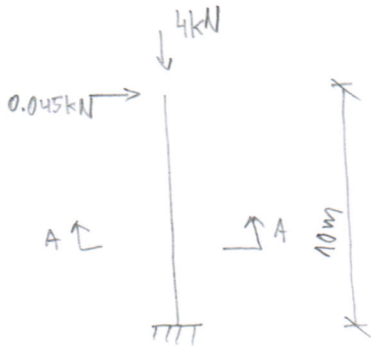


GOAL: Illustrate P-Delta and Large Displacements on a simple fixed cantilever model.

MODEL:

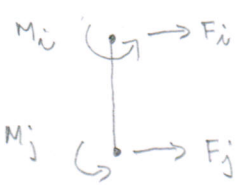


SECTION A-A

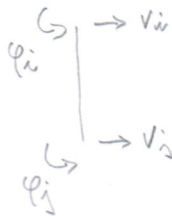
ELASTIC AND GEOMETRIC MATRICES:

Edward L. Wilson: static & dynamic analysis of structures, 2004

p. 120-121



FORCES



DISPLACEMENTS

$$K_e = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & -6L \\ 6L & 4L^2 & -6L & -2L^2 \\ -12 & -6L & 12 & -6L \\ -6L & -2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K_g = \frac{T}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}$$

$$\{F\} = \begin{bmatrix} F_i \\ M_i \\ F_j \\ M_j \end{bmatrix}, \quad \{V\} = \begin{bmatrix} v_i \\ \phi_i \\ v_j \\ \phi_j \end{bmatrix} \Rightarrow \{F\} = [k_e + k_g] \{V\}$$

LINEAR ANALYSIS

$$\text{solve: } \underbrace{[k_e]_{2 \times 2}} \begin{Bmatrix} v_i \\ \varphi_i \end{Bmatrix} = \begin{Bmatrix} F_i \\ M_i \end{Bmatrix}$$

2x2 submatrix of  
full K matrix on page 1

calculate element end forces:

$$\{F\}_{4 \times 1} = [k_e]_{4 \times 4} \{v\}_{4 \times 1}$$

NONLINEAR ANALYSIS WITH P-DELTA EFFECTS

$$\text{solve: } [k_e + k_g]_{2 \times 2} \begin{Bmatrix} v_i \\ \varphi_i \end{Bmatrix} = \begin{Bmatrix} F_i \\ M_i \end{Bmatrix}$$

$$\text{calculate element end forces: } \{F\}_{4 \times 1} = [k_e + k_g]_{4 \times 4} \{v\}_{4 \times 1}$$

- since  $k_g$  changes with applied load, the response should be obtained from multiple load increments

NONLINEAR ANALYSIS WITH D-DELTA EFFECTS AND LARGE DISPLACEMENTS

- similar to P-Delta analysis, but equilibrium equations are assembled and solved for deformed configuration using updated Lagrangian formulation