on the bolts; the forces on the bolts are proportional to their distance from the centroid of the bolt group.



F1x
F1y

y max = 200mm

x max = 75mm

Figure 9.18 Shear Forces on Bolts Due to Moment

$$\frac{F_1}{d_1} = \frac{F_2}{d_2} = \frac{F_3}{d_3}$$

$$M^* = 4 F_1 d_1 + 4 F_2 d_2 + 2 F_3 d_3$$

$$M^* = \frac{F_1}{d_1} \times \left[4 d_1^2 + 4 d_2^2 + 2 d_3^2\right] = \frac{F_1}{d_1} \times \Sigma \ d^2$$
But  $d_1 = d_{\text{max}}$ 

Hence

$$F_1 = \frac{M^* d_{\text{max}}}{\sum d^2}$$
 (Max shear per bolt due to moment)

$$F_1 = \frac{139.5 \times 10^3 \times 213.6}{4 \times 213.6^2 + 4 \times 125^2 + 2 \times 75^2}$$
  
F<sub>1</sub> = 116.28 kN

Resolving F<sub>1</sub> into two components:

$$F_{1x} = F_1 \cos\theta = 116.28 \text{ x} \cos 20.56^{\circ} = 108.88 \text{ kN}$$
  
 $F_{1y} = F_1 \sin\theta = 116.28 \text{ x} \sin 20.56^{\circ} = 40.83 \text{ kN}$ 

#### **Alternatively**

$$F_{1x} = \frac{M^* y_{\text{max}}}{\Sigma x^2 + \Sigma y^2} = \frac{139.5 \times 10^3 \times 200}{10 \times 75^2 + 4 \times 100^2 + 4 \times 200^2} = 108.88 \text{ kN}$$

$$F_{1y} = \frac{M^* x_{\text{max}}}{\Sigma x^2 + \Sigma y^2} = \frac{139.5 \times 10^3 \times 75}{10 \times 75^2 + 4 \times 100^2 + 4 \times 200^2} = 40.83 \text{ kN}$$

$$F_2 = \frac{M^* d_2}{\sum d^2}, F_{2x} = \frac{M^* y_2}{\sum x^2 + \sum y^2}, F_{2y} = \frac{M^* x_2}{\sum x^2 + \sum y^2}$$

Since all the bolts have the same x

$$\therefore F_{1y} = F_{2y} = F_3 = 40.83 \text{ kN}$$

Maximum Resultant Shear on Bolt  $(F_1)$  (Vector sum of shear)

$$V_R^* = \sqrt{108.88^2 + (40.83 + 23.25)^2}$$
  
 $V_R^* = 126.34 \text{ kN}$ 

Try 10 M24 8.8/TB Bolts on each face (Bolts are in single shear)

Shear capacity per bolt (threads included)  $\phi V_f = 133 \text{ kN} > V_R^* = 126.34 \text{ kN}$  OK

# Check Bearing Capacity of the column flange at the bolt holes:

### Check Tearing Capacity of the column flange:

The maximum vertical bolt force  $F_{1y \text{ (Resultant)}} = 40.83 + 23.25 = 64.08 \text{ kN}$ , this force can cause tearing between the bolt holes.

$$a_{ey} = 100 - d_{hole} + r_{bolt} = 100 - 26 + 12 = 86 \text{ mm}$$
  
 $\phi V_b = 0.9 \text{ a}_e t_p f_{up}$  AS4100 Cl.9.3.2.4(2)  
 $\phi V_b = 0.9 \times 86 \times 15.4 \times 440 \times 10^{-3} = 524.46 \text{ kN} > F_{1y \text{(Resultant)}} = 64.08 \text{ kN}$  OK

Use 10 M24 Bolts grade 8.8 / TB on each face

# Bracket:

Try 540 x 20 plate grade 300 steel shaped as shown in Figure 9.17 on each face.

$$M^* = 232.5 \times 0.6 = 139.5 \text{ kNm}$$
 (Max. Bending Moment in Bracket)

$$\phi M_{\rm s} = \frac{bh^2}{4} \times f_y = \frac{20 \times 540^2}{4} \times 300 = 437.4 \text{ kNm} > M^*$$
 OK

*Comment*: No need to check tearing and bearing in the bracket at the bolt holes because the bracket is thicker than the column flanges.

# 9.4.2.4 Bolted Splice Connection

Design a bolted cover plate splice for 460UB 67.1 in grade 300 steel with limited slip in the serviceability limit state. The splice is subject to the following design actions:

$$M^* = 230 \text{ kNm}$$
$$V^* = 115 \text{ kN}$$

$$V^* = 115 \text{ kN}$$

For serviceability limit state the splice is to carry:

$$M_{\rm s}^* = 120 \text{ kNm}, V_{\rm s}^* = 76.7 \text{ kN}$$

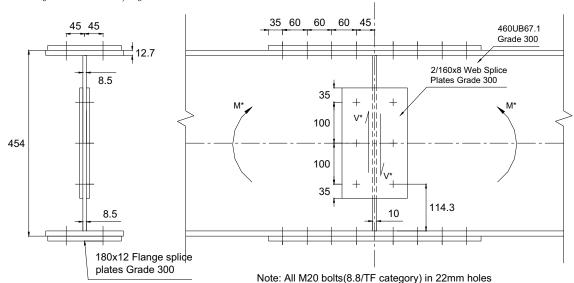


Figure 9.19 Bolted Cover Plate Splice

### Solution

Assume that the flange splices carry all of the moment and that the web splice only carries the shear.

# (1) Flange Splice

## Serviceability Limit State:

The moment acting on the connection is replaced by a couple whose forces act at the centroid of the beam flanges

Flange force = 
$$M_s^*/(d - t_f) = 120 \times 10^3/(454-12.7) = 271.92 \text{ kN}$$

Try 12x180x520 mm plate, grade 300 ( $f_{yp}$ = 310 MPa,  $f_{up}$  = 430 MPa)

Try M20 bolts in 8.8/TF category, bolts are in single shear.

$$V_{\rm sf} = \mu \, n_{\rm ei} \, N_{\rm ti} \, k_{\rm h}$$
 AS4100 Cl. 9.3.3.1   
  $V_{\rm sf} = 0.35 \times 1 \times 145 \times 1 = 50.75 \, \rm kN$   $\phi V_{\rm sf} = 0.7 \times 50.75 = 35.53 \, \rm kN$  (Slip Resistance / Bolt)

Number of Bolts required = 271.92 / 35.53 = 7.6

Try 4 rows of 2 x M20 bolts in each flange (each side of joint).

### Strength Limit State Check:

The moment acting on the connection is replaced by a couple whose forces act at the centroid of the beam flanges.

$$N_{\rm ft}^* = N_{\rm fc}^* = M^* / (d-t_{\rm f}) = 230 \times 10^3 / (454 - 12.7) = 521.2 \text{ kN}$$
  
 $V_{\rm f}^* = 521.2 / 8 = 65.15 \text{ kN}$  (Shear force / Bolt)

Design shear capacity per bolt for threads included in single shear plane

$$\phi V_{\rm f} = 0.8 \times 0.62 f_{\rm uf} k_{\rm r} A_{\rm c}$$
 AS4100 Cl. 9.3.2.1

*Note*: threads would normally be assumed included in the shear plane for both the flange and the web splices.

$$L_j$$
 = the distance from first to last bolt on each side of splice  $L_j$  = 60 x 3 = 180 mm < 300 mm  $k_r$  = 1.0   
 $\phi V_f$  = 0.8 x 0.62 x 830 x 1 x 1 x 225  $\phi V_f$  = 92.6 kN >  $V_f^*$  = 65.15 kN  $OK$ 

# Check Bearing Capacity of the Flange Splice Plate at the Bolt-Holes:

 $V_f^* = 65.15 \text{ kN}$  (bolt force acting on the plate at the bolt hole)

$$\phi V_b = 0.9 \ (3.2 \ d_f t_p f_{up})$$

$$\phi V_b = 0.9 \ x \ 3.2 \ x \ 20 \ x \ 12 \ x \ 430 \ x \ 10^{-3}$$

$$\phi V_b = 297.2 \ kN$$

$$\phi V_b = 297.2 \ kN > V_f^* = 65.15 \ kN$$
 $OK$ 

# Check Tearing of the Flange Splice Towards an Edge:

Bolt force acting on the plate at the bolt hole and that can cause tearing towards an edge  $V_f^* = 65.15 \text{ kN}$ 

$$\phi V_b = 0.9 \text{ a}_e t_p f_{up}$$

$$\phi V_b = 0.9 \times 34 \times 12 \times 430 \times 10^{-3}$$

$$\phi V_b = 157.9 \text{ kN} > V_f^* = 65.15 \text{ kN}$$

$$OK$$
AS4100 Cl.9.3.2.4(2)

Use 4 rows of 2xM20 bolts 8.8/TF category in each flange (each side of joint).

#### *Check the tensile capacity of plates:*

$$\begin{split} \phi N_t &= \phi A_g f_y & \text{(yielding limit state)} \\ \phi N_t &= 0.9 \text{ x } 180 \text{ x } 12 \text{ x } 310 \text{ x } 10^{-3} \\ \phi N_t &= 602.64 \text{ kN} > N_{\rm ft}^* = 521.2 \text{ kN} & OK \\ \phi N_t &= \phi 0.85 k_t A_n f_u & \text{(fracture limit state)} \\ \phi N_t &= 0.9 \text{ x } 0.85 \text{ x } 1 \text{ x } (180 \text{ x } 12 - 2 \text{ x } 22 \text{ x } 12) \text{ x } 430 \text{ x } 10^{-3} \\ \phi N_t &= 536.85 \text{ kN} > N_{\rm ft}^* = 521.2 \text{ kN} & OK \end{split}$$

Use 2 x (12x180x520) mm grade 300 flange splice plates

# (2) Web Splice:

Try 2 x (8x160x270mm) grade 300 web splice plates.

Try using 3M20 bolts 8.8/TF category, on each side of the joint (Bolts are in double shear).

# Serviceability Limit State:

$$V_{\rm s}^* = 76.7 \text{ kN}$$
  
Shear force / Bolt =  $76.7/3 = 25.3 \text{ kN}$   
Slip resistance / Bolt =  $\phi V_{\rm sf} = \mu n_{\rm ei} N_{\rm ti} k_{\rm h} = 0.7 \times 0.35 \times 2 \times 145 \times 1$   
 $\phi V_{\rm sf} = 71.1 \text{ kN} > V_{\rm sf}^* = 25.3 \text{ kN}$  *OK*

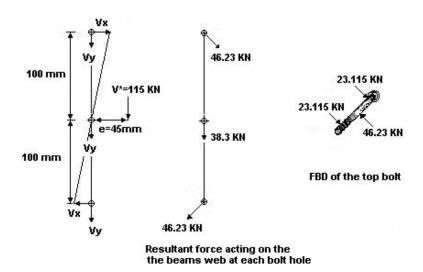
#### Comment

In connections where no slip requirement in the serviceability limit state is essential, a special bolt category 8.8 / TF is used. These bolts are designed so that service shear loads are transmitted by friction between the plies under the bolt head, not through bearing between the bolts and the internal surface of the holes in the plates, and shearing on the bolts; that's why they require a special check under service loads, when the slip load is reached these bolts will transmit the shear forces exactly like any bearing type bolt, therefore AS 4100 requires to check the capacity of these bolts as bearing type bolts at the ultimate limit state.

# Strength Limit State Check:

There are two bolt groups (one each side of the centre line of the splice) each having one column of three bolts.

Equal eccentricity (e = 45mm) is taken on each bolt group.



Horizontal shear force on bolt due to moment due to eccentricity

$$V_{fx}^* = \frac{V^* \times e \times r_i}{\Sigma r_i^2} = \frac{115 \times 45 \times 100}{2 \times 100^2}$$
  
 $V_{fx}^* = 25.9 \text{ kN}$ 

$$V_{fy}^* = 115 / 3 = 38.3 \text{ kN}$$
 (vertical shear force per bolt)  
 $V_f^* = \sqrt{38.3^2 + 25.92} = 46.23 \text{ kN}$  (resultant shear force on top and bottom bolt)

This force is trying to shear off the bolt on two shear planes (i.e. the bolt is in double shear)

$$\phi V_{\rm f} = 2 \times 92.6$$
 (Shear capacity / Bolt (threads included))

$$\phi V_f = 185.2 \text{ kN} > V_f^* = 46.23 \text{ kN}$$

# Check Bearing Capacity of the Beam's Web at the Bolt-Holes:

$$V_f^* = 46.23 \text{ kN}$$
 (maximum bolt force acting on the web at the bolt hole)

$$\phi V_b = 0.9 \ (3.2 \ d_f t_p f_{up})$$
 AS4100 Cl.9.3.2.4(1)

$$\phi V_b = 0.9 \times 3.2 \times 20 \times 8.5 \times 440 \times 10^{-3}$$

$$\phi V_b = 215.42 \text{ kN} > V_f^* = 46.23 \text{ kN}$$
 OK

# Check Bearing Capacity of the Web Plates at the Bolt-Holes:

$$V_{fb}^* = 46.23 / 2 = 23.1 \text{ kN}$$
 (maximum bolt force acting on one web plate at the bolt hole)

$$\phi V_b = 0.9 \times 3.2 \times 20 \times 8 \times 430 = 198.14 \text{ kN}$$

$$\phi V_b = 198.14 \text{ kN} > V_{fp}^* = 23.1 \text{ kN}$$
 OK

# Check Tearing Capacity of the Beam's Web at the Bolt-Holes:

# (a) Towards an edge

$$V_{fx}^{*} = 25.9 \text{ kN}$$

$$a_{ex} = 45 - 10/2 - r_{hole} + r_{holt} = 40 - 11 + 10 = 39 \text{ mm}$$

$$\phi V_b = 0.9 \text{ a}_e t_p f_{up}$$
AS4100 Cl.9.3.2.4(2)

$$\phi V_b = 0.9 \times 39 \times 8.5 \times 440 \times 10^{-3}$$

$$\phi V_b = 131.3 \text{ kN} > V_{fx}^* = 25.9 \text{ kN}$$
 OK

$$V_{fv}^* = 38.3 \text{ kN}$$

$$a_{\text{ey}} = 114.3 - r_{\text{hole}} + r_{\text{bolt}} = 114.3 - 11 + 10 = 113.3 \text{ mm}$$

$$\phi V_b = 0.9 \text{ x } 113.3 \text{ x } 8.5 \text{ x } 440 \text{ x } 10^{-3} = 381.4 \text{ kN} > V_{fy}^* = 38.3 \text{ kN}$$
 OK

# (b) Between Bolt holes:

$$V_{fv}^* = 38.3 \text{ kN}$$

$$a_{ev} = 100 - d_{hole} + r_{hole} = 100 - 22 + 10 = 88 \text{ mm}$$

$$a_{\text{ey}} = 100 - d_{\text{hole}} + r_{\text{hole}} = 100 - 22 + 10 = 88 \text{ mm}$$
  
 $\phi V_b = 0.9 \times 88 \times 8.5 \times 440 \times 10^{-3} = 296.2 \text{ kN} > V_{fy}^* = 38.3 \text{ kN}$  OK

#### Check Tearing Capacity of the Plates at the Bolt-Holes:

$$V_{fpx}^* = 25.9 / 2 = 12.95 \text{ kN}$$

$$a_{ex} = 34 \text{ mm}$$

$$V_{fpy}^* = 38.3 / 2 = 19.15 \text{ kN (govern)}$$

$$a_{ev} = 34 \text{ mm}$$

$$\phi V_b = 0.9 \times 34 \times 8 \times 430 \times 10^{-3} = 105.3 \text{ kN} > V_{fpy}^* = 19.15 \text{ kN}$$
 OK

Use 3 M20 grade 8.8 /TF bolts at 100 mm pitch in web each side of joint.

Use 2 / (160 x 8) grade 300 web splice plates.

# 9.4.2.5 Bolted End Plate Connection (Standard Knee Joint)

Design a bolted end plate to connect a haunched 530UB92.4 rafter to the flange of 800WB122 column. The connection is subjected to the design actions listed in Table 9.2.5 and is shown in Figure 9.20.All steel is grade 300.

Table 9.2.5

| Load Combination  | $M^*$ (kNm) | $N^*$ (kN)  | $V^*(kN)$ |
|-------------------|-------------|-------------|-----------|
| LC1 $1.2G + 1.5Q$ | -961.25     | 188.24 (C)  | 194.64    |
| LC2 0.9G + Wu     | 411.44      | -113.92 (T) | -91.23    |

The shear force in the column just below the bottom flange of haunch is:  $V_c^* = 131.74$  kN for LC1 and 105.8 kN for LC2

#### Solution

## Derived design action for design of bolts, end plate and stiffeners

The bending moments, axial forces and shear forces that correspond to the above load cases are shown in Figure 9.20. AISC Connection Manual [2] states that for the design of bolts, end plate and stiffeners, it is conventional practise to assume that all of the force above and below the neutral axis is concentrated at the flanges.

# (a) Maximum tension in top flange and maximum compression in bottom flange LC1

$$M^* = 961.25 \text{ kNm (at the column face)}$$

$$N^* = 188.24 \text{ kN (compression)}$$

$$V^* = 194.64 \text{ kN}$$

$$N_{\text{ft}}^* = \frac{M^*}{d_H - t_{f_H}} \times \cos \theta - \frac{N^*}{2} \times \cos \theta + \frac{V^*}{2} \times \sin \theta$$

$$N_{\text{ft}}^* = \frac{961.25 \times 10^3}{1040 - 15.6} \times \cos 14.93^\circ - \frac{188.24}{2} \times \cos 14.93^\circ + \frac{194.64}{2} \times \sin 14.93^\circ$$

$$N_{\text{ft}}^* = 841 \text{ kN}$$

$$N_{\text{fc}}^* = \frac{M^*}{d_H - t_{f_H}} \times \cos \theta + \frac{N^*}{2} \times \cos \theta - \frac{V^*}{2} \times \sin \theta$$

$$N_{\text{fc}}^* = \frac{961.25 \times 10^3}{1040 - 15.6} \times \cos 14.93^\circ + \frac{188.24}{2} \times \cos 14.93^\circ - \frac{194.64}{2} \times \sin 14.93^\circ$$

$$N_{\text{fc}}^* = 973 \text{ kN}$$

$$V_{\text{vc}}^* = V^* \mathbf{x} \cos\theta + N^* \mathbf{x} \sin\theta$$
  
 $V_{\text{vc}}^* = 194.64 \mathbf{x} \cos 14.93^\circ + 188.24 \mathbf{x} \sin 14.93^\circ$ 

$$V_{\rm vc}^{\ \ *} = 237 \ \rm kN$$

# (b) Maximum tension in bottom flange and maximum compression in top flange LC2

$$M^* = 411.44 \text{ kNm}$$

$$N^* = 113.92 \text{ kN (tension)}$$

$$V^* = 91.23 \text{ kN}$$

$$N_{\text{fi}}^* = \frac{M^*}{d_H - t_{f_H}} \times \cos\theta + \frac{N^*}{2} \times \cos\theta - \frac{V^*}{2} \times \sin\theta$$

$$N_{\rm ft}^* = \frac{414.44 \times 10^3}{1040 - 15.6} \times \cos 14.93^\circ + \frac{113.92}{2} \times \cos 14.93^\circ - \frac{91.23}{2} \times \sin 14.93^\circ$$

$$N_{\rm ft}^* = 431 \, \rm kN$$

$$N_{\text{fc}}^* = \frac{M^*}{d_H - t_{f_H}} \times \cos \theta - \frac{N^*}{2} \times \cos \theta + \frac{V^*}{2} \times \sin \theta$$

$$N_{\text{fc}}^* = \frac{411.44 \times 10^3}{1040 - 15.6} \times \cos 14.93^\circ - \frac{113.92}{2} \times \cos 14.93^\circ + \frac{91.23}{2} \times \sin 14.93^\circ$$

$$N_{\text{fc}}^* = 333 \text{ kN}$$

$$V_{vc}^* = V^* \times \cos\theta + N^* \times \sin\theta$$
  
 $V_{vc}^* = 91.23 \times \cos 14.93^\circ + 113.92 \times \sin 14.93^\circ$   
 $V_{vc}^* = 118 \text{ kN}$ 

#### **Bolts**

Maximum tensile force acting on the top two rows of bolts  $N_{\rm ft}^*$  = 841 kN (LC1) Increase this force by 30% to allow for additional bolt force due to prying and divide it by 4 to get the design tensile force per bolt.

$$N_{\rm ftp}^*$$
 / Bolt = 841 x 1.3 / 4 = 273.3 kN

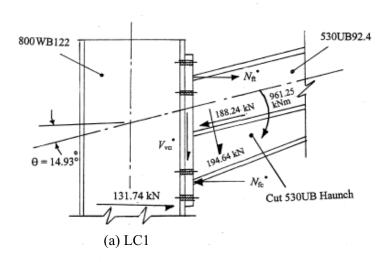
Try 4 – M30 8.8 / TB at the top flange (Tension flange for LC1) 
$$\phi N_{\text{tf}} = 0.8 \ A_{\text{s}} f_{\text{uf}} = (0.8 \ \text{x} \ 561 \ \text{x} \ 830) \ \text{x} \ 10^{-3} = 373 \ \text{kN} > N_{\text{ftp}}^{*} / \text{Bolt} = 273.3 \ \text{kN}$$
  $OK$ 

**Note:** *It is suggested in the* AISC structural steelwork connections manual [2] *to allow an increase of* 20 *to* 33% *in the bolt tension force to account for prying action.* 

A commonly accepted assumption made in a number of references states that the shear force acting on the connection is taken by the compression flange bolts.

Try 4 – M30 8.8 / TB at the bottom flange (Compression flange for LC1) Design shear force per bolt 
$${V_{\rm vc}}^*$$
 / Bolt = 237/4 = 59.25 kN (LC1) Design shear capacity of a bolt with threads included in the shear plane  $\phi V_{\rm f} = 0.8 \times 0.62 \ A_{\rm c} f_{\rm uf} = 0.8 \times 0.62 \times 519 \times 830 = 214 \ {\rm kN} > V_{\rm vc}^*$  / Bolt = 59.25 kN  $\ OK$ 

### Hence adopt M30 8.8 / TB bolts



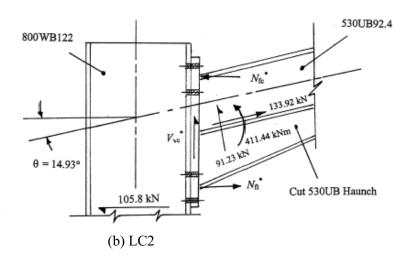


Figure 9.20 Design Actions for Knee Joint (a)LC 1 (b)LC 2

# End Plate

The end plate thickness must be such that

$$N_{\rm ft}^* \leq [\phi N_{\rm pb}, \phi V_{\rm ph}]$$

Try 240 x 32 rolled edge flat x 1310 mm deep in grade 300 steel.

## (a) Check Flexural Capacity

Refer to Figure 9.21  $\phi N_{pb} = \phi f_{yi} \times b_i t_i^2 / a_{fe}$ Where  $b_i$ ,  $t_i$ ,  $f_{yi}$  = plate width, thickness and yield stress respectively.  $a_{fe} = a_f - (d_f/2)$   $a_f = (S_p - t_{fb}) / 2 = (150 - 15.6) / 2 = 67 \text{mm}$   $a_{fe} = 67 - (30/2) = 52.2 \text{mm}$  $\phi N_{pb} = 0.9 \times 280 \times 240 \times 32^2 / 52 = 1191 \text{ kN} > N_f^* = 841 \text{ kN}$  *OK* 

**Note:** The strength of the end plate in bending is based on the assumption of double curvature.

# (b) Check Shear Capacity

The shear stress distribution in a rectangular plate is parabolic with the maximum stress being 1.5 times the average stress. AS 4100 makes allowances for non-uniform shear stress in a web with the formula,

$$V_{\rm v} = [2V_{\rm u}/(0.9 + (f_{\rm um}^*/f_{\rm va}^*))] \le V_{\rm u}$$
 AS4100 Cl.5.11.3

where  $V_{\rm u}$  is the nominal shear capacity of a web with uniform stress distribution  $f_{\rm vm}^*$ ,  $f_{\rm va}^*$  = the maximum and average shear stresses in the web respectively.

#### Hence,

$$\begin{aligned} &V_{\rm v} = [2\,V_{\rm u}\,/\,(0.9 + 1.5)] = 0.833\,\,V_{\rm u} \\ &V_{\rm v} = 0.833\,\,\mathbf{x}\,\,0.6\,f_{\rm y}A_{\rm w} = 0.5\,f_{\rm y}A_{\rm w} \\ &\phi V_{\rm ph} = 2\,\,\mathbf{x}\,\,\phi 0.5\,f_{\rm y}A_{\rm w} \\ &\phi V_{\rm ph} = 2\,\,\mathbf{x}\,\,0.9\,\,\mathbf{x}\,\,0.5\,\,\mathbf{x}\,\,280\,\,\mathbf{x}\,\,240\,\,\mathbf{x}\,\,32 = 1935.4\,\,\mathrm{kN} > {N_{\rm ft}}^* = 841\,\,\mathrm{kN} \quad OK \end{aligned}$$

**Note:** The maximum shear force acting on the endplate is half the design flange force  $N_{\rm ft}^*$  and therefore the designer can either multiply the shear capacity by 2 and compare the results with  $N_{\rm ft}^*$  or divide the design flange force  $N_{\rm ft}^*$  by 2 and compare the results with the shear capacity of the end plate <u>above</u> the top flange.

Hence adopt 240 x 32 rolled edge flat x 1310 mm deep in grade 300 steel.

Sec.4.8.3.4(a) [2]

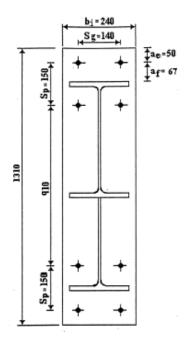


Figure 9.21

# **Check need for Tension Stiffeners:**

Tension stiffeners are needed if

$$N_{\rm ft}^* > \phi R_{\rm t}$$

$$\phi R_{t} = [\phi R_{t1}, \, \phi R_{t2}]_{\min}$$

$$\phi R_{t1} = \phi f_{ycf} t_{fc}^2 \mathbf{x} [3.14 + (2a_c + S_p - d_h) / a_d]$$

Where the notation is that adopted in AISC Connection Manual [2].

$$\phi = 0.9$$

$$f_{\text{ycf}} = 300 \text{ MPa}$$
  
 $S_g = 140 \text{ mm}$ 

$$S_{a} = 140 \text{ mm}$$

$$S_p = 150 \text{ mm}$$

### Check Geometry Restrictions:

$$S_g \le b_{fb} - d_f = 209 - 30 = 179 \text{ mm}$$

$$S_{g \text{ (used)}} = 140 \text{ mm} < 179 \text{ mm } OK$$

 $a_c$  = bolt edge distance for column flange

$$a_c = (b_{fc}-S_g)/2 = (250-140)/2 = 55 \text{ mm} > 1.25 d_f = 1.25 \times 30 = 37.5 \text{ mm } OK$$

$$d_h = d_f + 2 \text{ mm} = 30 + 2 = 32 \text{ mm}$$

$$a_d = \frac{S_g - t_{wc} - 2r_c}{2} = \frac{140 - 10 - 2 \times 0}{2} = 65 \text{ mm}$$

$$\phi R_{t1} = 0.9 \times 300 \times 16^2 \times [3.14 + (2 \times 55 + 150 - 32) / 65]$$

$$\phi R_{t1} = 460 \text{ kN}$$

$$\phi R_{12} = \phi \left[ f_{ycf} t_{fc}^{2} \times \left[ \frac{3.14 \times (a_{d} + a_{c}) + 0.5S_{p}}{(a_{d} + a_{i})} \right] + 4 \left[ \frac{a_{i}}{(a_{d} + a_{i})} \right] N_{ff}^{*} \right]$$
 Sec.4.8.3.4 [2]

 $a_i$  = bolt edge distance for end plate

$$a_i = (b_i - S_g)/2 = (240 - 140)/2 = 50 \text{ mm} > 1.5 d_f = 1.5 \times 30 = 45 \text{ mm } OK$$

 $N_{\rm tf}^*$  = maximum design bolt force in tension = 841 / 4 = 210.2 kN

*Note*: No prying action should be included in  $N_{\rm tf}$ .

$$\phi R_{12} = 0.9 \text{ x} \left[ 300 \times 16^2 \times \left[ \frac{3.14 \times (65 + 55) + 0.5 \times 150}{(65 + 50)} \right] \times 10^{-3} + 4 \left[ \frac{50}{(65 + 50)} \right] \times 210.2 \right]$$

 $\phi R_{t2} = 601 \text{ kN}$ 

 $\phi R_t = [460,601]_{min}$   $\phi R_t = 460 \text{ kN} < N_{ft}^* = 841 \text{ kN} \quad NG$ 

Hence tension stiffeners are required for the column flange at the tension flange of haunch.

## **Design Tension Stiffeners**

Stiffeners are proportioned to carry the excess tension force the column flange can't take.

$$N_{\text{ts}}^* \le \phi N_{\text{ts}}$$
 Sec.4.8.3.5(a) [2]

Where,

 $N_{\rm ts}^*$  = stiffener design force at the tension flange =  $N_{\rm ft}^*$  -  $\phi R_{\rm t}$  = 841 - 460 = 381 kN

 $\phi N_{ts}$  = stiffener capacity in tension =  $\phi f_{vs} A_{s}$ 

Try 2-100 x 8 plates in grade 300 steel

Since under different loading the stiffeners will be under compression they should satisfy:

$$b_{es} \le \frac{15 \ t_s}{\sqrt{\frac{f_{ys}}{250}}} = \frac{15 \times 8}{\sqrt{\frac{320}{250}}} = 106 \,\text{mm}$$

 $b_{es} = 100 \text{ mm} < 106 \text{ mm } OK$ 

$$\phi N_{ts} = 0.9 f_{vs} A_{s} = 0.9 \text{ x } 320 \text{ x } 2 \text{ x } 100 \text{ x } 8 = 461 \text{ kN} > N_{ts}^{*} = 381 \text{ kN } OK$$

### **Stiffeners Welds:**

Across column flange at tension flange, welded both sides for each stiffener.

Try 6 E48XX SP fillet welds

Total run of fillet weld across column flange at tension flange of haunch = 2 x 2 x 100 = 400

Weld capacity =  $\phi V_w \mathbf{x}$  total run of weld = 0.978  $\mathbf{x}$  400 = 391 kN >  $N_{ts}^*$  = 381 kN OKHence adopt 6 E48XX SP fillet welds.

# **Check Need for Doubler Plates:**

Once the need to provide tension stiffeners to the column flange has been established, it is necessary to check that the stiffened flange is strong enough [2]

Doubler Plates are required if,

$$N_{\rm ft}^* > \phi R_{\rm ts}$$
 Sec.4.8.3.4(d) [2]

Where,

 $\phi R_{ts}$  = capacity of the stiffened column flange.

$$\Phi R_{ts} = \Phi f_{ycf} t_{fc}^{2} \left[ \frac{2w_{2} + 2w_{1} - d_{h}}{a_{d}} + \left( \frac{1}{w_{1}} + \frac{1}{w_{2}} \right) (2a_{d} + 2a_{c} - d_{h}) \right]$$

$$w_{1} = \sqrt{a_{d} \times (a_{d} + a_{c} - 0.5d_{h})} = \sqrt{65 \times (65 + 55 - 0.5 \times 32)} = 82 \text{ mm}$$

$$w_{1} = S_{p} - t_{s} - 2t_{w} \leq w_{s}$$

$$w_{2} = \frac{S_{p} - t_{s} - 2t_{w}}{2} \le w_{1}$$

$$w_{2} = \frac{150 - 8 - 2 \times 6}{2} = 65 \text{ mm} < 82 \text{ mm } OK$$

$$\phi R_{ts} = 0.9 \text{ x} \quad 300 \times 16^{2} \left[ \frac{2 \times 65 + 2 \times 82 - 32}{65} + \left( \frac{1}{82} + \frac{1}{65} \right) (2 \times 65 + 2 \times 55 - 32) \right]$$

$$\phi R_{ts} = 675 \text{ kN} < N_{ft}^{*} = 841 \text{ kN} \quad NG$$

Hence flange doubler plates are required.

## **Design Doubler Plates**

If flange doubler plates are used instead of conventional stiffeners, the following must be satisfied:

$$N_{\rm ft}^* > \phi R_{\rm td}$$

Where.

$$\phi R_{\text{td}} = \phi \left( t_{fc}^{2} f_{ycf} + \frac{t_{d}^{2} f_{yd}}{2} \right) \times \left( \frac{S_{p} + 4a_{d} + 1.25a_{c}}{a_{d}} \right)$$
 Sec.4.8.3.4(d) [2]

Try 2-115 x 12 plates in grade 300 steel butt welded to the column web in lieu of the two conventional stiffeners

$$\phi R_{td} = 0.9 \text{ x } \left( 16^2 \times 300 + \frac{12^2 \times 300}{2} \right) \times \left( \frac{150 + 4 \times 65 + 1.25 \times 55}{65} \right)$$

$$\phi R_{\rm td} = 652 \text{ kN} < N_{\rm ft}^* = 841 \text{ kN} \quad NG$$

Hence increase the thickness of the doubler plates or use them in combination with conventional stiffeners. As compression stiffeners will probably be required at the haunch top flange for load combination LC2, use doubler plates in combination with conventional stiffeners.

No formula is recommended in the AISC Connection Manual [2] for the case where both doubler plates and conventional stiffeners are used, but it is suggested by the Design of Portal Frame Buildings [4] that the expression for  $\phi R_{ts}$  to be used with  $(t_{fc} + t_d)$  substituted for  $t_{fc}$ .

Hence,

$$\phi R_{\text{ts}} = \frac{(16+12)^2}{16^2} \times 675 = 2067 \text{ kN} > N_{\text{ft}}^* = 841 \text{ kN} \text{ OK}$$

# **Check Need for Compression Stiffeners**

Compression stiffeners are required if

$$N_{\rm fc}^* > \phi R_{\rm c} = [\phi R_{\rm c1}, \phi R_{\rm c2}]_{\rm min}$$

The following methods can be used to calculate  $\phi R_{c1}$  and  $\phi R_{c2}$ 

- (i) AISC Connection Manual [2] method which is based on actual tests on moment connections.
- (ii) AS 4100 [1] method.

The first method is adopted in this example; nevertheless the second method is presented in italics for comparison.

# (i) AISC Connection Manual [2] method:

Design Bearing Yield Capacity  $\phi R_{c1}$ 

$$\phi R_{\rm c1} = \phi f_{\rm ycw} t_{\rm wc} \left( t_{\rm fb} + 5k_{\rm c} + 2t_{\rm i} \right)$$

$$\phi R_{c1} = 4.5 f_{ycw} t_{wc} k_c + 0.9 f_{ycw} t_{wc} (t_{fb} + 2t_i) = k_9 + k_{10} (t_{fb} + 2t_i)$$

 $k_c$  = distance from outer face of column flange to inner end of root radius

$$k_c = t_{fc} + r_c = 16 + 0 = 16 \text{ mm}$$

$$k_9 = 4.5 f_{ycw} t_{wc} k_c = 4.5 \times 310 \times 10 \times 16 = 223.2 \text{ kN}$$

$$k_{10} = 0.9 f_{ycw} t_{wc} = 0.9 \times 310 \times 10 = 2.79 \text{ kN/mm}$$

$$\phi R_{c1} = 223.2 + 2.79 (15.6 + 2 \times 32) = 445.3 \text{ kN}$$

Design Bearing Buckling Capacity  $\phi R_{c2}$ 

$$\phi R_{c2} = \frac{0.9 \times 10.8 \ t_{wc}^{3} \ \sqrt{f_{ycw}}}{d_{wc}}$$

$$d_{\text{wc}} = d_{\text{c}} - 2 \times k_{\text{c}} = 792 - 2 \times 16 = 760 \text{ mm}$$

$$\phi R_{c2} = \frac{0.9 \times 10.8 \times 10^3 \sqrt{310}}{760} = 225.2 \text{ kN}$$

$$\phi R_c = [445.3, 225.2]_{\text{min}} = 225.2 \text{ kN} < N_{fc}^* = 973 \text{ kN} \ NG$$

Hence compression stiffeners are required for the column's web at the bottom flange of the haunch,

## (ii) Alternative AS 4100 [1] method

*Design Bearing Yield Capacity*  $\phi R_{c1}$ 

$$\phi R_{c1} = 0.9 (1.25 \ b_{bf} t_{wc} f_{yw})$$

$$b_{bf} = t_{fb} + 5t_{fc} + 2t_{i} = 15.6 + 5 \times 16 + 2 \times 32 = 159.6 \text{ mm}$$

$$\phi R_{c1} = 0.9 \text{ x } 1.25 \text{ x } 159.6 \text{ x } 10 \text{ x } 310 = 556.6 \text{ kN}$$

Design Bearing Buckling Capacity

$$\phi R_{c2} = 0.9 (\alpha_c k_f A_{wc} f_{ycw})$$

AS4100 Cl. 5.13.4

AS4100 Cl. 5.13.3

where.

 $k_{\rm f}$  = 1.0 since local buckling is not a design consideration

$$A_{\rm wc} = b_{\rm b} \times t_{\rm w}$$

$$b_{\rm b} = b_{\rm bf} + d_2$$

 $d_2$  = twice the clear distance from the neutral axis to the compression flange =  $d_1$  for a symmetrical section

 $\alpha_c$  = the member slenderness reduction factor

The web is treated like a column of a cross section  $A_{wc}$  and a length  $d_1$ 

$$b_b = 159.6 + 760 = 919.6 \text{ mm}$$

$$A_{\rm wc} = 919.6 \text{ x } 10 = 9196 \text{ mm}^2$$

$$\alpha_c = 2.5 d_1 / t_w = 2.5 \times 760 / 10 = 190$$

AS 4100 Cl. 5.13.4

$$f_{\text{yew}} = 310 \text{ MPa}$$

$$\alpha_b = 0.5$$
 (other sections not listed)

*AS*4100 *Table* 6.3.3(1)

$$\lambda_{\rm n} = (L_{\rm e}/{\rm r})\sqrt{k_{\rm f}}\sqrt{(f_{\rm yw}/250)}$$

$$\lambda_n = 190 \times 1 \times \sqrt{(310/250)} = 211.58$$

$$\alpha_c = 0.152$$

AS4100 Table 6.3.3(3)

$$\phi R_{c2} = 0.9 \times 0.152 \times 1 \times 310 \times 9196 = 390 \text{ kN}$$

$$\phi R_c = [556.6, 390]_{\text{min}} = 390 \text{ kN} < N_{fc}^* = 973 \text{ kN} \quad NG$$

## **Design Compression Stiffeners**

The following methods can be used to design compression stiffeners:

- (iii)AISC Connection Manual [2] method which is based on actual tests on moment connections.
- (iv) AS 4100 [1] method.

The first method is adopted in this example; nevertheless the second method is presented in italics for comparison.

# (i) AISC Connection Manual [2] method:

Stiffeners are proportioned to carry the excess load so that

$$N_{\rm cs}^* \leq \phi N_{\rm cs}$$

Sec.4.8.3.5 (b) [2]

$$N_{cs}^*$$
 = stiffener design force at compression flange  
=  $N_{fc}^*$  -  $\phi R_c = 973 - 225.2 = 747.8 kN$ 

Try 2-120 x 12 plates in grade 300 steel

# Check Outstand

$$b_{es} \le \frac{15 t_s}{\sqrt{\frac{f_{ys}}{250}}} = \frac{15 \times 12}{\sqrt{\frac{310}{250}}} = 162 \text{ mm but not more than } (b_{fc} - t_{wc}) / 2 = (250-10) / 2 = 120 \text{ mm}$$

$$b_{es} = 120 \text{ mm} OK$$

$$\phi N_{\text{ts}} = 0.9 f_{\text{vs}} A_{\text{s}} = 0.9 \times 310 \times 2 \times 120 \times 12 = 803.52 \text{ kN} > N_{\text{cs}}^* = 747.8 \text{ kN } OK$$

# Check Strength of Stiffened Web

The stiffened web may be considered satisfactory if

$$N_{\rm fc}^* \leq \phi R_{\rm cs}$$

$$\phi R_{\rm cs} = \phi f_{\rm ys} A_{\rm s} + 1.47 f_{\rm ycw} t_{\rm fc} \sqrt{b_{fc} t_{\rm wc}}$$

$$\phi R_{cs} = 803.52 + 1.47 \times 310 \times 16 \times \sqrt{(250 \times 10)}$$

$$\phi R_{cs} = 1168.08 \text{ kN} > N_{fc}^* = 973 \text{ kN } OK$$

Hence adopt 2-120 x 12 compression stiffeners for the columns web at the bottom flange of haunch.

# **Stiffeners Welds:**

Along web of column flange to resist stiffener design force at compression flange, welded both sides for each stiffener.

Try 6 E48XX SP fillet welds

Total run of fillet weld along web of column at compression flange =  $2 \times 2 \times 760 = 3040 \text{ mm}$  Weld capacity =  $\phi V_w \times 1000 \text{ total}$  run of weld =  $0.978 \times 3040 = 2973 \text{ kN} > N_{cs}^* = 747.8 \text{ kN}$  OK Hence adopt 6 E48XX SP fillet welds.

# (ii) Alternative AS 4100[1] method

For comparison with the AISC method, the capacity of the web stiffened with 2-120 x 12 stiffeners will be calculated in accordance with AS4100.

## Check Outstand

$$b_{es} \le \frac{15 \ t_s}{\sqrt{\frac{f_{ys}}{250}}} = \frac{15 \times 12}{\sqrt{\frac{310}{250}}} = 162 \,\text{mm}$$
 but not more than  $(b_{fc} - t_{wc}) / 2 = (250 - 10) / 2 = 120 \,\text{mm}$ 

$$b_{es} = 120 \,\text{mm} \quad OK$$

#### Yield Capacity

$$\phi R_{\text{sy}} = \phi R_{\text{by}} + \phi f_{\text{ys}} A_{\text{s}}$$
  
 $\phi R_{\text{by}} = \phi R_{\text{c1}} = 556.6 \text{ kN}$   
 $\phi f_{\text{ys}} A_{\text{s}} = 0.9 \times 310 \times 2 \times 120 \times 12 = 803.52 \text{ kN}$   
 $\phi R_{\text{sy}} = 556.6 + 803.52 = 1360.12 \text{ kN} > N_{\text{fc}}^* = 973 \text{ kN } OK$ 

#### **Buckling Capacity**

The two stiffeners and part of the column web will behave as a cross shaped member. The width of the web included in this member has a width on each side of the centreline taken as the lesser of:

a) 
$$L_w = 17.5 t_w / \sqrt{(f_y/250)} = 17.5 \times 10 / \sqrt{(310/250)} = 157 \text{ mm}$$

b) S/2 which is not applicable in this case.

$$\begin{array}{l} \textit{A} = 2 \text{ x } 120 \text{ x } 12 + 2 \text{ x } 157 \text{ x } 10 = 6020 \text{ mm}^2 \\ \textit{I}_y = 12 \text{ x } (120 + 120 + 10)^3 / 12 + (157 + 157 - 12) \text{ x } 10^3 / 12 = 15.65 \text{ x } 10^6 \text{ mm}^4 \\ \textit{r}_y = \sqrt{(\textit{I}_y / A)} = \sqrt{(15.65 \text{ x } 10^6 / 6020)} = 51 \text{ mm} \\ \textit{I}_e / \textit{r} = \textit{I}_d / \textit{r} = 760 / 51 = 14.90 \\ \lambda_n = (\textit{I}_e / \textit{r}) \sqrt{\textit{k}_f} \sqrt{(\textit{f}_y / 250)} = 14.9 \text{x} \sqrt{1} \text{x} \sqrt{(310 / 250)} \\ \lambda_n = 16.6 \\ \alpha_b = 0.5 \\ \alpha_c = 0.984 \\ \phi \textit{N}_c = 0.9 \text{ } \alpha_c \textit{k}_f \textit{A}_n \textit{f}_y \end{array}$$

$$\phi N_c = 0.9 \text{ x } 0.984 \text{ x } 1\text{x } 6020 \text{ x } 310 = 1652.70 \text{ kN} > N_{fc}^* = 973 \text{ kN } OK$$

#### **Check Load Combination (2)**

Under load combination (2) the bottom flange of haunch is under tension while the top flange is under compression.

#### **Check need for Tension Stiffeners:**

 $\phi R_t = 460 \text{ kN}$  (as previously calculated)  $> N_{\rm ft}^* = 431 \text{ kN}$  *OK* Hence tension stiffeners are not required at the bottom flange of haunch.

# **Check Need for Compression Stiffeners**

$$\phi R_c = 225.2 \text{ kN} < N_{fc}^* = 333 \text{ kN } NG$$

Hence compression stiffeners are required for the column's web at the top flange of the haunch,

# **Design Compression Stiffeners**

Check the strength of the 2-100 x 8 plates used as tension stiffeners in LC1, using the AISC Connection Manual [2] method:

Stiffeners are proportioned to carry the excess load so that

$$N_{\rm cs}^* \le \phi N_{\rm cs}$$
 Sec.4.8.3.5 (b) [2]  $N_{\rm cs}^* = {\rm stiffener\ design\ force\ at\ compression\ flange}$   $= N_{\rm fc}^* - \phi R_{\rm c} = 333 - 225.2 = 107.8\ kN$ 

$$\phi N_{cs} = 0.9 f_{vs} A_{s} = 0.9 \times 320 \times 2 \times 100 \times 8 = 461 \text{ kN} > N_{cs}^{*} = 107.8 \text{ kN } OK$$

#### Check Strength of Stiffened Web

The stiffened web may be considered satisfactory if

$$N_{\rm fc}$$
  $\leq \phi R_{\rm cs}$ 

$$\phi R_{cs} = \phi f_{ys} A_s + 1.47 f_{ycw} t_{fc} \sqrt{b_{fc} t_{wc}}$$

$$\phi R_{cs} = 461 + 1.47 \times 310 \times 16 \times \sqrt{(250 \times 10)} \times 10^{-3}$$

$$\phi R_{cs} = 825.6 \text{ kN} > N_{fc}^* = 333 \text{ kN } OK$$

Hence adopt 2-100 x 8 compression stiffeners for the column's web at the top flange of the haunch.

# Stiffeners Welds:

The top two stiffeners will be under compression in LC2 and under tension in LC1 and therefore they should be welded along the web of the column to resist stiffener design force at compression flange (i.e. top flange in LC2) and across the column flange to resist stiffener design force at tension flange of haunch (i.e. top flange in LC1).

#### Try 6 E48XX SP fillet welds

Total run of fillet weld along web of column at compression flange =  $2 \times 2 \times 760 = 3040 \text{ mm}$ Weld capacity =  $\phi V_w \times 1000 \times 10000 \times 1000 \times 1000$  Hence adopt 6 E48XX SP fillet welds.

# **Check Need for Shear Stiffeners**

Shear force in column just below the haunch bottom flange  $V_c^* = 131.74$  kN (LC1) Resultant design shear force at bottom flange of haunch,  $V^* = N_{\rm fc}^* - V_{\rm c}^* = 973 - 131.74 = 841.26 \text{ kN}$ 

The web is required to satisfy,

$$V^* \leq \phi V_{v}$$

In this case the shear stress distribution can be assumed to be approximately uniform, so Clause 5.11.2 of AS 4100 applies.

$$d_{\rm p}/t_{\rm w} = d_{\rm 1}/t_{\rm w} = 760/10 = 76 > 82/\sqrt{(310/250)} = 73.64$$
 and therefore  $V_{\rm u} = V_{\rm b} = \alpha_{\rm v}/V_{\rm w} \le V_{\rm w}$ 

$$\alpha_{\rm v} = \left[\frac{82}{(d_p/t_{\rm w})\sqrt{(f_{\rm yew}/250)}}\right]^2 = \left[\frac{82}{76 \times \sqrt{(310/250)}}\right]^2 = 0.94 \qquad AS\,4100\,Cl.\,5.11.5.1$$

$$V_{\rm w} = 0.6\,f_{\rm yew}A_{\rm w} = 0.6\,{\rm x}\,310\,{\rm x}\,760\,{\rm x}\,10 = 1413.6\,{\rm kN}$$

$$V_{\rm u} = \alpha_{\rm v}\,V_{\rm w} = 0.94\,{\rm x}\,1413.6 = 1328.8\,{\rm kN}$$

$$\phi V_{\rm u} = 0.9\,{\rm x}\,1328.8 = 1996\,{\rm kN} > V^* = 841.26\,{\rm kN}\,OK$$

Hence shear stiffeners are not required.

### Weld Design:

For the design of the flange and web welds the AISC Connection Manual [2] assumes that the proportion of the bending moment transmitted by the web is  $k_{\text{mw}}$  while the proportion of the bending moment transmitted by the flanges is  $(1-k_{\rm mw})$  provided that the applied bending moment is equal to or less than the design moment capacity for the one set of yielding at the extreme fibbers  $(\phi M_v)$ , if the applied bending moment is more than  $\phi M_v$  the proportion of the bending moment transmitted by the flanges is  $M_{\rm f}^* = 0.9 \, f_{\rm yf} \, {\rm x}$  flange area  ${\rm x} \, (d_{\rm b} - t_{\rm fb})$  and the proportion of the bending moment transmitted by the web  $M_{\rm w}^* = M^* - 0.9 \, f_{\rm yf}$  x flange area x  $(d_b-t_{fb})$ . The flanges and the web transmit a share of the design axial force  $N^*$ , the proportion taken by each being proportional to their contribution to the total cross sectional area. The web weld transmits the design shear force  $V^*$ .

### Flange welds:

The flange weld must be such that:

$$[N_{\rm ft}^*, N_{\rm fc}^*] < \phi N_{\rm w}$$

 $[N_{\rm ft}^*, N_{\rm fc}^*] \le \phi N_{\rm w}$ Except that if  $N_{\rm ft}^*$  and  $N_{\rm fc}^* > 0.9 f_{\rm yf} b_{\rm f} t_{\rm f}$ 

then set  $N_{\rm ft}^*$  and  $N_{\rm fc}^*$  equal to  $(0.9 f_{\rm vf} b_{\rm f} t_{\rm f})$  for the purposes of weld design only and the weld must be SP category [2].

$$M_{\rm x}^* = 961.25 \text{ kNm}$$
 $N^* = 188.24 \text{ kN}$ 
 $V^* = 194.64 \text{ kN}$ 
 $k_{\rm mw} = \frac{I_w}{I_w + I_f} = \frac{I_w}{I_{total}}$ 

where I<sub>w</sub> is the second moment of area of the web (ignoring the middle flange of the haunched section) and I<sub>f</sub> is the second moment of area of the flanges alone

$$\begin{split} &\mathbf{I_w} = 10.2 \text{ x } (1040 - 2 \text{ x } 15.6)^3 \, / \, 12 = 872.64 \text{ x } 10^6 \text{ mm}^4 \\ &\mathbf{I_f} = \ 2 \times 209 \times 15.6 \times \left[ \frac{1040 - 15.6}{2} \right]^2 = 1710.22 \times 10^6 \text{ mm}^4 \\ &\mathbf{I_{total}} = 872.64 \text{ x } 10^6 + 1710.22 \text{ x } 10^6 = 2583.4 \text{ x } 10^6 \text{ mm}^4 \\ &k_{mw} = \ \frac{872.64 \times 10^6}{2583.4 \times 10^6} = 0.34 \end{split}$$

$$k_{\rm w} = \frac{web \ area}{total \ area} = \frac{10.2 \times (1040 - 2 \times 15.6)}{10.2 \times (1040 - 2 \times 15.6) + 2 \times 209 \times 15.6}$$
$$k_{\rm w} = 0.612$$

Hence.

$$N_{\text{ft}}^* = \frac{M^*}{d_H - t_{f_H}} \times (1 - k_{mw}) - \frac{N^*}{2} \times (1 - k_w)$$

$$N_{\text{ft}}^* = \frac{961.25 \times 10^3}{1040 - 15.6} \times (1 - 0.34) - \frac{188.24}{2} \times (1 - 0.612)$$

$$N_{\rm ff}^{\ \ *} = 583 \ \rm kN$$

$$N_{\text{fc}}^* = \frac{M^*}{d_H - t_{fu}} \times (1 - k_{mw}) + \frac{N^*}{2} \times (1 - k_w)$$

$$N_{\text{fc}}^* = \frac{961.25 \times 10^3}{1040 - 15.6} \times (1 - 0.34) + \frac{188.24}{2} \times (1 - 0.612)$$

$$N_{\rm fc}^* = 656 \text{ kN}$$

$$\phi N_{
m w} = {
m design}$$
 capacity of the butt weld =  $\phi f_{
m yf} b_{
m f} t_{
m f} = (0.9 \ {
m x} \ 300 \ {
m x} \ 209 \ {
m x} \ 15.6) \ {
m x} \ 10^{-3} \ \phi N_{
m w} = 880.31 \ {
m kN} > N_{
m ft}^* \ {
m o} K$ 

Use full penetration butt weld E48XX SP category

# Web weld:

The web weld must be such that:

$$v_{\text{resultant}}^* = \sqrt{v_z^{*2} + v_y^{*2}} \le \phi v_w$$

The web weld is assumed to transmit  $V^*$ ,  $M_w^*$ ,  $N_w^*$ 

 $L_w$  = total run of the fillet weld along one side of the web

$$L_w = 1040 - 3 \times 15.6 = 993 \text{ mm}$$

$$v_z^* = \frac{N_w^*}{2L_w} + \frac{3M_w^*}{L_w^2} = \frac{k_w N^*}{2L_w} + \frac{3k_{mw} M^*}{L_w^2} = \frac{0.612 \times 188.24}{2 \times 993} + \frac{3 \times 0.34 \times 961.25 \times 10^3}{993^2}$$

$$v^* = 1.05 \text{ kN/mm}$$

$$v_y^* = \frac{V^*}{2L_{...}} = \frac{194.64}{2 \times 993} = 0.1 \text{ kN/mm}$$

$$v_{\text{resultant}}^* = \sqrt{1.05^2 + 0.1^2} = 1.05 \text{ kN/mm}$$

Use 8mm fillet weld E48XX SP category  $\phi v_w = 1.3 \text{ kN/mm} > v_{\text{resultant}}^* = 1.05 \text{ kN/mm} OK$ 

## 9.4.2.6 Bolted End Plate Connection (Non-Standard Knee Joint)

Design a bolted end plate to connect a haunched 610UB125 rafter to the flange of 800WB122 column. All steel is grade 300.

The connection is subjected to the following design actions:

 $M^* = 1189.65 \text{ kNm}$ 

 $N^* = 220.61 \text{ kN (Compression)}$ 

 $V^* = 245.11 \text{ kN}$ 

The shear force in the column just below the bottom flange of haunch is:

 $V_{\rm c}^* = 150.3 \text{ kN}$ 

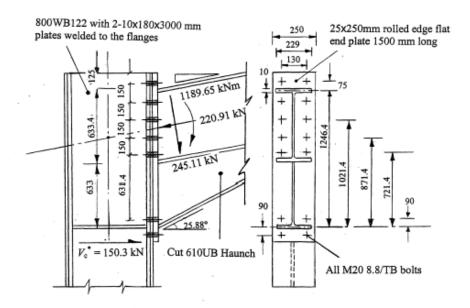


Figure 9.22 Bolted End Plate for Eaves Connection of Pitched Roof Portal Frame

# Solution

# Derived design action for design of bolts, end plate and stiffeners

No design model is given in the AISC Connection Manual [2] for this connection and therefore the design model given in the Structural Steelwork Connections [3] which follows the British Practise will be used here. According to Ref [3] the connection is assumed to 'pivot' about the hard spot at the bottom flange (haunch) and the loads in the bolts are assumed to be proportional to their distance from the centre of bottom flange. However, some

account is taken of the greater flexibility of the cantilever end plate supporting bolts  $F_1$ , compared with the portion of the plate supporting bolts  $F_2$ , which is stiffened by the beam web, by assuming that the loads in bolt  $F_1$  and  $F_2$  are equal.

*Note*: the horizontal component of the shear force and the axial force is assumed to act at the centroid of the rafter section at the connection (i.e. ignoring the haunch) although this assumption is conservative it's on the safe side. A less conservative assumption which is probably more accurate is that the horizontal component of the shear force and the axial force acts at the centroid of the whole section at the connection (i.e. the rafter and the haunch).

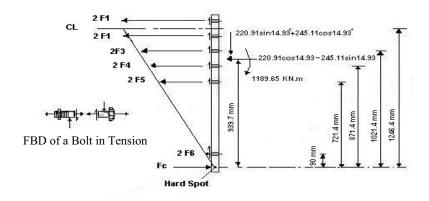


Figure 9.23 Free Body Diagram of the End Plate

```
By taking moments about the centroid of the bottom flange \sum\!M_c=0 we get: 1189.65\times10^3-(220.91cos14.93^\circ-245.11\sin14.93^\circ)\times939.7 -(2F1+2F2) x 1246.4 - 2F3 x 1021.4 - 2F4 x 871.4-2F5 x 721.4 - 2F6 x 90 = 0 From triangular symmetry 2F_1/1246.4=2F_3/1021.4=2F_4/871.4=2F_5/721.4=2F_6/90 2F_1/1246.4)\times[2\times1246.4^2+1021.4^2+871.4^2+721.4^2+90^2]=1048.41\times10^3 F_1=F_2=120.15\ kN F_3=98.46\ kN F_4=84\ kN F_5=69.54\ kN F_6=8.68\ kN
```

As a simple rule the max bolt force can be obtained from:

$$F_1 = M / [4d_e + 2 (\Sigma y_i^2 / d_e)]$$

Where:

 $d_e$  = distance from the centreline of the compression flange to a point midway between the two top rows of bolts.

M = is the moment applied at the connection plus or minus the moment of the shear force and the axial force components about the compression flange centroid.

$$\Sigma F_x = 0$$
  
 $F_c + 245.11 \sin 14.93^\circ - 220.91 \cos 14.93^\circ - 2 (120.15 + 120.15 + 98.46 + 84 + 69.54 + 8.68) = 0$   
 $F_c = 220.91 \cos 14.93^\circ - 245.11 \sin 14.93^\circ + 2(2 \times 120.15 + 98.46 + 84 + 69.54 + 8.68)$ 

 $F_c = 1152.26$  kN which is the horizontal component of the Bottom flange (Haunch – flange) force.

# End Plate:

# (a) Check Flexural Capacity

Assume that the end plate bends in double curvature between the bolt line and the flange weld with a sagging yield line at the line of the bolts and a hogging yield line at the weld.

The end plate thickness must be such that:

$$N_{\text{ft}}^* \le [\phi N_{\text{pb}}, \phi V_{\text{ph}}]$$
  
Where  $N_{\text{ft}}^* = 4 \text{ F}_1 = 4 \text{ x } 120.15 = 480.6 \text{ kN}$ 

Try 250 x 25 mm rolled edge flat end plate x 1500mm deep grade 300 steel

```
Refer to Figure 9.22 N_{\rm pb}^* = \phi f_{\rm yi} \times b_{\rm i} t_{\rm i}^2 / a_{\rm fe} Where b_{\rm i}, t_{\rm i}, f_{\rm yi} = plate width, thickness and yield stress respectively. a_{\rm fe} = a_{\rm f} - (d_{\rm f}/2) a_{\rm f} = (S_{\rm p} - t_{\rm fb}) / 2 = (150 - 19.6) / 2 = 65.2 {\rm mm} a_{\rm fe} = 65.2 - (20/2) = 55.2 {\rm mm} \phi N_{\rm pb} = 0.9 \times 280 \times 250 \times 25^2 / 55.2 = 713.3 {\rm kN} > N_{\rm ft}^* = 480.6 {\rm kN} OK
```

### (b) Check Shear Capacity

The shear stress distribution in a rectangular plate is parabolic with the maximum stress being 1.5 times the average stress. AS 4100 makes allowances for non-uniform shear stress in a web with the formula,

$$V_{\rm v} = [2V_{\rm u}/(0.9 + (f_{\rm um}^*/f_{\rm va}^*))] \le V_{\rm u}$$
 AS4100 Cl.5.11.3

Where  $V_{\rm u}$  is the nominal shear capacity of a web with uniform stress distribution  $f_{\rm vm}^*$ ,  $f_{\rm va}^*$  = the maximum and average shear stresses in the web respectively.

#### Hence,

Use 250x25 mm, rolled edge flat end plate x 1500 mm deep grade 300 steel.(Plate - component could be substituted).

#### **Bolts**

Try 14 - M20~8.8 / TB distributed as shown in Figure 9.22 Maximum tensile force acting on each bolt in the top two rows of bolts  $F_1 = 120.15~kN$ 

Increase this force by 30% to allow for additional bolt force due to prying.

$$N_{\rm ftp}^*$$
 / Bolt = 120.15 x 1.3 = 156.2 kN   
  $\phi N_{\rm tf} = 0.8 \ A_{\rm s} f_{\rm uf} = (0.8 \ {\rm x} \ 245 \ {\rm x} \ 830) \ {\rm x} \ 10^{-3} = 163 \ {\rm kN} > {N_{\rm ftp}}^*$  / Bolt = 156.2 kN  $OK$ 

*Note*: It is suggested in the AISC Connection Manual [2] to allow an increase of 20 to 33% in the bolt tension force to account for prying action.

A commonly accepted assumption made in a number of references states that the shear force acting on the connection is taken by the bolts closest to the compression flange.

Shear/Bolt = 
$$V_{\rm f}^*$$
 = (220.91x sin14.93° + 245.11x cos14.93°) /4 = 293.75 / 4 = 73.44 kN Design shear capacity of an M20 bolt with threads included in the shear plane is:  $\phi V_{\rm f}$  = 0.8 x 0.62  $A_{\rm c} f_{\rm uf}$  = 0.8 x 0.62 x 225 x 830 = 92.6 kN >  $V_{\rm f}^*$  = 73.44 kN  $OK$ 

Hence adopt M20 8.8 / TB bolts.

*Note*: for the design of stiffeners and weld the procedure outlined in Example 9.4.2.4 can be used.

### 9.5 REFERENCES

- 1. Standards Australia (1998). AS 4100 Steel Structures.
- 2. Hogan T.J., Thomas I.R., (1994). *Design of Structural Connections*, 4<sup>th</sup> edn. Australian Institute of Steel Construction.
- 3. Owens, G.W. and Cheal, B.D., (1989) Structural Steelwork Connections.
- 4. Bradford M.A., Kitipornchai S., Woolcock S.T., (1999) *Design of Portal Frame Buildings* Third edition (to AS 4100).