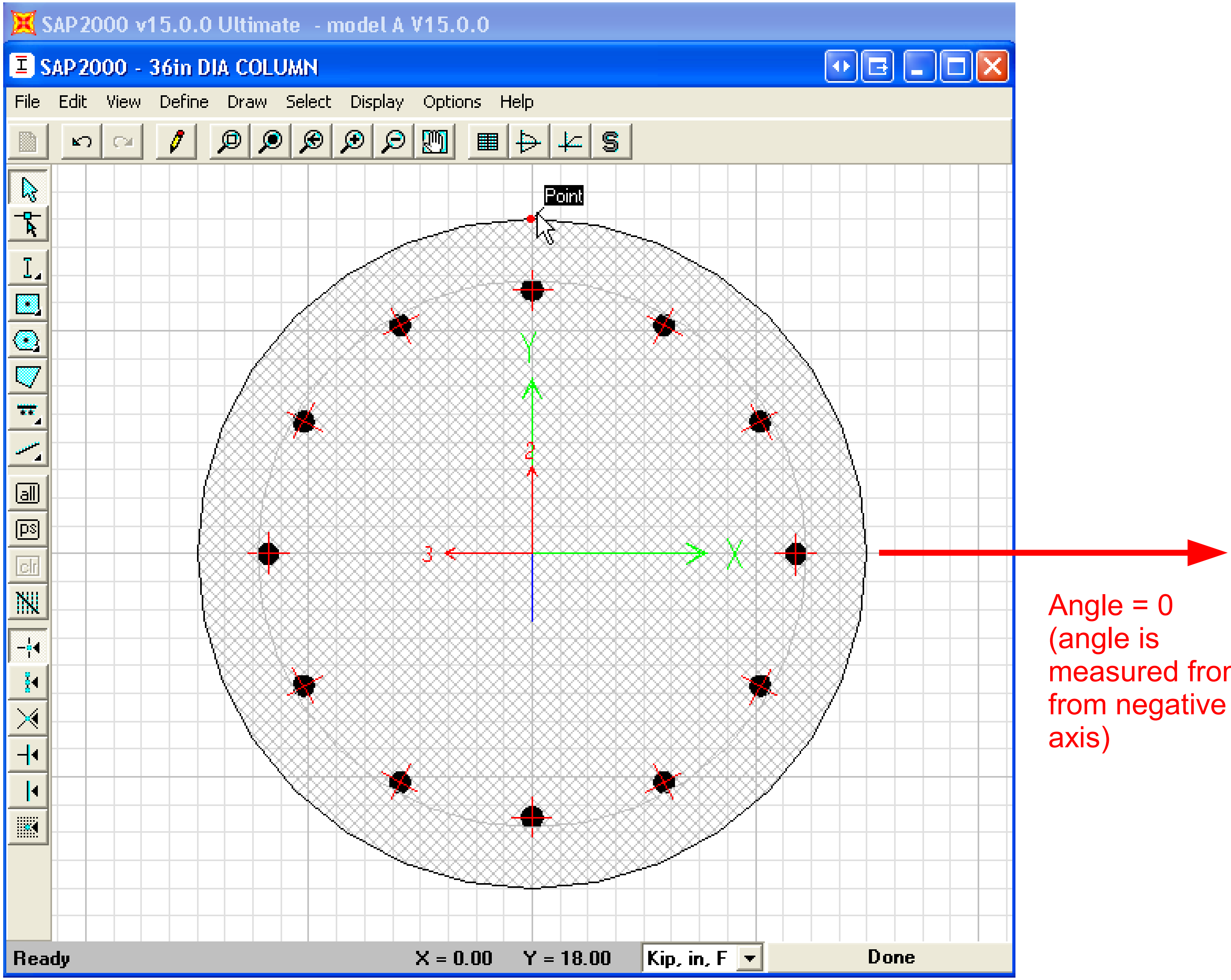
Moment curvature and cracked moment of inertia

- The purpose of this test problem is to explain the meaning of various parameters reported for the moment curvature curve and illustrate how is the cracked moment of inertia calculated.
- The moment curvature analysis is performed for circular column section described in detail in Example 2 of Reference [1].

Cross-Section

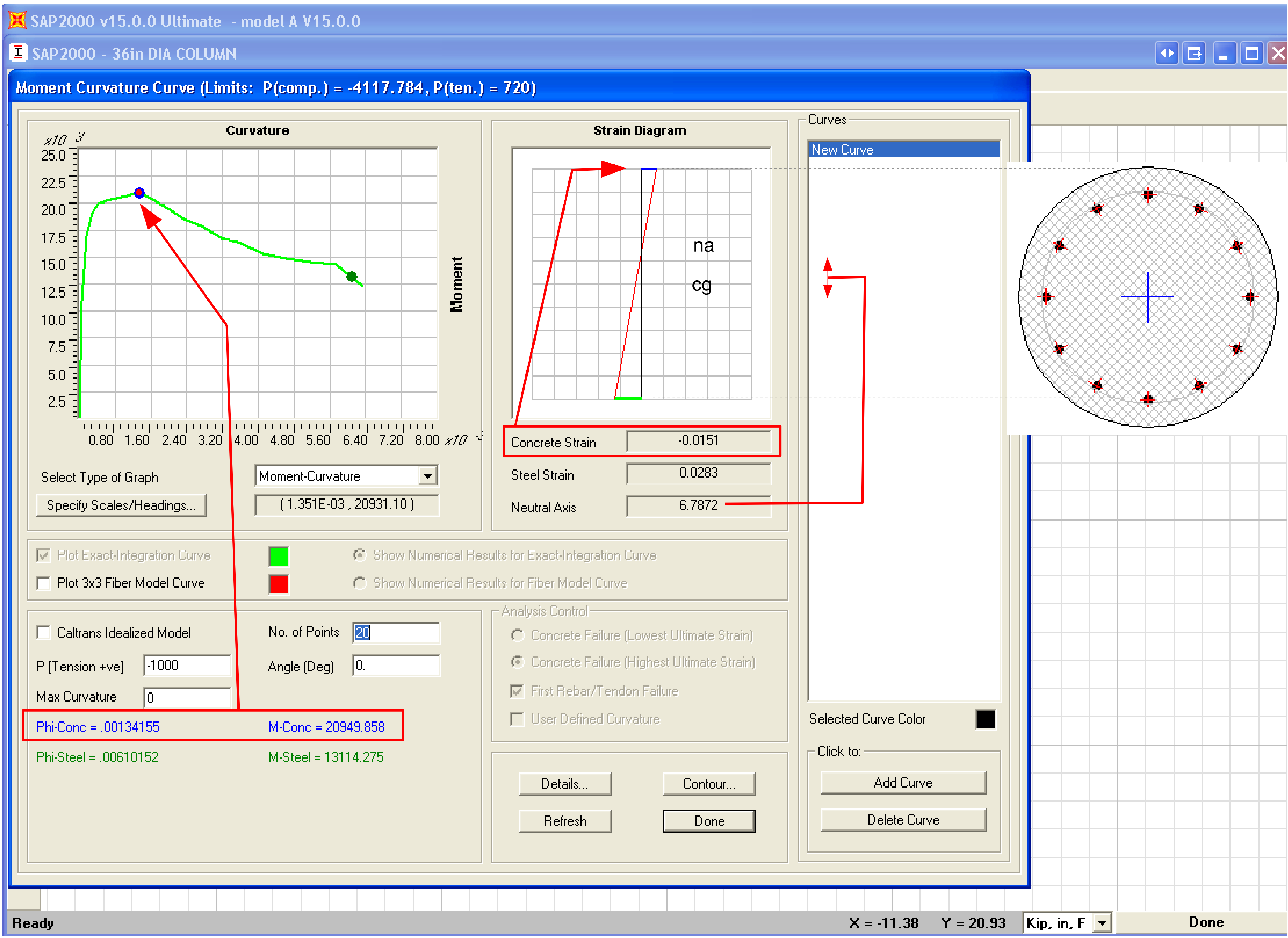


Angle = 0
(angle is
measured from
from negative 3
axis)

Explanation of Moment-Curvature Output Parameters

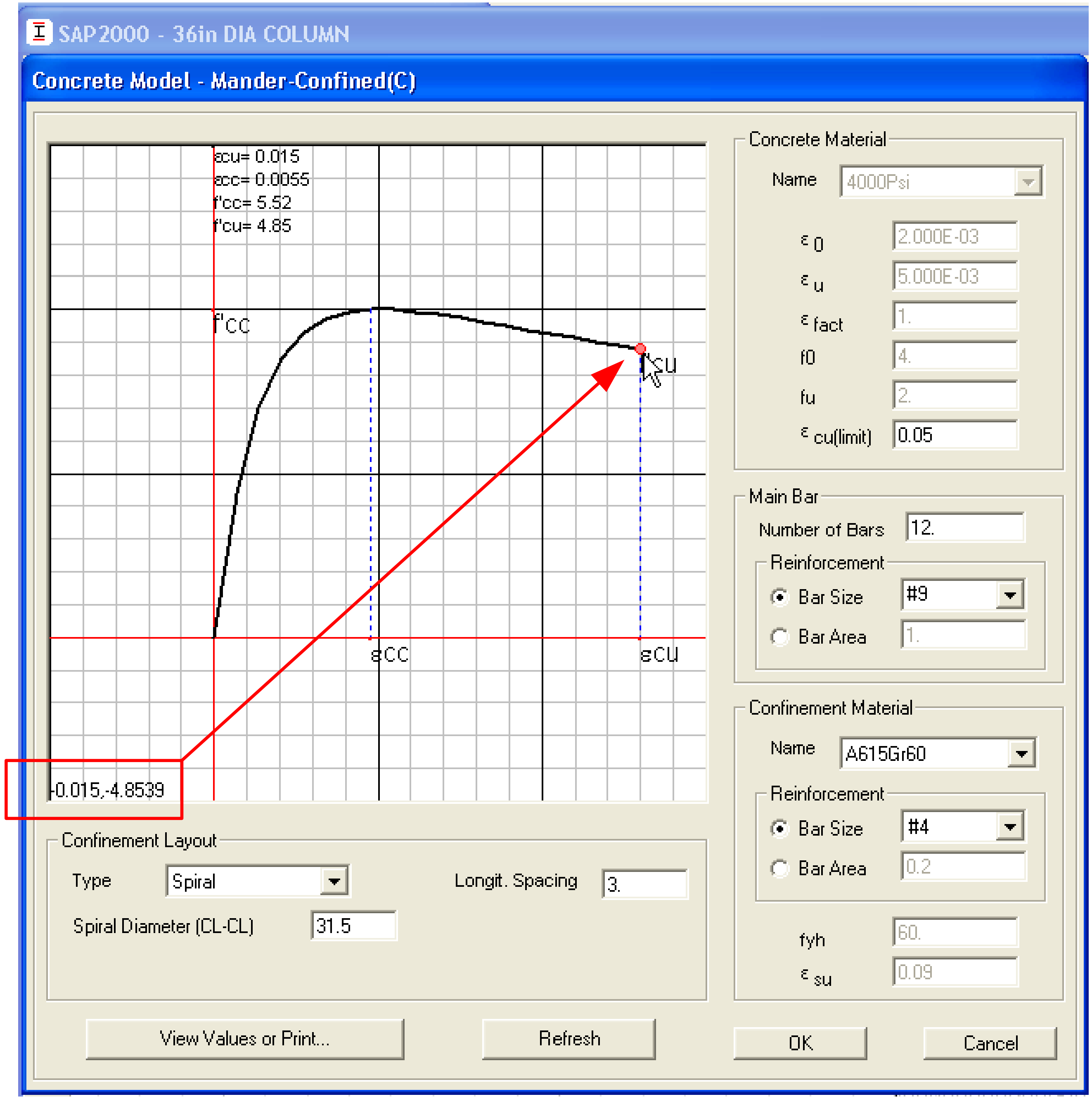
Exact Integration Curve Phi-Conc, M-Conc

- Phi-Conc is the curvature and M-Conc is the moment for which ultimate concrete strain is reached in extreme compressive fiber



SAP2000 v15.0.0 Ultimate - model A V15.0.0
SAP2000 - 36in DIA COLUMN
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)
Curvature
Strain Diagram
Curves
New Curve
Moment
na
cg
Concrete Strain
-0.0151
Steel Strain
0.0283
Neutral Axis
6.7872
Select Type of Graph
Moment-Curvature
Specify Scales/Headings...
(1.351E-03 , 20931.10)
Plot Exact-Integration Curve
Plot 3x3 Fiber Model Curve
Show Numerical Results for Exact-Integration Curve
Show Numerical Results for Fiber Model Curve
Caltrans Idealized Model
No. of Points
20
P [Tension +ve]
-1000
Angle (Deg)
0.
Max Curvature
0
Analysis Control
Concrete Failure (Lowest Ultimate Strain)
Concrete Failure (Highest Ultimate Strain)
First Rebar/Tendon Failure
User Defined Curvature
Phi-Conc = .00134155
M-Conc = 20949.858
Phi-Steel = .00610152
M-Steel = 13114.275
Selected Curve Color
Click to:
Add Curve
Delete Curve
Details...
Contour...
Refresh
Done
Ready
X = -11.38
Y = 20.93
Kip, in, F
Done

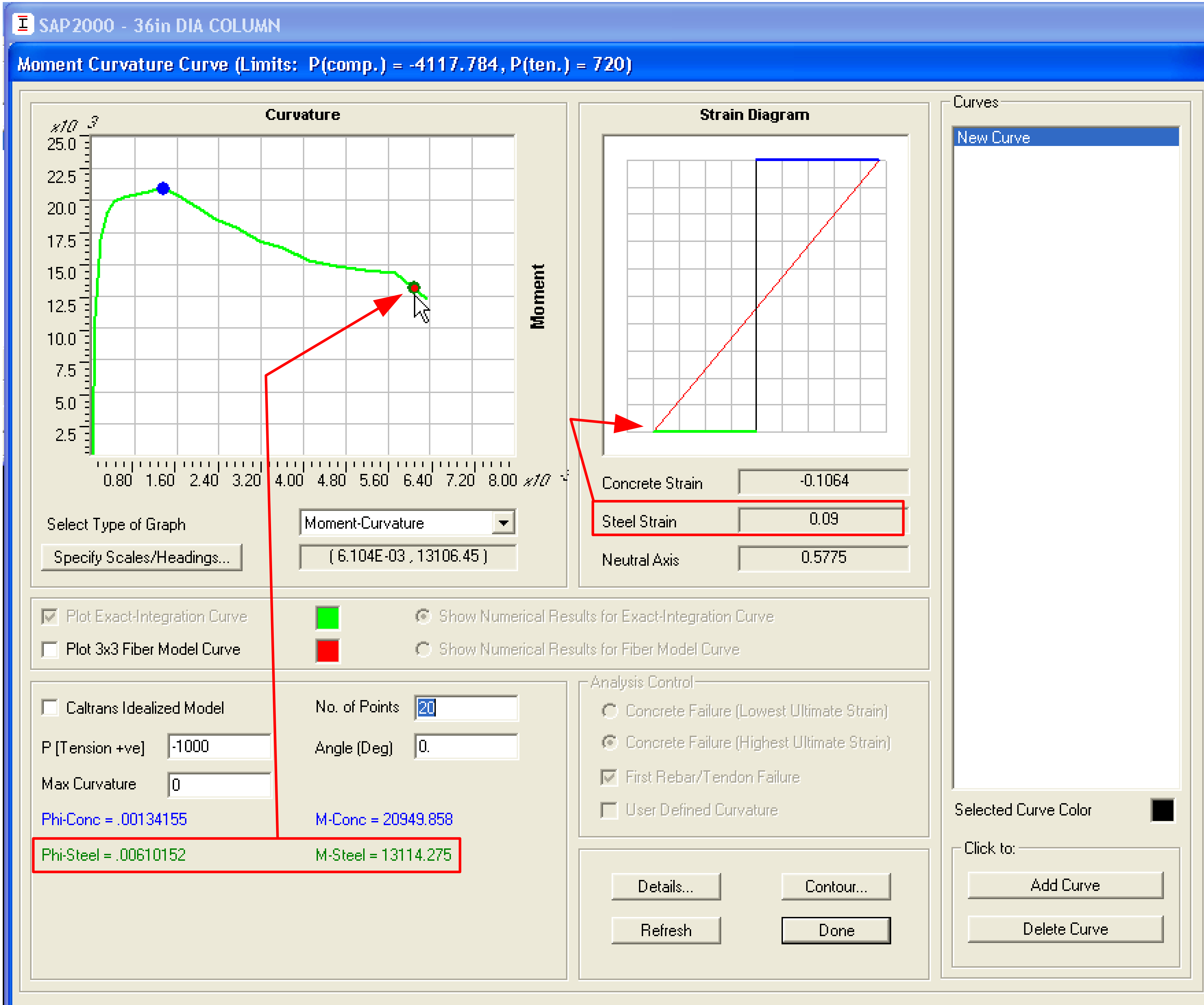
To obtain the concrete stress-strain curve shown below, right click on the material region in Section Designer and select "C Model":

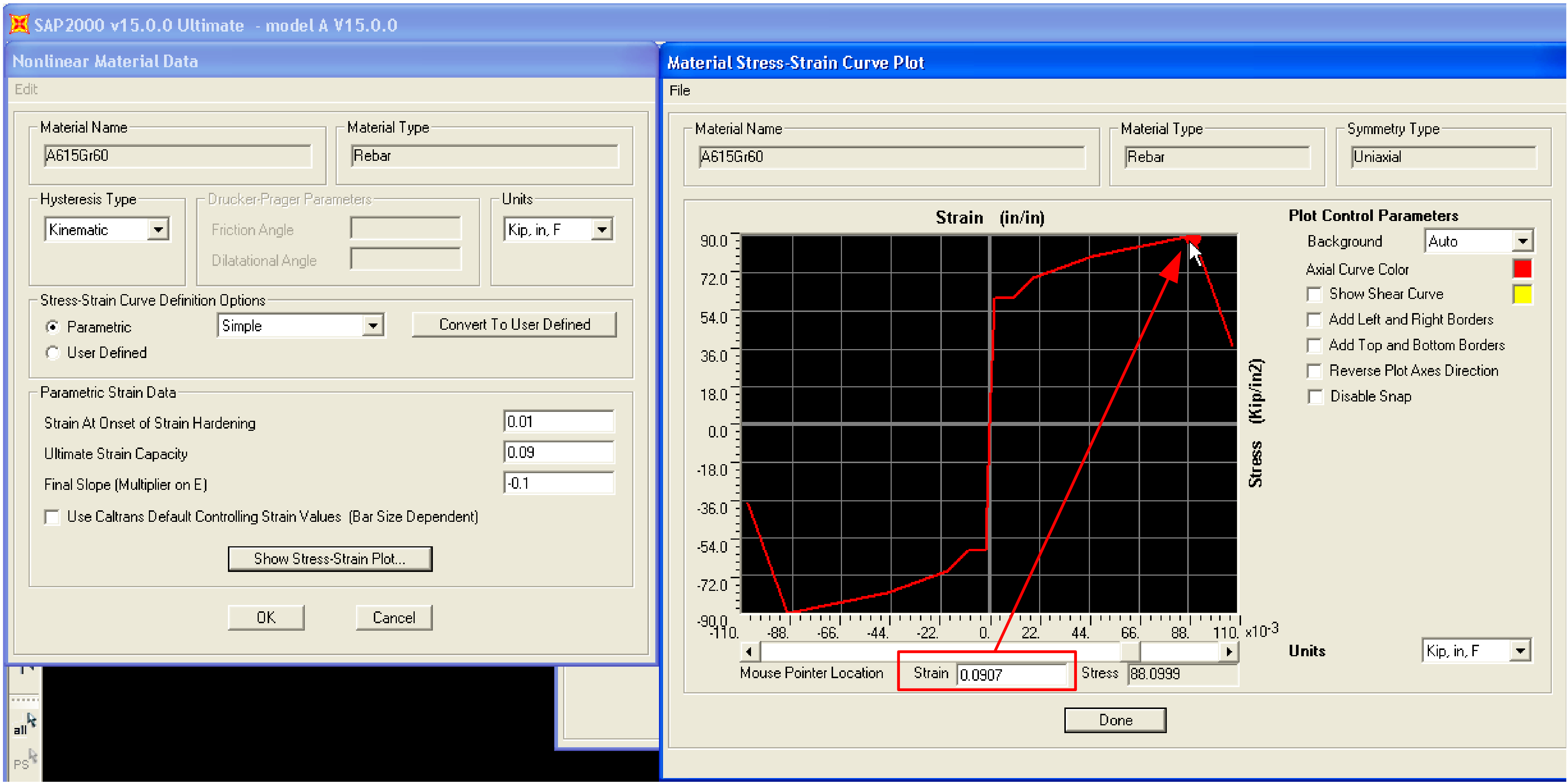


The ultimate concrete strain of -0.015 corresponds to the concrete strain shown on the previous page

Exact Integration Curve Phi-Steel, M-Steel

- Phi-Steel is the curvature and M-Steel is the moment for which failure reinforcement strain/stress is reached in the any reinforcement bar

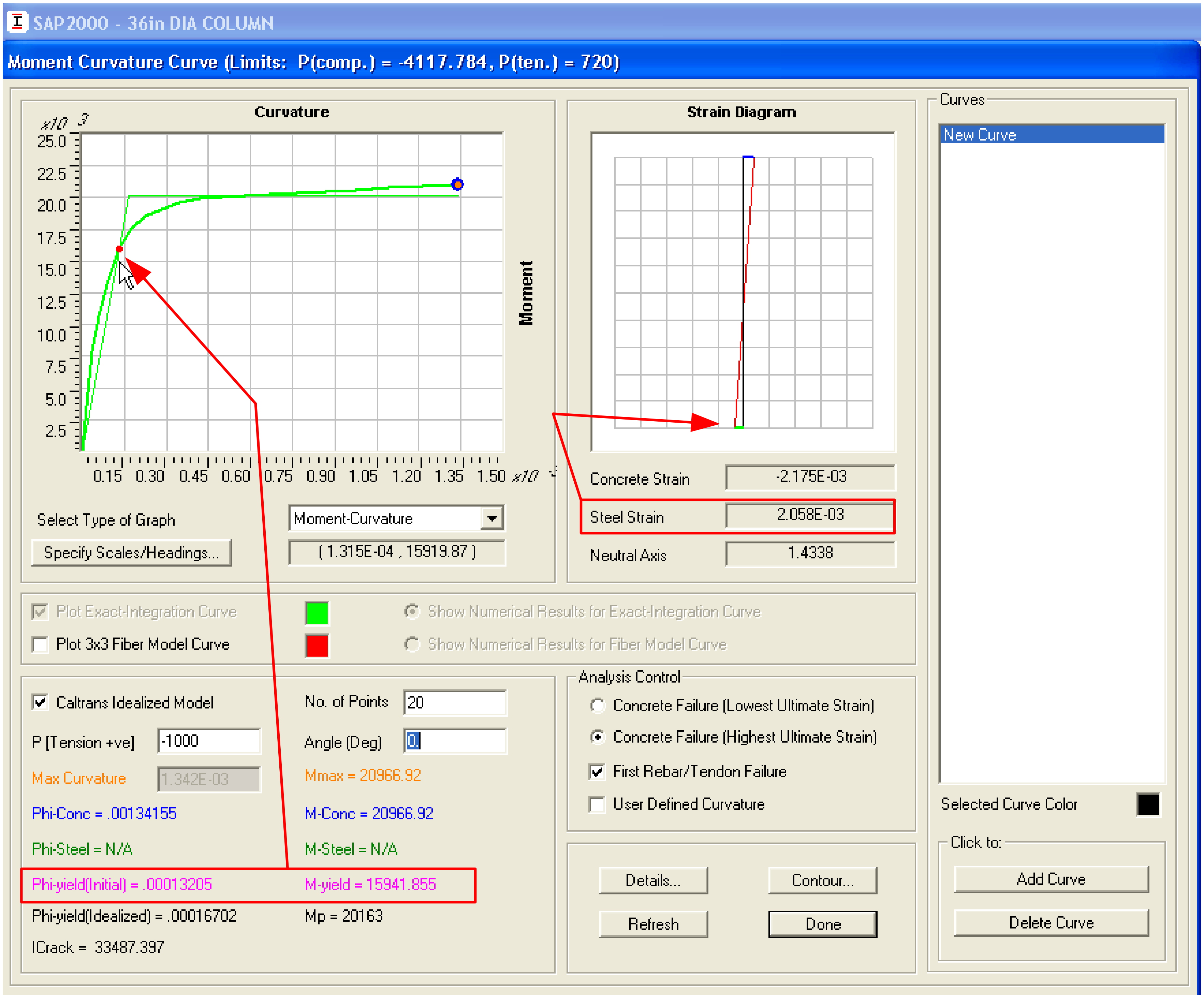

SAP2000 - 36in DIA COLUMN
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)
Curvature
x10 -3
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.80 1.60 2.40 3.20 4.00 4.80 5.60 6.40 7.20 8.00 x10 -3
Moment
Strain Diagram
Concrete Strain
-0.1064
Steel Strain
0.09
Neutral Axis
0.5775
Curves
New Curve
Select Type of Graph
Moment-Curvature
Specify Scales/Headings...
(6.104E-03 , 13106.45)
Plot Exact-Integration Curve
Plot 3x3 Fiber Model Curve
Show Numerical Results for Exact-Integration Curve
Show Numerical Results for Fiber Model Curve
Caltrans Idealized Model
No. of Points
20
P [Tension +ve]
-1000
Angle (Deg)
0.
Max Curvature
0
Phi-Conc = .00134155
M-Conc = 20949.858
Phi-Steel = .00610152
M-Steel = 13114.275
Analysis Control
Concrete Failure (Lowest Ultimate Strain)
Concrete Failure (Highest Ultimate Strain)
First Rebar/Tendon Failure
User Defined Curvature
Details...
Contour...
Refresh
Done
Selected Curve Color
Click to:
Add Curve
Delete Curve

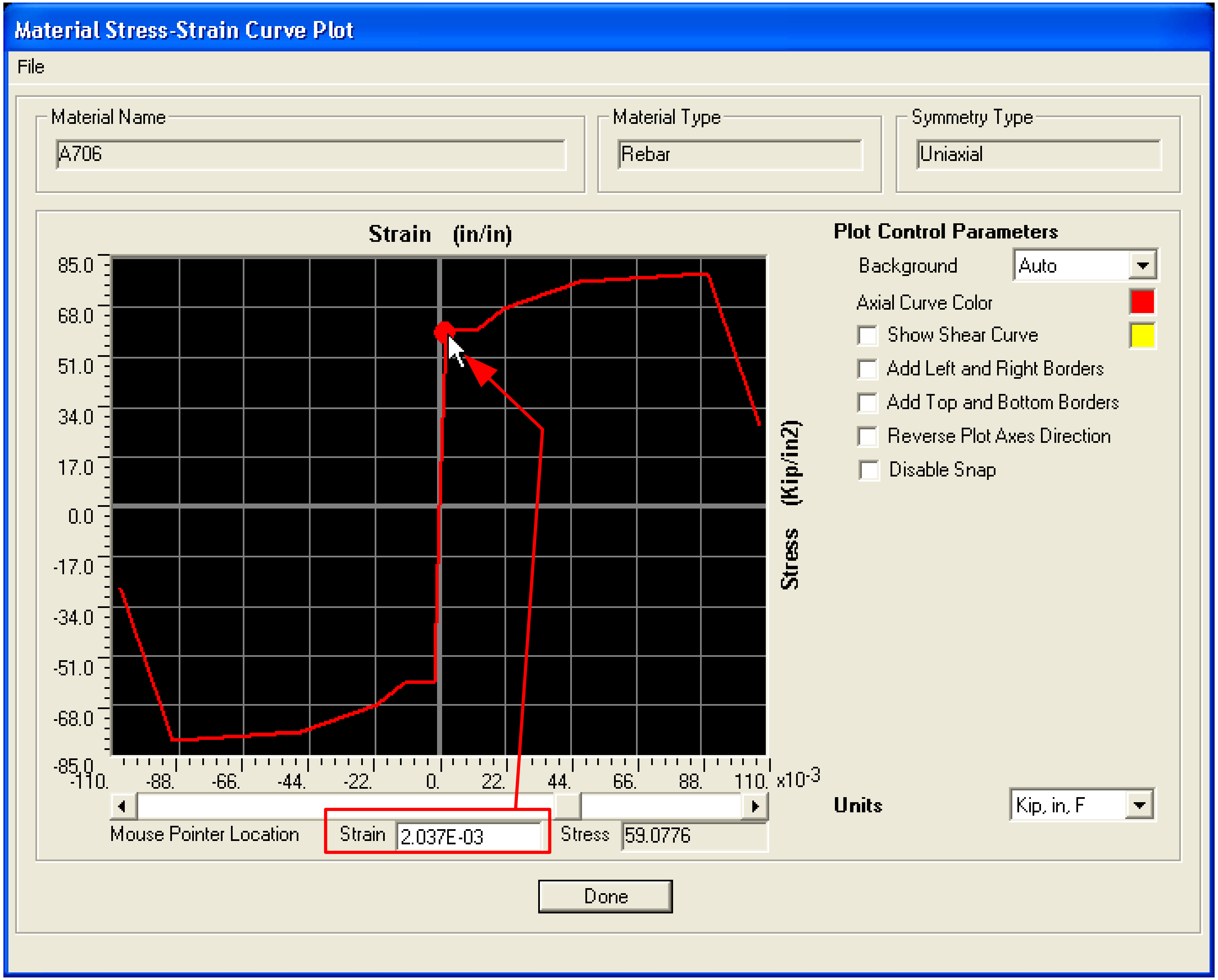


The ultimate steel strain of 0.0907 corresponds to the steel strain shown on the previous page

Caltrans Model
Phi-yield(initial), M-yield

- Phi-yield(initial) is the curvature and M-yield is the moment for the the first rebar in the cross-section starts to yield

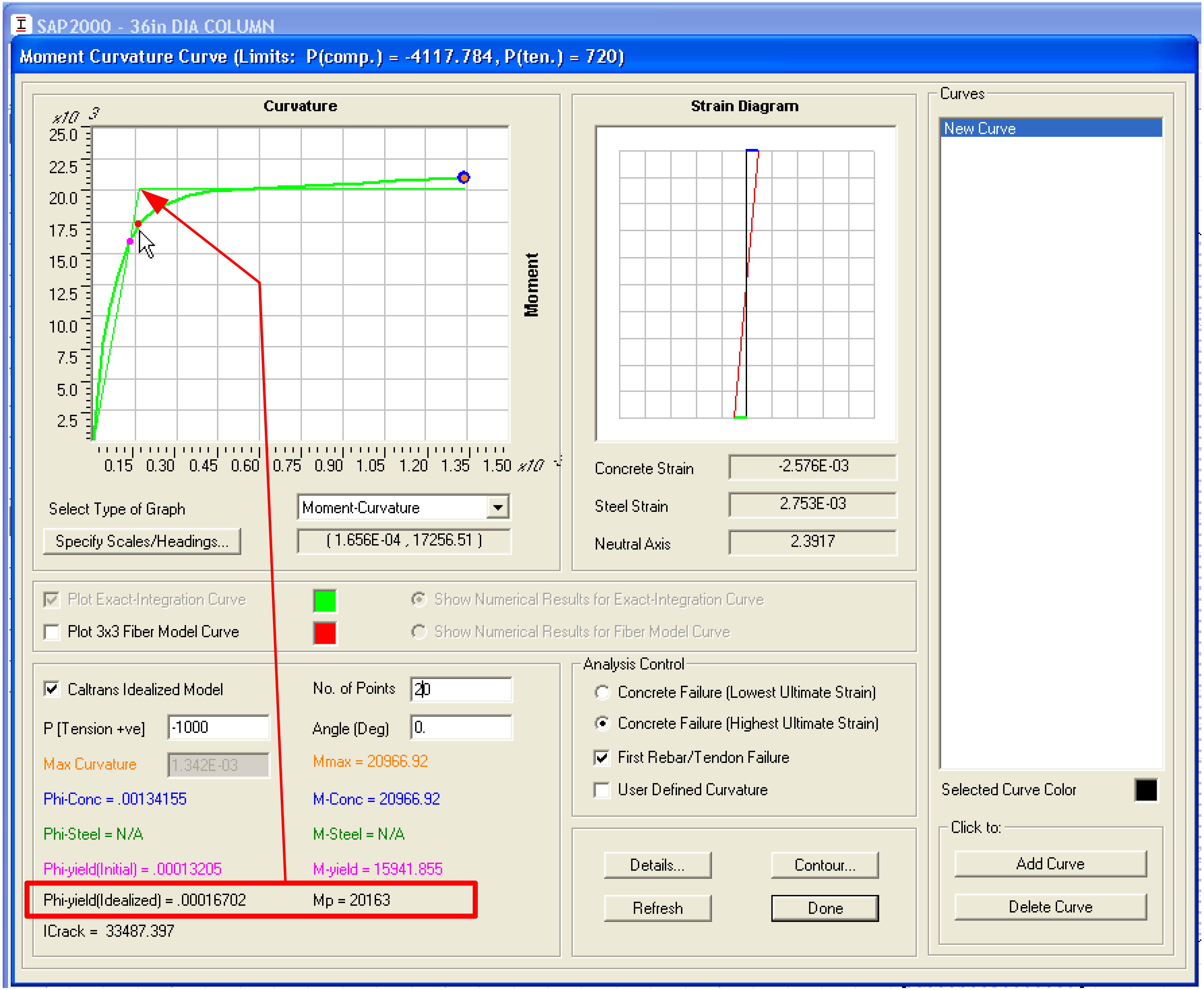

SAP2000 - 36in DIA COLUMN
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)
Curvature
x10 3
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.15 0.30 0.45 0.60 0.75 0.90 1.05 1.20 1.35 1.50 x10 -3
Moment
Strain Diagram
Concrete Strain
-2.175E-03
Steel Strain
2.058E-03
Neutral Axis
1.4338
Curves
New Curve
Select Type of Graph
Moment-Curvature
Specify Scales/Headings...
(1.315E-04 , 15919.87)
Plot Exact-Integration Curve
Plot 3x3 Fiber Model Curve
Show Numerical Results for Exact-Integration Curve
Show Numerical Results for Fiber Model Curve
Caltrans Idealized Model
No. of Points
20
P [Tension +ve]
-1000
Angle (Deg)
0
Max Curvature
1.342E-03
Mmax = 20966.92
Phi-Conc = .00134155
M-Conc = 20966.92
Phi-Steel = N/A
M-Steel = N/A
Phi-yield(Initial) = .00013205
M-yield = 15941.855
Phi-yield(Idealized) = .00016702
Mp = 20163
ICrack = 33487.397
Analysis Control
Concrete Failure (Lowest Ultimate Strain)
Concrete Failure (Highest Ultimate Strain)
First Rebar/Tendon Failure
User Defined Curvature
Details...
Contour...
Refresh
Done
Selected Curve Color
Click to:
Add Curve
Delete Curve



The yield steel strain of 0.002037 corresponds to the steel strain shown on the previous page

Caltrans Model
Phi-yield(idealized), Mp

- Phi-yield(idealized) is the curvature and Mp is the moment for the initial yield on the Caltrans idealized moment curvature curve.
- The idealized curve is obtained as per Caltrans Seismic Design Criteria [2].


SAP2000 - 36in DIA COLUMN
Moment Curvature Curve (Limits: P(comp.) = -4117.784, P(ten.) = 720)
Curvature
x10 3
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.15 0.30 0.45 0.60 0.75 0.90 1.05 1.20 1.35 1.50 x10
Moment
Strain Diagram
Curves
New Curve
Concrete Strain
-2.576E-03
Steel Strain
2.753E-03
Neutral Axis
2.3917
Select Type of Graph
Moment-Curvature
Specify Scales/Headings...
(1.656E-04 , 17256.51)
Plot Exact-Integration Curve
Plot 3x3 Fiber Model Curve
Show Numerical Results for Exact-Integration Curve
Show Numerical Results for Fiber Model Curve
Caltrans Idealized Model
No. of Points
20
P [Tension +ve]
-1000
Angle (Deg)
0.
Max Curvature
1.342E-03
Mmax = 20966.92
Phi-Conc = .00134155
M-Conc = 20966.92
Phi-Steel = N/A
M-Steel = N/A
Phi-yield(Initial) = .00013205
M-yield = 15941.855
Phi-yield(Idealized) = .00016702
Mp = 20163
ICrack = 33487.397
Analysis Control
Concrete Failure (Lowest Ultimate Strain)
Concrete Failure (Highest Ultimate Strain)
First Rebar/Tendon Failure
User Defined Curvature
Details...
Contour...
Refresh
Done
Selected Curve Color
Click to:
Add Curve
Delete Curve

The idealized Mp on the previous page is determined as specified in Caltrans Seismic Design Criteria, June 2006, Version 1.4:

3.3 Plastic Moment Capacity for Ductile Concrete Members

3.3.1 Moment Curvature ($M{-}\phi$) Analysis

The plastic moment capacity of all ductile concrete members shall be calculated by $M{-}\phi$ analysis based on expected material properties. Moment curvature analysis derives the curvatures associated with a range of moments for a cross section based on the principles of strain compatibility and equilibrium of forces. The $M{-}\phi$ curve can be idealized with an elastic perfectly plastic response to estimate the plastic moment capacity of a member's cross section. The elastic portion of the idealized curve should pass through the point marking the first reinforcing bar yield. The idealized plastic moment capacity is obtained by balancing the areas between the actual and the idealized $M{-}\phi$ curves beyond the first reinforcing bar yield point, see Figure 3.7 [4].

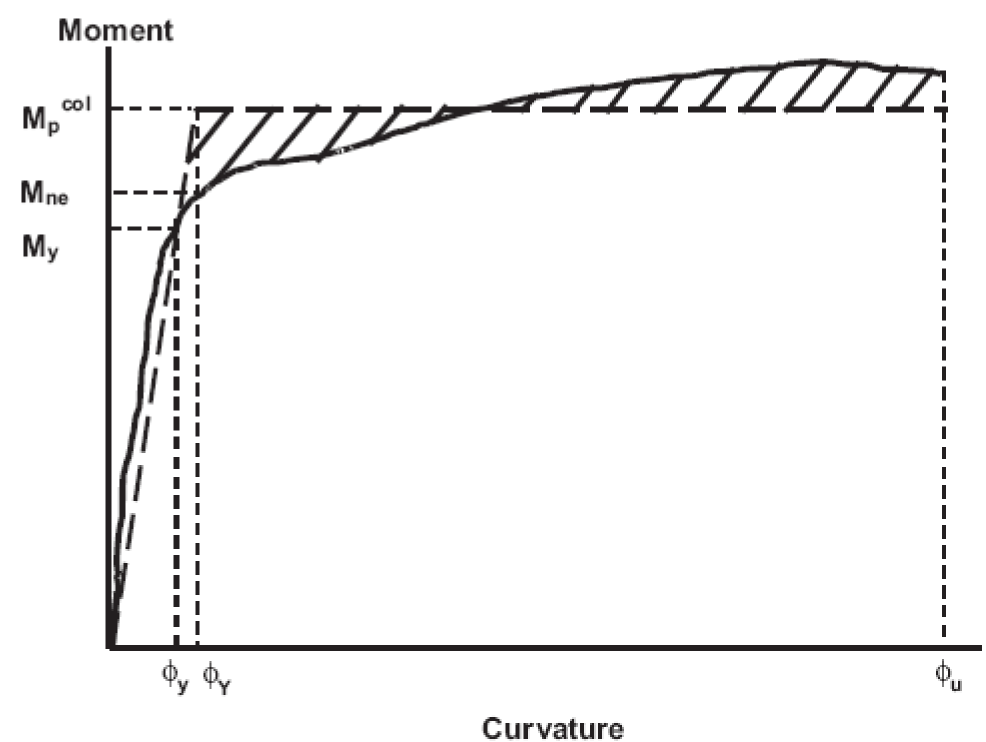


Figure 3.7 Moment Curvature Curve

Caltrans Model
Icrack

Icrack is calculated as follows:

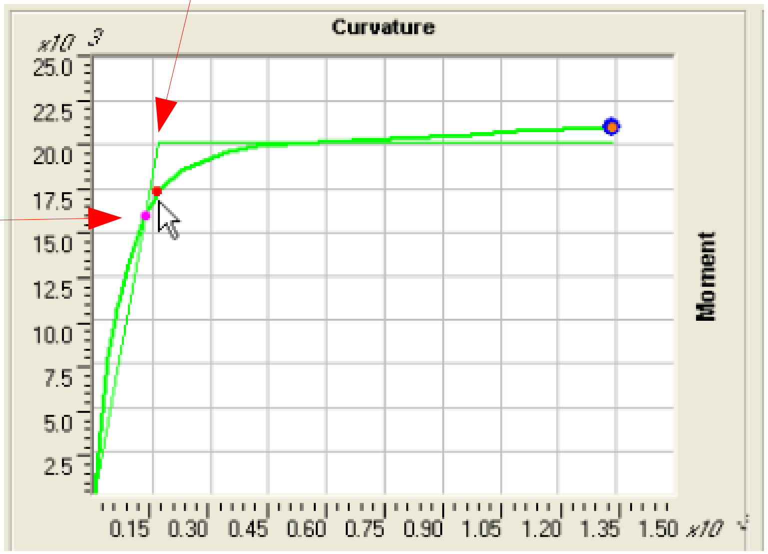
Basic equation:

$$\phi = \frac{M}{EI}$$

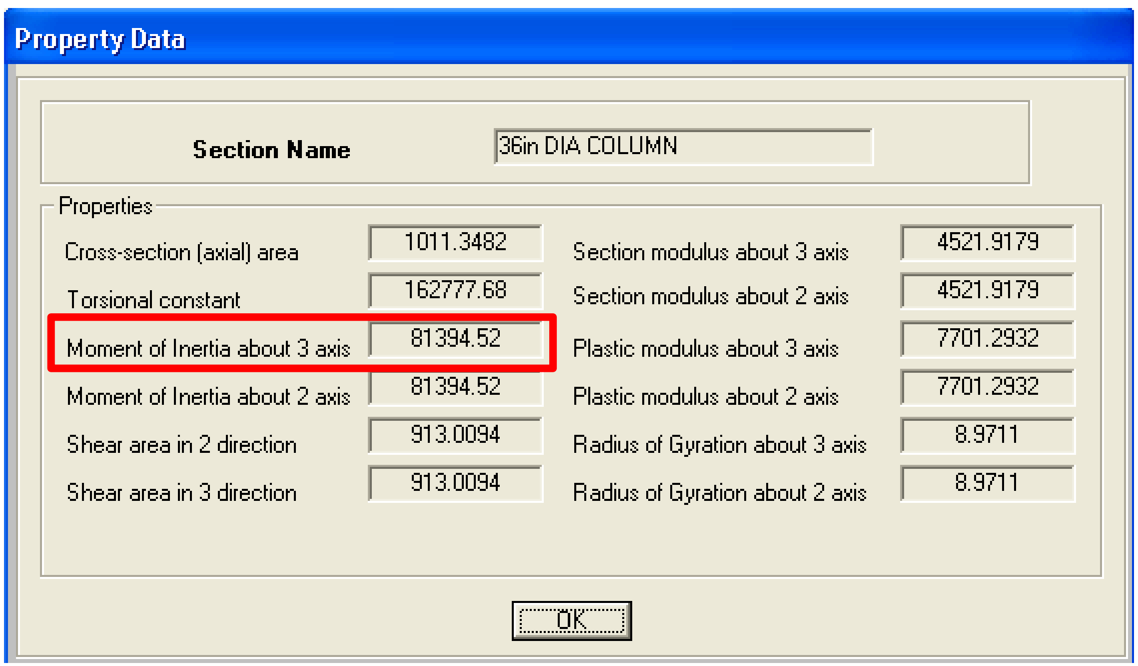
Caltrans idealized moment-curvature equations:

$$I_{crack} = \frac{M_p}{C_p E} = \frac{20163\,\text{kip-in}}{(0.00016702\,\text{rad/in})(3604\,\text{ksi})} = 33483\,\text{in}^4$$

$$I_{crack} = \frac{M_y}{C_y E}$$



- Compare Icrack = 33483 in^4 with gross moment of inertia of 81394 in^4



References

[1] Robert Mathews: Moment Curvature, 2001.
http://www.structsource.com/pdf/Momcurv_web.pdf

[2] Caltrans Seismic Design Criteria, Version 1.4
http://www.dot.ca.gov/hq/esc/techpubs/manual/othermanual/other-engin-manual/seismic-design-criteria/sdc.html